STEADY STATE NUMBER OF THE EXTINCT COMETS IN HIGH-INCLINATION ORBITS

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Abstract

Steady state number of the short-period(SP) comets captured from nearly parabolic(NP) orbits with $60° \leqslant i$(inclination)$\leqslant 120°$ and $0.5 \leqslant q$ (perihelion distance)$\leqslant 1.5$ AU is calculated. Due to smallness of the q and slowness of the capture process, almost all these SP comets become completely extinct. Combining annual flux of the observed NP comets of high inclination with the capture probability from NP to SP orbits and the ejection rate in SP orbits obtained by Monte Carlo simulations, we find that the steady state number of the extinct SP comets in high-inclination orbits is at least 1-2 hundred. If this number can be considered to correspond to the observed one of Apollo-type objects with $i \geqslant 60°$, it is concluded that only less than a few percent of extinct comets leaves sizable non-volatile cores or shells. The number of asteroid-like bodies deduced from the extinct comets with small perihelion distance and high inclination has less ambiguity than that of low inclination, because contribution from asteroid belt is negligible and the source comets are visible(free from observational selection) through the whole course of orbital evolution.

It is of primary importance to properly estimate the fraction of the comets which leave sizable core or shell of stony material after losing volatile components(extinct comets), both in order to determine the dominant source of Apollo-Amor(AA) objects and to get information on internal structure of cometary nuclei. Major contributions to this problem have been made by Öpik(1963), Wetherill(1979), Kresák(1980), Rickman and Froeschlé(1980), Levin and Simonenko(1981) and others.

The purpose of this paper is to estimate an upper limit of the fraction of asteroid-like survivors among extinct(EX) comets without ambiguity of the source as far as possible. We also intend to understand the population of EX comets within the framework of orbital evolution of long-period(LP) comets. For those purposes, we restrict our treatment to the LP comets with $0.5 \leqslant q \leqslant 1.5$ AU and $60° \leqslant i \leqslant 120°$. The reasons and advantage for this restriction are :

V. V. Markellos and Y. Kozai (eds.), Dynamical Trapping and Evolution in the Solar System, 97–104.

1) contribution from asteroid belt to high-inclination AA objects is negligible,
2) the effects of observational selection with respect to q are minimum among various LP comets,
3) physical disintegration is active and at a nearly constant rate during the whole course of orbital evolution, and
4) the ejection rate is comparatively small in spite of slowness of orbital change.

As is shown later, the observed number of the AA objects of high inclination is extremely small. This prevents us from defining observationally the orbital region of the EX comets of high inclination. Therefore we regarded somewhat arbitrarily the region of a(semi-major axis) $\lesssim 10$ AU, $0 < q \lesssim 1.5$ AU and $60° \lesssim i \lesssim 120°$ as the place where EX comets are expected to be found. This selection, however, is justified from the viewpoint of orbital evolution of LP comets(Nakamura 1981).

The steady state number of the EX comets is given by n/λ (e.g., Wetherill 1979), where n is the annual injection rate into this region and λ is the expulsion rate per comet per year by Jovian perturbations. Since the value of n cannot be known from observation, it must be estimated from n_0 the observed annual rate of Oort-cloud(NP) comets and the dynamical capture efficiency of NP comets via LP comets into short-period(SP) comets. n_0 was found to be 0.195 objects/yr, by picking up 14 objects(/135 yr) of the NP comets with $1/a_{orig} \leqslant 50 \times 10^{-6}$ AU^{-1}, $0.5 \leqslant q \leqslant 1.5$ AU and $60° \leqslant i \leqslant 120°$ from the list of 200 original orbits of LP comets (Marsden et al., 1978), and by correcting the number by a factor of 375/200, the ratio of the total number of the LP comets which appeared past 135 years to the 200 original LP comets.

The capture efficiency of the NP comets was calculated by a means of Monte Carlo simulation of the orbital evolution due to perturbations of Jupiter. The method adopted here is similar to that described in Nakamura(1981): orbital evolution is traced as a random walk in the 3-dimensional space of Kepler energy, total angular momentum and its z-component, where each step was chosen following the trivariate distributions of Jovian perturbations calculated beforehand. About 2.3 % of the comets which start from the inner Oort cloud region is found to reach the region of $a \lesssim 5.2$ AU after the mean revolution of nearly 8000. Then we have $n = 0.023 n_0 = 4.5 \times 10^{-3}$ objects/yr.

