

Proposal for Load Adaptive Design of Microlattice Structures Suitable for PBF-LB/M Manufacturing

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Abstract

In this paper, a proposal for a new method to design load-adaptive microlattice structures for PBF-LB/M manufacturing is presented. For this purpose, a method was developed to stiffen microlattice structures in particular by using self-similar sub-cells to ensure their manufacturability. The quality of the stiffness increase was investigated and verified by finite element simulations. Subsequently, the simulation results were critically discussed with respect to their potential for future design processes for architected materials.

Keywords: architected materials, 3D printing, additive manufacturing, lattice structures, design methods

1. Introduction

For several years, lattice structures have been a focus of structural and materials research for lightweight and functional design in mechanical engineering. This is based on the possibilities of programmable structural response through the targeted arrangement of material in volume, which allows the realization of high specific stiffness combined with low mass (Rehme and Emmelmann, 2006). On the one hand, attempts are made to investigate cell structures from nature with focus on their design and load capacity and to integrate them into the technical design process. On the other hand, possibilities for optimizing existing lattice structures or generating new structures by changing the arrangement of material in volume are analyzed. Thereby, the manipulation of the structural behavior of microlattices represents a focus of the current research of architected materials (Pan et al., 2020; Kadic et al., 2019; Mines, 2019; Singh et al., 2015; Valdevit et al., 2018). One research goal is to manipulate lattice structures with feature sizes in the micrometer scale and cell sizes of a few millimeters by topological or geometrical modifications in order to program the structural response for specific load cases (Mines, 2019). The specific manipulability of such structures allows a targeted and multidimensional spectrum of structural responses for different loading conditions, opening up new possibilities, especially in the medical field, for the design of more load-friendly and bone-protecting or osseointegrated bone replacement implants (Wang et al., 2018).

However, the manufacturing of metallic microlattices is still a challenge on conventional laser-based powder bed fusion of metals (PBF-LB/M) printing systems. In particular, the production quality of these lattices is still a focus of additive manufacturing, since even small variations in the manufacturing parameters or unfavorable exposure strategies have a significant impact on the production quality of the lattice struts (Korn et al., 2018a; Korn et al., 2018b; Koch et al., 2018). The manufacturing of approximately round and cylindrical struts in microlattices remains very challenging, since powder adhesion and necking effects in struts and nodal areas have a significant influence on the load-bearing capacity of microlattices. In studies, this problem has already been improved by modified exposure strategies

and adapted slicing algorithms for microlattices (Korn et al., 2018a; Korn et al., 2018b). However, the boundary conditions apply that the exposure strategies for thin struts must be kept as constant as possible. Due to this resulting sensitivity of the manufacturing quality of microlattices, existing manipulation strategies developed to influence the stiffness of microlattices can only be used to a limited extent or not at all in the manufacturing process on conventional equipment. This results in a negative impact on the establishment of microlattices in the technical design process.

Based on this problem, this paper presents a proposal for a method to manipulate microlattices in a manufacturing-friendly manner with consistent exposure strategies for PBF-LB/M machines. Thereby, the stiffness of single cells can be increased to enable a local and load-adaptive design of microlattices. In doing so, this paper presents a cell design method that uses self-similar sub-cells to stiffen individual cells of a microlattice. Following, the presented design method is critically analyzed by numerical simulations and discussed with regards to its plausibility.

2. Materials and Methods

2.1. Methods for load-adaptive lattice designs

In order to use lattice structures more flexibly as design structures, methods for manipulating the physical properties of such structures have been investigated for several years (Pan et al., 2020). Here, the adaptation of the mechanical properties is of great interest for the load-adaptive generation of lattice structures as a component in the engineering design process. Over the past years, this aspect has been a main focus in the research of homogeneous periodic lattice structures. Due to the large number of parameters to be varied, a wide range of methods for stiffness adjustment have been developed (Al Nashar and Sutradhar, 2021; Meza et al., 2015; Sha et al., 2018; Wang et al., 2020; Zhu et al., 2021; Wu et al., 2021a; Bai et al., 2021; Wu et al., 2021b; Pan et al., 2020). These methods generally modify either the geometry or the topology of the lattice structure via the unit cell. Either individual cells or entire structures are adapted to the specified loads. One method to manipulate the geometry of individual lattice cells or entire structures is to change the cross-section or diameter of the modified struts along the stress, depending on the load case. Such methods grade the diameter of individual struts along the applied load (Figure 1) (Zhu et al., 2021).

