OCR A level Further Mathematics Statistics (A) by Jean-Paul Muscat, pp. 362, £24, ISBN 978-1-5104-1451-8, Hodder Education (2018)

As with the Mechanics textbook, this treatment assumes no prior knowledge apart from the probability and statistics in GCSE, and candidates are not expected to be using the Mathematics for Year 2 textbook from Hodder.

The first chapter is a general introduction to sampling techniques and makes a valuable distinction between random variables and data values. The exercises allow the reader to practise various important calculations from sets of data. However, the introduction of both mean and variance is rather strange. In the definition of $\bar{x}$, oddly given as $\sum_i \frac{x_i}{n}$, readers are expected to know what the i is doing. There is no mention of the fact that this is equal to $\frac{1}{n}\sum x_i$, which might be 'obvious', but making this assumption is presumptive. The definition of variance of a sample is given as $s^2 = \frac{S_{xx}}{n}$ where $S_{xx} = \sum (x_i - \bar{x})^2$. Now the n is in a more sensible place, but the subscript xx will strike readers as curious. A few lines later we are told that $S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - n\bar{y}^2$ and this is closely followed by the bivariate form. The two different forms are stated in the single-subject Year 2 book, but it is far from obvious that the two forms of S_{xx} are in fact equal and their equivalence is not shown. What we seem to have here is a collection of 'facts' without very much justification for why they are useful or even true. This strikes me as a compromise which is only going to get worse when we work with estimators and encounter the divisor $n - 1$. It also seems unnecessary to make things more complicated than they really are at the same time as requiring the reader to believe something because they are told it is true. Given that this text is aimed at students studying Further Mathematics, a little more emphasis on how sigma algebra works, and justifications of the various facts which are to be memorised, would seem sensible.

The second chapter focuses on random variables, introduces some useful terminology, and focuses on the concepts of expectation and variance. Unfortunately it suffers from the same weaknesses as the preceding chapter, and students are presented with statements such as $\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$ and $\mathrm{E}(X - \mu)^2 = \sum (r - \mu)^2 \mathrm{P}(X = r)$ without any justification – not even a mention that the sum is taken over values of r. Later in the same chapter, there is a section on sums and differences of independent random variables. So far as I can see, this is the first mention of the term 'independent' in the text; the index points to later pages 79 and 117, but neither reference is relevant. A few pages later we are told that $\mathrm{E}(X_1 + X_2) = \mathrm{E}(X_1) + \mathrm{E}(X_2)$ is true even when the variables are not independent and that $\mathrm{Var}(X_1 + X_2) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2)$ when they are, with a statement that the justification of this is beyond the scope of the book. Given that there is a later chapter on correlation, this strikes me as a cop-out. It must be clear by now that I am beginning to have some reservations about this text.

The treatment of the binomial distribution does state assumptions for the model to be appropriate and it justifies the formulae for expectation and variance. For the Poisson distribution, four modelling conditions are stated, but the first two are trivial and the fourth might be better stated as events occurring at a constant average rate. There is a cogent discussion of this in [1]. The chapter on bivariate distributions and correlation introduces the PMCC and makes a valiant effort to explain the tricky concept of degrees of freedom. It also, somewhat surprisingly, introduces the notions of a test statistic and hypothesis testing. Rank correlation and the Spearman coefficient figure later in the same chapter, and the chi-squared test for goodness of fit appears in the next. This tendency to treat statistics as a series of rules might appeal to pupils who enjoy using recipes, but I would find it rather unnerving as a teacher.

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There is a workmanlike chapter on counting techniques before continuous random variables, including the Normal distribution, are introduced. This leads to the central limit theorem and its various uses. The next chapter is on confidence intervals, and now we encounter the thorny issue of estimating variance. In order to confront the fact that the sample variance is a biased estimator for population variance, one needs a theoretical framework which, unless a teacher is prepared to go well beyond the syllabus, is beyond the scope of a textbook. The current author simply says what happens.

The general theoretical framework of hypothesis testing is not taken beyond the level of Year 1 core until the penultimate chapter. My own practice was to introduce the concepts of Type 1 and Type 2 errors in the context of binomial tests, so that the calculations are relatively straightforward and yet the concepts can be understood thoroughly. However, the author's treatment of this, and non-parametric testing, seems to be very sound.

This is not the book I would have written if anyone had the misfortune to commission one from me. Perhaps I am expecting a somewhat more theoretical approach to what is, after all, a difficult subject to teach. However, it contains plenty of excellent questions and everything you need to know is there, even it is presented as a *fait accompli* rather than justified from first principles. Use it by all means, but be prepared for some searching questions from that awkward pupil who insists on knowing why things work rather than just how to do the problems.

Reference

1. Owen Toller, *The mathematics of A-level statistics*, The Mathematical Association (2009).

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OCR A level Further Mathematics Discrete (A) by Nick Geere, pp. 162, £21, ISBN 978-1-5104-3337-3, Hodder Education (2018)

This slim volume completes the set of five books which cover the syllabus for the OCR Further Mathematics examination. I have to admit that discrete mathematics is not an area I have taught and, apart from combinatorics, not one I am expert in. Moreover, my interest is motivated by problem-solving rather than by familiarity with particular aspects of discrete mathematics such as network algorithms, critical path analysis, linear programming or game theory. So I shall take it as read—I think with reason—that the author of the book knows the syllabus intimately, has covered all the necessary topics and has offered much useful advice on tackling the questions which will appear on the examination paper. The focus of this review, therefore, will mainly be on the first two chapters and, in particular, on concept development and problem-solving.

The first chapter provides a synopsis of what this subject is about, beginning with a useful classification of problems into the four (possibly overlapping) categories of existence, construction, enumeration and optimisation. The specific items covered include the pigeonhole principle, set theory with Venn diagrams (and the inclusion-exclusion principle) and permutations and combinations. Derangements are mentioned but not pursued beyond four objects. I have no criticisms of this treatment.