As for high-inclination comets, there are good reasons to believe that argument of perihelion ω may play an important role in their orbital evolution. This situation does not apply to low-inclination case. Geometrical considerations suggest that the LP comets of $q \lesssim 1$ AU with $\omega \sim 90°$ or $\sim 270°$ may not interact with Jupiter strongly enough to be brought into SP orbits. Dynamical considerations, on the other hand, indicate that secular perturbation on ω may gradually change its value, resulting in close approaches to Jupiter within an appropriate period of time. Only detailed numerical simulations will be able to decide which is true. However, an orbital statistics of high-inclination LP comets seems to support the former. Fig.1 represents the distributions

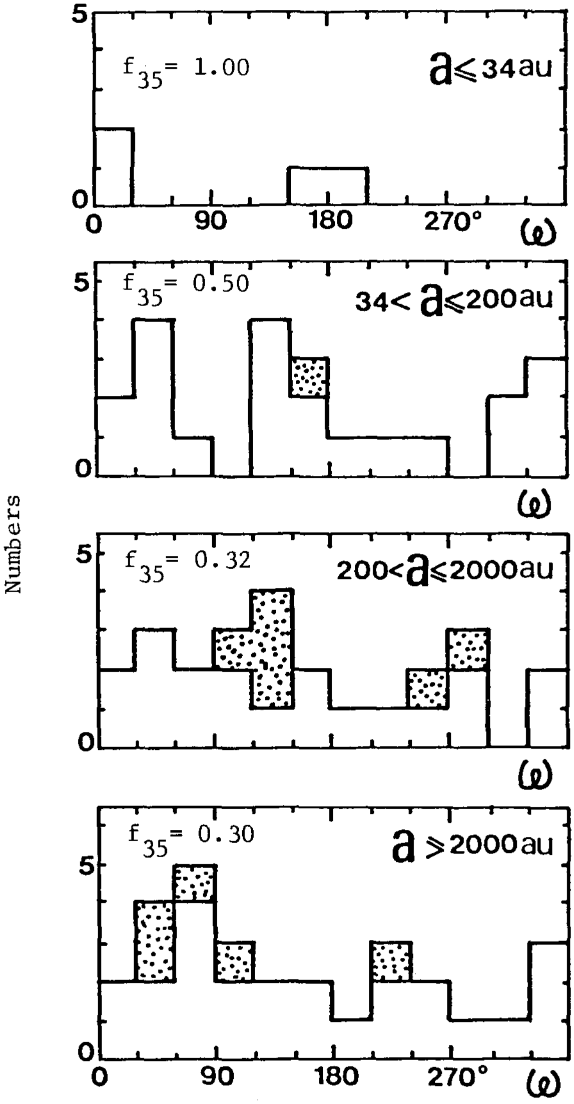

Fig.1. Distributions of argument of perihelion for LP comets with $60° \leq i \leq 120°$ and $0 < q \leq 2$ AU(dotted blocks are for $1.5 < q \leq 2.0$ AU).

of ω for the LP comets with $0 < q \leq 2$ AU and $60° \leq i \leq 120°$, in order of a_{orig}, which can be regarded as an indicator of degree of orbital evolution. The data are from Marsden(1979) and Marsden et al.(1978). It is clearly seen that along with the decrease of a_{orig}, the comets with ω ~90° or~270° are removed and finally only the comets with ω ~ 0° or ~180° are left. Adoption of a factor f_{35} will make the situation clearer. f_{35} is the ratio of the comets whose ω lie both between -35° and 35° and between 145° and 215° to all the comets considered. Fig.1 will be interpreted as: the NP comets other than those with ω ~ 0° and~180° cannot evolve into shorter-period ones by Jovian perturbations. Then, we should modify the value of n by f_{35}; namely, n = $4.5 \times 10^{-3} f_{35}$ = 1.75×10^{-3} objects/yr, in which a flat distribution of ω for the LP comets is assumed.

In Fig.2 are shown the mean paths of orbital evolution of the LP comets with high inclination. On an average the increases of i were less than 20-30°.

N	100	500	1000	2000	5000	(rev)
$\bar{a}$	156	56	34	21	11	(AU)

This is a relation of mean semi-major axis($\bar{a}$) versus revolution(N) for the initial q of 1 AU.