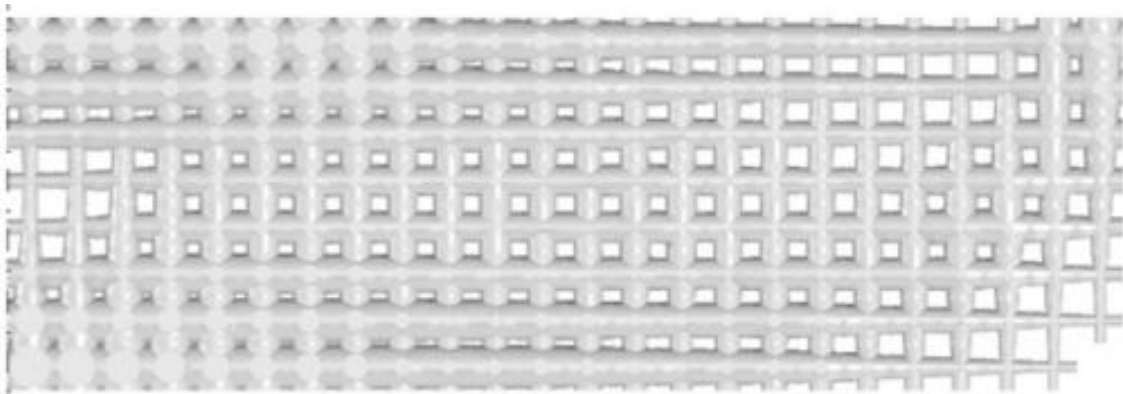


Figure 1. Stress adjusted graded lattice structures (Zhu et al., 2021)

In addition to the manipulation of the strut geometry, the struts of cells can also be aligned along calculated stress trajectories (Figure 2) (Wu et al., 2021b). In this process, the struts are aligned along the occurring tensile and compressive stresses, thus preventing dominant bending loads in the struts. In addition to the alignment, the cell density can be increased in heavily loaded areas by reducing the cell dimensions. Thereby, the cell dimensions are adjusted with the help of stress-based field functions. This leads to a better load distribution and loading of individual struts as well as to a higher stiffness of the lattice structure (Wu et al., 2021b). In addition to these two prominent approaches, there are other ones in which, for example, the strut cross-section has been adapted to the occurring loads or the nodal regions of individual cells have been designed with the use of large radii to reduce the notch effect of

colliding struts in order to increase the structural load-bearing capacity (Zhu et al., 2021; Wu et al., 2021b; Bai et al., 2021; Wang et al., 2021; Bai et al., 2019).

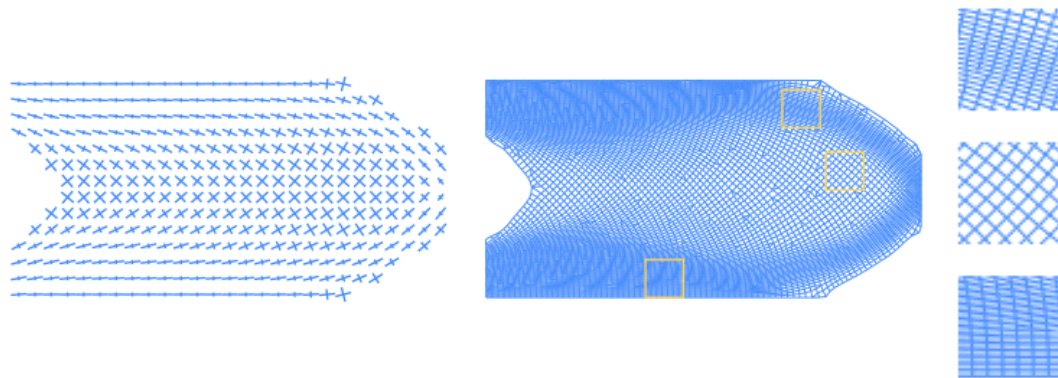


Figure 2. Lattice generation on field-based cell orientation (Wu et al., 2021b)

