graduate students. After two introductory chapters on sets and functions, and integers and the rational numbers, the author develops elementary group theory; elementary ring theory; theory of modules, tensor products and algebras (including the decomposition of finitely generated modules over principal ideal domains); theory of vector spaces and canonical forms of matrices; theory of algebraic and transcendental extensions of fields; and Galois theory with classical applications. Thus the material is more or less classical (except for the chapter on modules it is roughly the same as the first volume of the English edition of van der Waerden), but the language is modern.

The book has a number of good features. In particular, I found the author's digressions on crucial points helpful. There are numerous good examples and exercises (with plentiful hints), and often the proofs of minor points are relegated to exercises. There are "Excursions" which deal with topics off the main path, and each chapter ends with a list of further references.

In spite of these features, I would not choose this book for a text. I believe that in general a student would find it unnecessarily hard work to read. Where notation and definitions should be used to clarify and simplify the subject, too often the notation obscures the essentially simple, and definitions are too numerous. A textbook should be written for the student's benefit, and there seems little excuse for a mass of symbols where a few extra words would clarify a statement and made the reading smoother. Categorical language has justified itself and the notion of a universal element seems to be especially valuable, but a student should be warned that (at this level at any rate) it <u>is</u> only a tool. Perhaps the irreverence of Serge Lang in referring to "generalized nonsense theory" is appropriate. This book contains some of the exciting and deep theorems in algebra; it is a pity if the student is left negotiating the shallows.

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<u>The variational theory of geodesics</u> by M.M. Postnikov. Translated from the Russian by Scripta Technica Inc. edited by Bernard R. Gelbaum. W.B. Saunders Company, Philadelphia and London, 1967. viii + 200 pages.

The general trend of this book, whose first Russian edition appeared in 1965, is modern throughout. Its primary concern is an important area of contact between the calculus of variations and differential geometry, namely the theory in the large of geodesics on Riemannian spaces.

The book as a whole is self-contained. It begins with a somewhat unusual introduction to the theory of differentiable manifolds, multilinear algebra and exterior differential forms. The next two chapters are respectively concerned with affine connections and Riemannian geometry.

Apart from a few basic results the development differs significantly from that of standard texts dealing with these topics. For a connected Riemannian manifold M the concept of the interior metric $\rho(p, q)$ is introduced almost immediately as follows: for any two points $p, q \in M$ one defines $\rho(p, q)$ as the greatest lower bound of the lengths of all piecewise smooth curves joining p, q; thus $\rho(p, q)$ transforms M into a metric space. This gives rise to the theory of normal convex neighbourhoods. Results such as the Hopf-Rinow theorem, or the following statements, are typical of these chapters: For any connected Riemannian space M the following properties are equivalent: (1) the space M is complete in the sense that each maximal geodesic on it is defined on the entire real axis [here a geodesic is an autoparallel curve with respect to the usual connection; it is maximal if it is not the restriction of any geodesic defined on a larger interval on the real axis]; (2) the space M is complete with respect to the interior metric ρ (i.e., every fundamental sequence converges to a point of M); (3) every closed subset of M which is bounded in respect of the metric ρ is compact.

The variational properties of geodesics [with fixed end-points $(\bar{p}, \bar{q})$] are then discussed. This includes a treatment of conjugate points, indices of points and intervals and their evaluation by means of the quadratic forms of Morse and Bott, so that some of the basic tools of Morse theory are automatically included. These results are generalized to the case in which $\bar{q}$ is replaced by some submanifold $N \subset M$, the geodesics concerned being orthogonal to N, which gives rise to a corresponding theory of focal points and their indices. The book concludes with the reduction theorem of Bott.

Because of some unorthodox features of the book one should hesitate before recommending it to a beginner; however, to the specialist interested in differential geometry or in the calculus of variations this work may well prove indispensable.

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Multiple integrals in the calculus of variations by C.B. Morrey. Springer-Verlag, New York, 1966. ix + 506 pages.

The theme of this book is, in the author's own words, "the existence and differentiability of the solutions of variational problems involving multiple integrals". Ever since Riemann formulated Dirichlet's principle over a hundred years ago, the connection between the calculus of variations and the analytical theory of differential equations has been an important strand in the fabric of analysis. Several topics of recent emergence in the theory of elliptic differential equations and systems, both linear and non-linear, are assembled and discussed in detail in this volume.