SIR,

Reply to comments on "Subglacial floods and the origin of low-relief ice-sheet lobes" by J. S. Walder

In commenting upon my paper (Shoemaker, 1992a), Walder (1994) asserted that thick water sheets are unconditionally unstable to formation of channels and strongly implied that this negates the possibility of water-sheet outburst floods. However, Walder's "fundamental conclusion" does not argue against the existence of water-sheet floods, as I will demonstrate.

Walder (1982) obtained the differential equation

$$(1/A)(\mathrm{d}A/\mathrm{d}t) = (1/s_1)(1 - \tfrac{2}{3}(k^2h^2)) - (1/s_2) - (1/s_3)(k^2h^2) \qquad (1)$$

by applying a standard perturbation analysis to the full equations for steady-state laminar flow of a water sheet. Here, A is the amplitude of growth of the perturbation h in the cross-section profile $z = h(1 + \epsilon \sin ky)$ of a water sheet, where h is the average water-sheet thickness, y is measured transverse to the flow, z is vertical to the flow and $\epsilon \ll 1$. With the last term in Equation (1) negligible (Walder, 1982), and considering only the case $kh \ll 1$, Equation (1) becomes

$$(1/A)(\mathrm{d}A/\mathrm{d}t) = 1/s_1 - 1/s_2 \qquad (2)$$

where

$$1/s_1 = \frac{(\rho_\mathrm{i} g)^2 \alpha^2 (1-\gamma) h^2}{4\mathrm{L}\eta_\mathrm{w}\rho_\mathrm{i}}, \quad 1/s_2 = \frac{(\rho_\mathrm{w} - \rho_\mathrm{i})g}{2k\eta_\mathrm{i}}. \qquad (3)$$

Here, α is the ice-surface slope (assuming a horizontal bed), η_w is the absolute viscosity of water, L is the specific latent heat of fusion, η_i is the effective viscosity of ice and ρ denotes density. The term $(1 - \gamma) \simeq \frac{2}{3}$.

If $1/s_1 - 1/s_2 > 0$, A grows exponentially in time and a broad water sheet will evidently collapse into many narrow water sheets which could eventually develop into channels operating at pressures less than the overburden pressure.

Table 1 values of $1/s_1$ are an extension of Walder's (1982) table which applied only to the range $\alpha \geq 0.01$. Apparently, Walder was considering valley glaciers. However, for ice sheets, a more appropriate range, corresponding to sites between 500 and 1500 km from the terminus, is $5 \times 10^{-4} < \alpha < 2 \times 10^{-3}$, for basal shear stresses between 8 kPa for a ponded soft bed (Shoemaker, 1991) and 50 kPa, the mean for contemporary ice sheets (Paterson, 1981). I have truncated Table 1 at $h = 70$ mm because, depending upon α, the transition to turbulent flow occurs for h between 30 and 70 mm. Assuming that the growth phase of a water-sheet outburst flood spans several weeks, it is clear from Table 1 that the water sheet can survive the laminar flood phase provided α is smaller than about 3×10^{-3}.

Table 1. Time constant $1/s_1$ for laminar flow

$h \setminus \alpha$ mm	$1/s_1(1\,\mathrm{d}^{-1})$ 10^{-2}	10^{-3}	10^{-4}
0.1	2.2×10^{-6}	2.2×10^{-8}	2.2×10^{-10}
1	2.2×10^{-4}	2.2×10^{-6}	2.2×10^{-8}
10	2.2×10^{-2}	2.2×10^{-4}	2.2×10^{-6}
50	0.55	5.5×10^{-3}	5.5×10^{-5}
70	1.1	1.1×10^{-2}	1.1×10^{-4}

Equation (2), with $1/s_2 = 0$, is easily obtained by considering two independent water sheets of rectangular cross-section with water thicknesses h_1 and h_2, respectively. Use the formula for fully developed laminar flow

$$Q = \rho_\mathrm{i} g \alpha h^3 / 12\eta_\mathrm{w} \qquad (4)$$

for flux Q $(\mathrm{m}^3(\mathrm{m\,s})^{-1})$. Now, equate the viscous power dissipation which goes into ice melting to the rate of change of enthalpy giving $(1-\gamma)g\alpha Q = L\mathrm{d}h/\mathrm{d}t$. Apply this independently to the two sheets and let $A = h_1 - h_2$. Substituting for Q from Equation (4) and using $({h_1}^3 - {h_2}^3) \simeq 3h^2(h_1 - h_2)$ with h fixed, as appropriate to the perturbation analysis, we arrive at Equation (2) with $1/s_2 = 0$. The exponential growth $A = \exp(t/s_1)$ represents the growth of the difference in water thickness of the two sheets.

To extend this simple analysis to the turbulent flood phase, replace Equation (4) by the Manning's equation as developed for a rectangular water sheet

$$Q = (\alpha\rho_\mathrm{i}/\rho_\mathrm{w})^{\frac{1}{2}}(h^{\frac{5}{3}})/(2^{\frac{2}{3}}n) \qquad (5)$$

where n(SI) is Manning's roughness factor. We obtain

$$(1/A)(\mathrm{d}A/\mathrm{d}t) = \frac{5(1-\gamma)g(\rho_\mathrm{i}\alpha)^{\frac{3}{2}}h^{\frac{2}{3}}}{(3)2^{\frac{2}{3}}n(\rho_\mathrm{w})^{\frac{1}{2}}L} = 1/s_1\,. \qquad (6)$$

Note that $1/s_1$ in Equation (3) is proportional to h^2 whereas $1/s_1$ in Equation (6) is proportional to $h^{\frac{2}{3}}$. The friction factor decreases less rapidly with increasing discharge in turbulent flow than in laminar flow.