In addition to local and global geometric approaches to modify the stiffness behaviour of lattice structures, there are also topological approaches to adjust the stiffness behaviour of lattice structures. In this paper, topological structural manipulations are defined as methods in which a change of stiffness in the structure is realized by manipulating the arrangement of struts in the unit cell or in the lattice. To achieve this, the cells can be trajectoryally aligned in the lattice to increase stiffness (Figure 2), or additional cells can be integrated into lattice structures as superposed or replacement cells to locally optimize the structural response. In this way, the method developed by Wu et al. (2021b) (Figure 2) is a combination of geometric and topological structure modification. Other prominent examples of topological changes to lattice structures are shown in Figures 3 and 4. Figure 3 shows a hierarchical topological manipulation approach in which the struts of a cell are replaced by lattice structures of the same cell type. The resulting mass reduction leads to an increase in flexibility. In addition, such structures can be used to achieve pseudo-hyperelastic material behaviour and provide high energy absorption. These kinds of lattices are particularly suitable for the development of structures for shock and impact absorption (Al Nashar and Sutradhar, 2021; Sha et al., 2018; Meza et al., 2015).

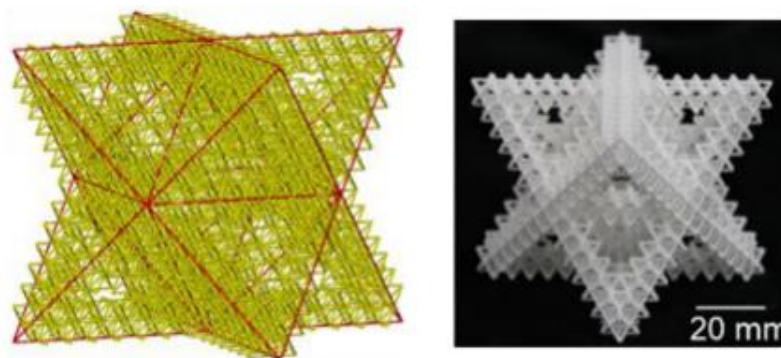


Figure 3. Hierarchical architected lattices (Sha et al., 2018)

On the other hand, a second design approach merges or replaces either single cells or whole structures with substructures of a different lattice type. In this way, ensemble structures or hybrid lattice structures are generated, which locally and load-adaptively exhibit significant stiffness increases (Figure 4) (Wang et al., 2020; Wu et al., 2021a). By adding a cube cell to a BCC cell in combination with a variation of the design parameters as shown in the figure, the structural stiffness could be increased significantly (Wang et al., 2020). Such heterogeneous lattice designs show an enormous potential by the multi-parametric control of the cell geometry for load-adaptive designed lattice structures.

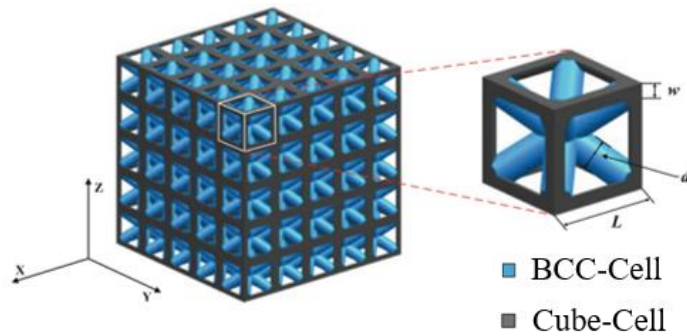


Figure 4. Hybrid lattice structures based of two different unit cells (Wang et al., 2020)

2.2. Proposal for a PBF-LB/M manufacturing-friendly load-adaptive lattice design

Due to the ongoing development of additive manufacturing processes, higher accuracies and shape fidelity are meanwhile achievable. Particularly in the case of metallic processes, such as laser beam melting, the quality can be increased through better hardware and powder quality, as well as through adaptive exposure strategies and adapted laser power, especially in the production of lattice structures. The manufacturing of microlattice structures with strut diameters in the micrometer scale up to the laser focus diameter is currently a research focus in additive manufacturing (Korn et al., 2018b; Korn et al., 2018a; Mines, 2019).

