The authors assume a knowledge of spectral theory, harmonic analysis, the theory of semi-groups and partial differential equations. The book contains applications to potential scattering, the Schrödinger equation and the acoustic equation with an indefinite energy form.

There are four appendices dealing with semi-groups of operators, energy decay, energy decay of star-shaped obstacles, and scattering theory for Maxwell's equations.

The text should be of great interest to mathematicians and theoretical physicists as it contains a lot of material which is highly original especially the authors' use of the Radon transform to give a new and more natural formulation of the Sommerfeld radiation condition which is also applicable to general hyperbolic equations.

C. Roth, McGill University.

Perturbation Methods in Applied Mathematics, by Julian D. Cole. Blaisdell Publishing Company, Waltham, Massachusetts, 1968. 260 pages. U.S.\$9.50.

This book is concerned with perturbation techniques in the theory of differential equations, written from the point of view of an applied mathematician, i.e. little attention is given to mathematical rigor and physical reasoning is often used to justify steps in the analysis. The main mathematical tools used are asymptotic expansions in terms of a parameter and the idea of matching so called "inner" and "outer" expansions at a boundary layer.

Chapter one is a very brief introduction to asymptotic sequences and expansions.

Chapter two is concerned with singular perturbation problems

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for ordinary differential equations, the singular nature arising from either the lowering of the order of the equations as a parameter tends to a limit or to difficulties associated with the behaviour of solutions near a boundary point. Examples are chosen from the theory of nonlinear oscillations, fluid mechanics and elasticity.

In chapter three, two variable expansion procedures are introduced in order to investigate physical problems which are characterized by the presence of a small force or disturbance which is active over a long time interval. The methods used are basically a generalization due to Kevorkian of Poincaré's idea for the calculations of periodic motions of slightly nonlinear oscillations, and examples are taken from this area. The classical problem of the approximate solution of a differential equation with a large parameter falls naturally into the discussion of this chapter.

In chapter four, the methods developed in chapter two are applied to partial differential equations. One problem that arises in the elliptic case is that when a segment of the boundary is subcharacteristic the boundary layer equation becomes a partial differential equation. For the hyperbolic case in order to assure stability it is necessary that the subcharacteristics be timelike and under this assumption the initial value and radiation problems are discussed. The rest of the chapter illustrates the general theory by considering examples from fluid mechanics, magneto-hydrodynamics, heat conduction and elastic-shell theory.

The last chapter is a continuation of chapter four in which various approximate equations (such as the transonic equation or nonlinear water-wave equation) involving several parameters are studied, and their solutions are examined for various "distinguished limits" of these parameters. In particular the concept of nearfield and far-field equations are introduced in order to examine various problems connected with the theory of wave propagation.

This book is a welcome addition to the literature on applied mathematics, having filled a gap which had previously been left vacant.

David L. Colton, McGill University.

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