Table 2 extends the exponential time constant $1/s_1$ into the turbulent range. From Tables 1 and 2, I conclude that a very thick water sheet, up to 100 m thick, can exist for weeks provided α is suitably small.

The term $1/s_2$ in Equations (2) and (6) (not shown) can be important. The effective ice viscosity η_i in Equation (3) decreases with increasing effective shear stress, τ (Paterson, 1981). The dominant contribution to τ in Walder's analysis was the basal shear stress, τ_b. But in the water-sheet problem the dominant stress after ice lift-off is the tensile stress, which could easily be an order of magnitude greater than τ_b (Shoemaker, 1992a, b). If τ increases by a factor of 10, η_i in Equation (3) decreases by a factor of 100 and $1/s_2 > 1/s_1$ at all but very short wavelengths. Others may wish to investigate the effect this has upon water-sheet stability.

In conclusion, once the turbulent phase is included in the analysis, Walder's main objection is refuted. Field evidence, particularly bed forms associated with turbulent flow, should be reviewed by those who reject water-sheet floods.

Table 2. Time constant $1/s_1$ *for turbulent flow*

$h \setminus \alpha$		$1/s_1(1\,d^{-1})$	
m	10^{-2}	10^{-3}	10^{-4}
0.05	0.011	3.3×10^{-4}	1.1×10^{-5}
0.1	0.017	5.3×10^{-4}	1.7×10^{-5}
1	0.078	2.5×10^{-3}	7.8×10^{-5}
10	0.36	0.011	3.6×10^{-4}
100	1.7	0.053	1.7×10^{-3}

Note. n equals 0.022 (SI). Tables apply to a horizontal bed.

Walder's second conclusion that a downstream decrease in α can be accommodated by an increase in channel cross-sectional area S is, of course, normally true. However, one can easily construct examples where this is not true. For these cases, a solution is obtained if a water sheet exists over one or more reaches. Walder's conclusion is incorrect.

Regarding Walder's third point that I ignored, plastic closure and other effects in predicting the hydrograph, this matter was better dealt with by Shoemaker (1992b) in which an argument was given for crudely estimating the duration of a flood. No hydrographic analysis was made because I have concluded that a meaningful analysis cannot be produced. Not enough is known, for example, about the mega subglacial lake that feeds a mega flood (Shoemaker, 1991).

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The accuracy of references in the text and in this list is the responsibility of the author, to whom queries should be addressed.

SIR,

Comments on "Analysis of glacier facies using satellite techniques" by Williams and others

A scheme of zones (facies) for glaciers, suggested by Benson (1959, 1961) and Müller (1962), was intended to cover all zones (facies) of glaciers anywhere in the world. This included ablation (ice) facies, superimposed-ice zone, slush zone, soaked (wet-snow) facies, percolation facies and dry-snow facies.

A Landsat 5 TM image (24 August 1986) of Brúarjökull, an outlet glacier from the Vatnajökull ice cap, east Iceland, was the source of a study by Williams and others (1991), from which they arrived at a revised scheme for glacier facies from remotely sensed data. Figures 5 and 6 in that paper show that there is a distinct difference of reflectance between areas on Brúarjökull on the above-mentioned date but, in the absence of direct and simultaneous ground observation, most of their physical properties remain speculative.

Williams and others (1991) pointed out that "the upper limit of wet snow corresponds approximately with the 1300 m contour line". By this, they did not imply the wet-snow facies, but this could have been misunderstood. As stated in the paper, there is no dry-snow facies on Vatnajökull and percolation facies cannot be detected on Landsat images. Rist (1961) and Theodórsson (1970) measured the temperature in drillholes at 1614, 1730 and 2000 m a.s.l. on the Vatnajökull ice cap and found that the glacier is temperate. Accordingly, there can be no upper limit to the wet-snow facies. This has been confirmed by Icelandic glaciologists during the cooling that started about 30 years ago. Generally, there are only two facies present on temperate glaciers, namely, ice facies and wet-snow facies. The presence of a slush zone would necessarily have to be confirmed by ground observation, but in this case it is absent.

Williams and others (1991) found that the transient snow-line is located between 1000 and 1100 m a.s.l. for most of the glacier. This might conceivably be correct, as there is a sharp contrast in the image at that elevation but it seems rather low for an ordinary year. However, another reason for that contrast might be the thin crust of porous ice (up to 10 cm thick) that is commonly superimposed on glacier ice over large areas on bright days in summer (probably condensed moisture from the atmosphere), giving the glacier an almost white appearance. Another type of white crust termed "weathering crust" has been described by Müller and Keeler (1969). These two different types of crust can probably be differentiated from snow on some bands in the satellite image.

Williams and others (1991) claimed to have found a slush zone immediately above the intended transient snow-line bordered up-glacier by the slush limit. As temperate glaciers above the equilibrium line are all within the wet-snow zone, there is no fine-grained snow in late summer that can be saturated with water. Consequently, there is usually no slush on Icelandic glaciers in late summer. This agrees with the experience of the numerous "Jeep" enthusiasts who travel all over the glaciers at all seasons of the year. An exception might be