The primary focus is on the software optimization of the manufacturing parameters of conventional PBF-LB/M systems in order to be able to produce microlattice structures on these systems, so that the use of such structures as design elements can be established in the design process. The manufacturing quality of single struts in the microlattice is still a problem. This can be attributed to the inaccurate generation of exposure hatches (Figure 5 Left) in the respective cross-sections and, as a result, an uneven local energy input in the struts. The result is constrictions in the struts (Figure 5 Right) which lead to a weakening of the structure (Korn et al., 2018a; Korn et al., 2018b).

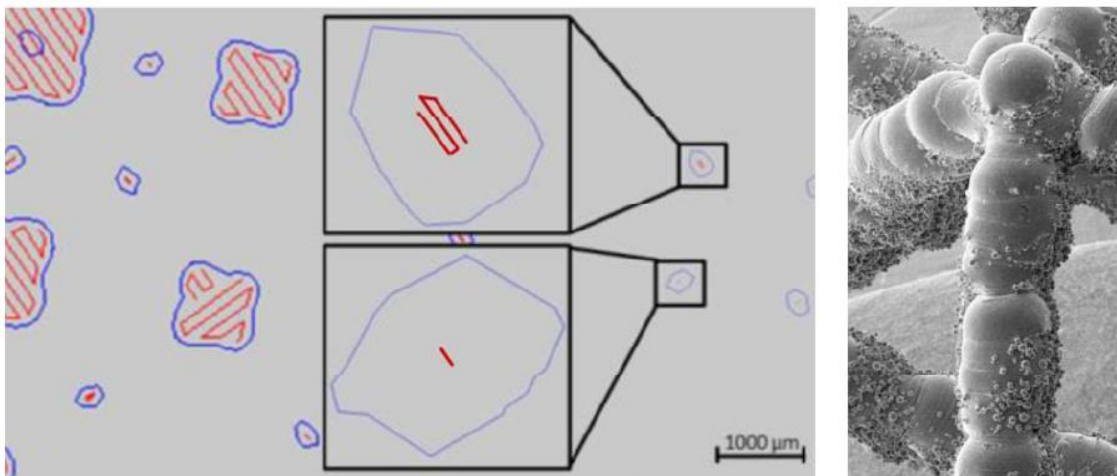


Figure 5. Manufacturing quality microstruts with common contour-hatch-scanning strategy lead to uneven energy input (Korn et al., 2018b)

These shortcomings can be reduced by adapting the exposure strategy in the strut and node cross sections. Point-, line- and cross-exposures are used, which allow a uniform energy input due to the adaptive control of the laser power (Korn et al., 2018a; Koch et al., 2018). Figure 6 shows the influence of the exposure strategy on the resulting lattice.

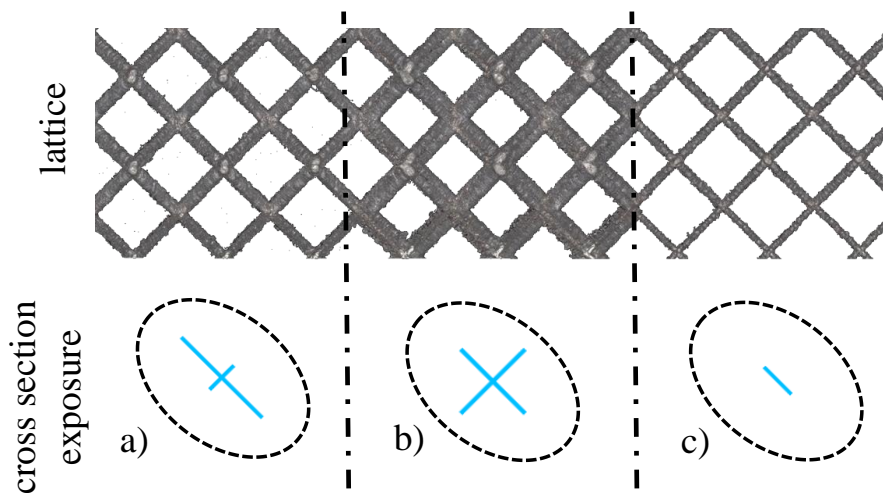


Figure 6. Adaptive exposure strategies for microlattice structures
 (Korn et al., 2018a; Koch et al., 2018).
 a) weighted cross-exposure, b) cross-exposure and c) line-exposure