According to sublimation theory of H_2O ice nucleus, about 1000 revolutions are a typical lifetime for a nucleus of 1 km radius and for q = 1 AU(Cowan and A'Hearn 1979, Weissman 1980). This figure will be slightly reduced if the effects of outburst and splitting are taken into account. Since the average diameter of LP comets is estimated to be 5 km or so(Whipple 1978, Rahe 1981), it is understood that, in combination with the result of the above table, most of the LP comets considered here will become completely extinct due to physical disintegration before they reach the orbital region of EX comets. This seems to be also the reason why we do not observe SP comets of high inclination (Fig.2).

Next we will discuss briefly non-gravitational(NG) effects on orbital elements. Table 1 is the net orbital changes per revolution due to jet reaction caused by sublimating gases on a nucleus for various q. These values were obtained by integrating numerically Lagrange equations of Gauss's form along parabolic orbits. $1/r^3 \cdot \exp(-r^2/2)$ is assumed as dependency of reactive force on heliocentric distance (Marsden et al., 1973). In this calculation $-0.98 \times 10^{-8} AU^4/day^2$ (retrograde rotation) is assigned to A_2 the tangential component of NG parameters, which is an average of the absolute values of nine A_2 of the LP comets tabulated in Marsden et al.(1973). Δ(1/a) in Table 1 shows that if NG effects act every return constantly in one sense --- this is no doubt improbable, it takes more than 5000 revolutions for a NP comet of q = 1 AU to

q(AU)	$\Delta(1/a)$ (AU^{-1})	Δ e	Δ q (AU)
0.2	1.91×10^{-3}	-3.82×10^{-4}	-3.48×10^{-5}
0.5	2.56 10^{-4}	-1.28	-1.85
0.7	1.09	-7.6 10^{-5}	-1.21
1.0	3.74 10^{-5}	-3.7	-6.2 10^{-6}
1.5	7.5 10^{-6}	-1.1	-1.8
2.0	1.5	-3.0 10^{-6}	-4.5 10^{-7}

Table 1.