On the basis of these observations, design constraints can be identified which affect both the design of microlattices and their possibilities for load-adaptive modifications. For reproducible manufacturing quality, the strut angles of all struts of a lattice must have a certain minimum angle to the printing field, since no support structures can be implemented for the manufacturing of lattice structures. Furthermore, reliable exposure vectors must be generated for good rod quality. Thus, there should be no change of exposure strategies within the manufacturing of single struts. As a result of these conditions, the methods presented above for the load-adaptive design of lattice structures cannot be transferred to PBF-LB/M microlattices, or can only be transferred within narrow parameter limits.

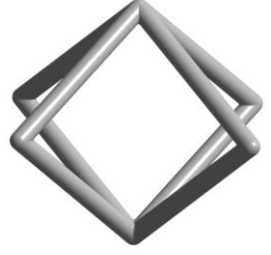
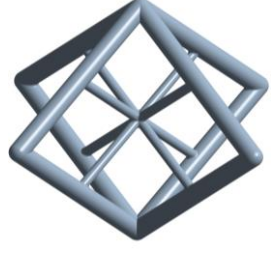
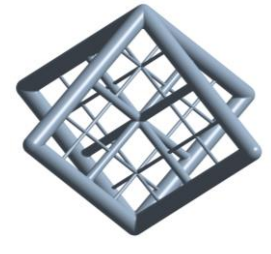
Therefore, a proposal for a new load-adaptive design approach for microlattice modifications was developed. In this method, the lattice structures are stiffened locally at the cell level, which allows the generation of load-adaptive structures from periodically arranged cells. To stiffen the unit cells of a structure, self-similar sub-cells are inserted, which are the same type as the unit cells. The dimensions of the sub-cell correspond to an integer divisor of the dimension of the unit cell. The sub-cell ensures that the occurring loads are better distributed in the cell which leads to a reduction of the stresses in the single struts. According to this rule, each further inserted sub-cell is oriented to the superordinate sub-cell already inserted. This ensures that homogeneous manufacturing sub-cells are built and that the resulting new cell consists of homogeneous struts with constant cross-sections and resulting constant exposure strategies. The approach of this method was carried out exemplarily on BCC cells. BCC cells do not have high compression stiffness due to their bending-dominant loading behavior in the struts (Maconachie et al., 2019). Consequently, this type of cell depends on stiffening methods to be used in the design process, since a great advantage of this type of cell is its good manufacturability, which is especially important in making microlattices (Ushijima et al., 2011; Feng et al., 2018).

For the analysis of the developed stiffening method, different cell designs in the microscale range were generated (Table 1). The base cell for the investigations is the BCC unit cell with a dimension of 6 mm in all directions and a strut diameter of 0.5 mm and is labeled HL_0 . Based on this cell, two more cells are designed. In the HL_1 cell, another sub-cell with the cell dimensions of 3 mm and a strut diameter of 0.25 mm is inserted. In this way, the parallel base struts of the HL_0 cell are stiffened by the struts of the HL_1 cell in their midpoint, with the purpose of reducing occurring deformations of the HL_0 cell struts and distributing occurring loads more effectively in the cell. In this way, the insertion of self-similar cells generates heterogeneous hierarchical structures defined by the hierarchy level. Consequently, the hierarchy level increases as more sub-cells are added. To investigate the influence of the support effect by inserting further sub-cells, a cell with two hierarchy levels HL_2 was generated. Based on cell HL_1 , additional sub-cells with the cell dimensions of 1.5 mm and a strut diameter of 0.1 mm were added. Cell HL_1 was expanded by additional 32 struts. It was ensured for all cells that the diameter - length ratio was not greater than 0.1. For all cells the relative density $\bar{\rho}_{HLi}$ was calculated according to (Equation 1), where V_{HLi} is the volume

of the particular cell and V_{voll} represents the total volume of the volume to be replaced with an edge length of 6 mm (Ushijima et al., 2011; Gümrük and Mines, 2013).

$$\bar{\rho}_{HLi} = \frac{\rho_{HLi}}{\rho_{voll}} = \frac{V_{HLi}}{V_{voll}} \quad (1)$$

Table 1. Hierarchical cell design

Cell	Cell Dimension [mm x mm x mm]	Hierarchical level	Strut diameter [mm]	Lattice volume V_{HLi} [mm ³]	Rel. density $\bar{\rho}_{HLi}$	Cell design
HL_0	6.0 × 6.0 × 6.0	0	0.500	7.876	0.036	
HL_1	6.0 × 6.0 × 6.0 & 3.0 × 3.0 × 3.0	1	0.250	8.826	0.041	
HL_2	6.0 × 6.0 × 6.0 & 3.0 × 3.0 × 3.0 & 1.5 × 1.5 × 1.5	2	0.100	9.067	0.042	

The proposal presented in this paper will be investigated by numerical simulations using the finite element method in the software ANSYS. The individual cells are calculated volumetrically in order to represent a more realistic load behavior (Ushijima et al., 2011). For this purpose, the struts are modulated with a cylinder, the nodes with spheres and they are combined to a homogeneous topology. Subsequently, the nodal regions are filleted with radii of 0.05 mm for HL_0 and HL_1 and 0.01 mm for HL_2 (Figure 7) to avoid singularities in the simulation and to improve the meshing quality. The radii of curvature have an influence on the stiffness of the structures, but this does not negatively affect this preliminary qualitative comparison of the cells with each other. However, this geometric adjustment must be taken into account for future validation with experimental data. Ti6-Al4-V is used as material for the cells and the material model is defined with an isotropic material behavior and a bilinear isotropic strain hardening (Table 2).

Table 2. Material properties

Material	Material properties	Young's modulus E [N/mm ²]	Poisson's ration ν	Yield strength σ_y [N/mm ²]	Ultimate strength σ_u [N/mm ²]	Tangent modulus E_{TAN} [N/mm ²]
Ti6-Al4-V	Isotropic	1.14 E+05	0.323	830	895	1250

For meshing, an average Jacobi ratio based on Gaussian points of 0.88 is set for element quality. All models are meshed using quadratic order 10-node tetrahedrons. In addition, a stress-based mesh refinement was defined to further increase the mesh quality. The purpose of this study is to determine the influence of self-similar sub-cells as components to stiffen the base cell of lattice structures. For this purpose, uniaxial compression tests are simulated to calculate the compressive stiffness of the single structures. For this the lower node of the cell is constrained by a fixed support in all degrees of freedom and a linear displacement of 0.1 mm in the Z-direction is defined at the upper node (Figure 7). The other four nodes of the cell were not restricted in any degree of freedom.

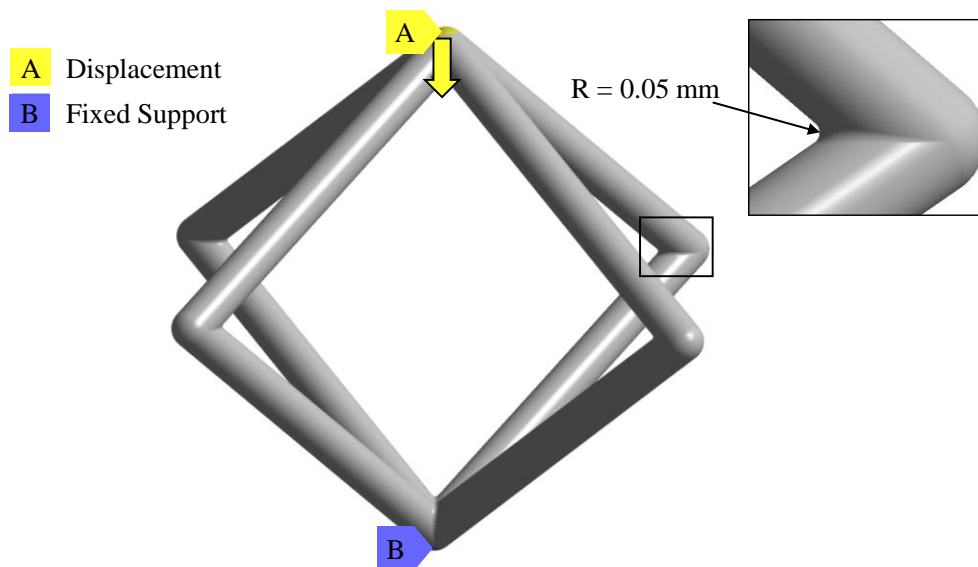


Figure 7. FE-model with boundary conditions using the design of HL_0 as example

3. Results

Based on the material and model conditions, the necessary force was calculated for each cell to achieve a displacement of 0.1 mm. The results are shown in Figure 8. From the graphs it can be seen that the relationship between force and displacement is linear for all cells. This means that all cells have elastic deformation behavior. Furthermore, it can be seen that the cell design has an influence on the force required for an equal displacement. The ratio of the change in force to the change in displacement according to (Equation 2) can be defined as compressive stiffness K_{HLi} . The compression stiffness corresponds to the slope of the linear graph shown in Figure 8. The specific compression stiffness K_{HLi} is subsequently normalized on the basis of K_{HL0} (the compression stiffness of the base cell) according to (Equation 3) and the relative density $\bar{\rho}_{HLi}$ is normalized on the basis of $\bar{\rho}_{HL0}$ (the relative density of the base cell) according to (Equation 4). These normalizations are subsequently brought into relation for the calculation of the relative compression stiffness \tilde{K}_{HLi} according to (Equation 5).

$$K_{HLi} = \frac{\Delta F_{HLi}}{\Delta u_z} \quad (2)$$

$$\bar{K}_{HLi} = \frac{K_{HLi}}{K_{HL0}} \quad (3)$$

$$\bar{\rho}_{HLi} = \frac{\bar{\rho}_{HLi}}{\rho_{HL0}} \quad (4)$$

$$\tilde{K}_{HLi} = \frac{K_{HLi}}{\bar{\rho}_{HLi}} \quad (5)$$

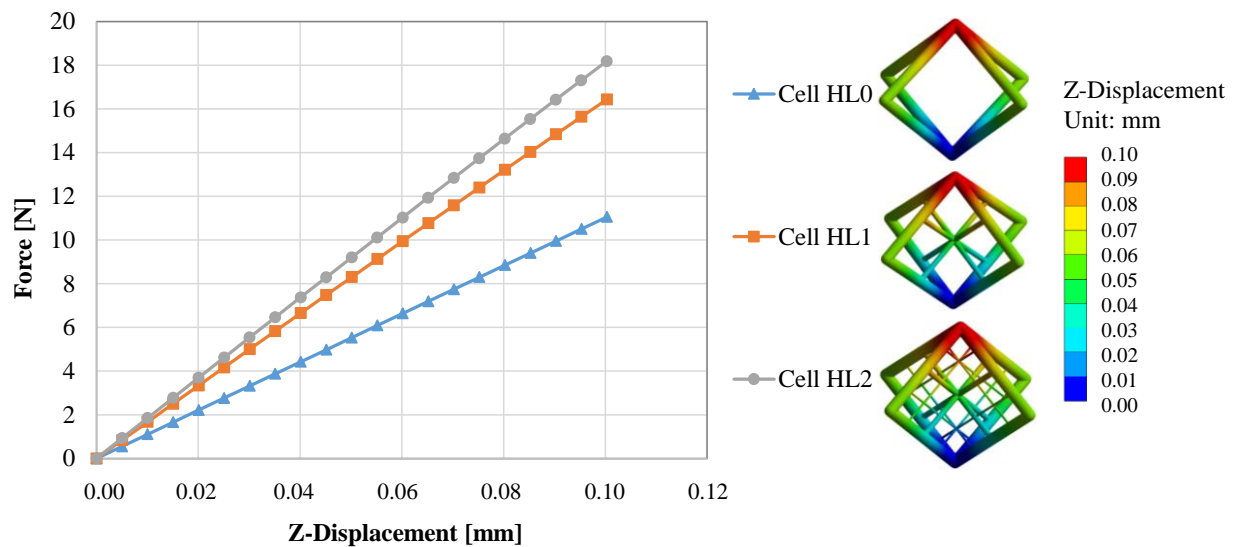


Figure 8. Compression behaviour of all cell designs

Based on the given equations and the results from FE simulation, the necessary values of each cell were calculated and listed in Table 3. From this table it can be seen that the insertion of self-similar cells has increased the relative density of the new cells with one hierarchical level by 14% and with two hierarchical levels by 17%. Thereby, the absolute compressive stiffness of the cells has improved with increasing hierarchy level by 49% and 64%, correspondingly. However, the absolute compressive stiffness must be considered in relation to the relative density increase. Thus, the relative compression stiffness \tilde{K}_{HL} of cell HL_0 has increased by 31% with the insertion of one additional self-similar cell with halved dimensions. By inserting another self-similar cell with dimensions equal to a quarter of the base cell HL_0 , the relative compressive stiffness has increased by 41%.

Table 3. Absolut, normalized and relative compression stiffness for all cell designs

Cell	$\bar{\rho}_{HLi}$	$\bar{\rho}_{HLi}$	F_{HLi} [N]	u_z [mm]	K_{HLi} [$\frac{N}{mm}$]	\bar{K}_{HLi}	\tilde{K}_{HLi}
HL_0	0.036	1.00	11.06	0.1	110.58	1.00	1.00
HL_1	0.041	1.14	16.44	0.1	164.35	1.49	1.31
HL_2	0.042	1.17	18.18	0.1	181.78	1.64	1.41

Here, it can be observed that the stiffness gradient from HL_0 to HL_1 is noticeably larger than the stiffness gradient from HL_1 to HL_2 . This is explained by the fact that the inserted sub-cells in the HL_2 design are very thin (0.1 mm) due to the constant diameter-length ratio. This is also represented by the increase in relative density. Due to these small diameters, the added resistance of bending of the individual struts is also very small. Furthermore, such thin struts cannot carry high loads. In addition, the large diameter difference between the struts of the base cell HL_0 and the struts of the second stiffening cell HL_2 is very large, which leads to an increased notch effect in the transition of the struts with such large differences in diameter and, as a result, increased mechanical stresses and incipient component failure are to be assumed at such areas.

From these results it can be assumed that the load distribution by introducing sub-cells has a positive effect on the stiffness behavior of the base cell as long as the difference in diameter between the self-similar cells is not too large. As a result, the relative compression stiffness of the cell under uniaxial compression loads was markedly increased. However, the gradient of stiffness improvement decreases

noticeably with introduction of further stage with diameter differences. From this primary analysis, it appears that the newly developed variant increases the stiffness of the cell noticeably with respect to the relative density, but further investigations must follow to bring out the full potential of this method.

4. Conclusion

In this paper a method for stiffening microlattices locally in a discrete and load-adaptive, manufacturing-friendly manner was presented. Thereby, it was investigated to which degree the stiffness of a unit cell is influenced by the insertion of further self-similar unit cells with reduced dimensions. The results showed that the insertion of a sub-cell with half the dimensions of the base cell and a constant diameter-length ratio of 0.1 increased the relative compression stiffness in relation to the relative density by 31%, and the insertion of a second sub-cell increased it by 41%. However, the cell design of two or more self-similar cells has to be considered critically, because the inserted struts lead to large diameter difference to the struts of the base cell due to a constant diameter-length ratio of 0.1. As a result, a large notch effect on the thin struts is assumed at the junction of the very small sub-cells to the base cell, which may lead to early failure and so possibly weaken the cell in reality by introducing the second level of hierarchy or higher. Therefore, further investigations are needed to fully explore the developed method and evaluate its potential qualitatively and quantitatively.

In future investigations, the cell design developed here should be manufactured and uniaxial compression tests should be performed in order to validate the simulation results calculated in this study. Furthermore, a full-factorial investigation for these cell designs seems to be reasonable, since it could be shown that an increase of the hierarchy level alone initially increases the stiffness, under otherwise identical conditions. However, too large diameter differences between the struts of the individual cells carry detrimental effects such as increased notch effects. Consequently, the simulation models must be extended to include material failure models in order to be able to investigate this aspect in more detail. This paper has shown that the method presented here for fabrication-friendly stiffening of cells in lattice structures has potential for further investigation. In the future, it will be possible to stiffen PBF-LB/M - fabricated lattice structures in a cell-discrete and load-adaptive way without major changes in the relative density of the unit cell.

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