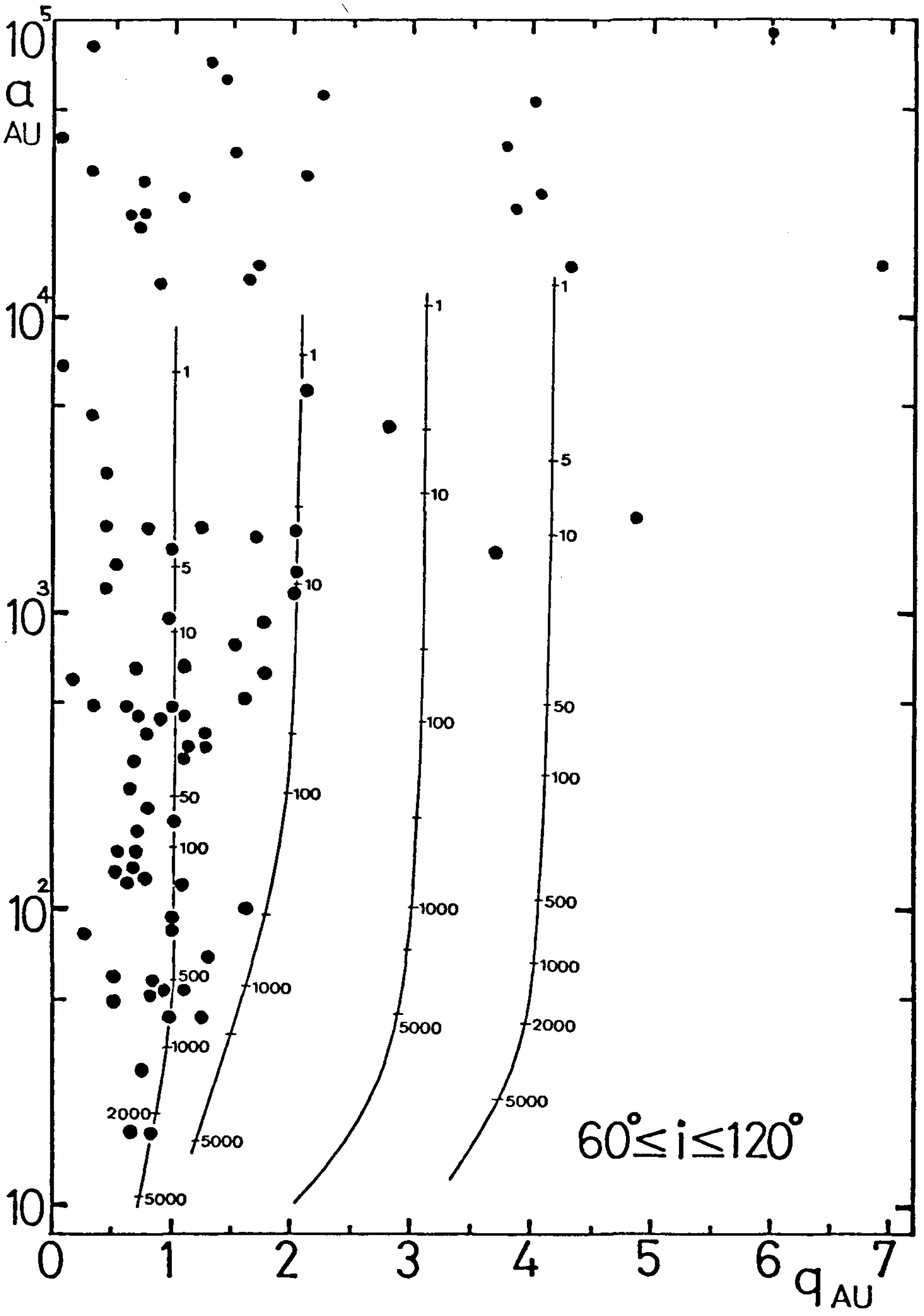

Fig.2. Orbital evolution of high-inclination LP comets. Figures attached to each curve represent mean revolutions. Filled circles are the observed comets(Marsden et al., 1978, Marsden 1979).

reach near the orbit of Jupiter. This suggests that inclusion of NG effects on 1/a does not change sharply the lifetime obtained in consideration of orbital evolution and physical disintegration.

The calculation of λ was carried out as follows. Many hypothetical EX comets were first scattered uniformly in the domain of $5.5 \leqslant a \leqslant 8.1$ AU, $0.5 \leqslant q \leqslant 1.5$ AU and $60° \leqslant i \leqslant 120°$ and their subsequent orbital evolution was traced by another Monte Carlo simulation. λ (/yr/comet) is expressed as

$$\lambda = -d\nu \;/\; \nu \;/(N_2 - N_1)/\; \overline{P},$$

where ν is the initial number of EX comets, dν the number ejected from the region of $0.5 \leqslant q \leqslant 1.5$ AU between N_2-th and N_1-th returns, and $\overline{P}$ the mean orbital period(yr) of EX comets. For $\nu = 210$, $N_2 - N_1 = 100$ (this simulation was traced up to 500 revolutions) and $\overline{P} = 17$ yrs, an averaged dν was 3.6; thus $\lambda = 1.0 \times 10^{-5}$/yr. This value might be a considerable overestimate, since the bodies once ejected from the region of $0.5 \leqslant q \leqslant 1.5$ AU are implicitly assumed to never come back to the same region. The loss rate due to planetary collision is shown to be about 10^4 times smaller than this λ (Wetherill 1979). Therefore, using the obtained values of n and λ, a steady state number of EX comets of 175 immediately results for the orbital range assigned above.

Now we are in a position to deduce the fraction of asteroidal survivors by comparing with the observed number of AA objects of high inclination. As the initial i of LP comets are limited to more than 60° and the orbital evolution has a trend to increase i, it would be then reasonable to regard the Apollos with $i \geqslant 60°$ as the candidates. Only two of such Apollos have been known so far:

No.	q(AU)	Q(AU)	i	ω
1973NA	0.88	3.98	68°	118°
1975YA	0.91	1.69	64	61 .

If we receive this number at its face value, the fraction of asteroidal survivors is about 1 %. However, as for 1975YA, it seems improbable that this is an EX comet, because its aphelion distance Q is too small to be accounted for by jet effects.

Attention must be called to the point that discovery condition of high-inclination bodies may be different from that of low inclination. Intuition predicts that the relative speed of high-inclination bodies to the earth will be larger than that of low inclination. Fig. 3 is a magnitude-observed daily motion diagram for the asteroids designated as "fast-moving objects" near their discovery in IAU Circulars past ten years. This diagram indicates that the intuition is correct to some extent, though exceptional cases are not rare. Large relative speed leads to failure of detection of faint objects, resulting in the raise of their limiting magnitude. We assume the size distribution of $dn \propto D^{-b}dD$ (D: asteroid's diameter and $b = 3 - 3.5$ for typical asteroids) and that the number density of silver grains in an asteroidal

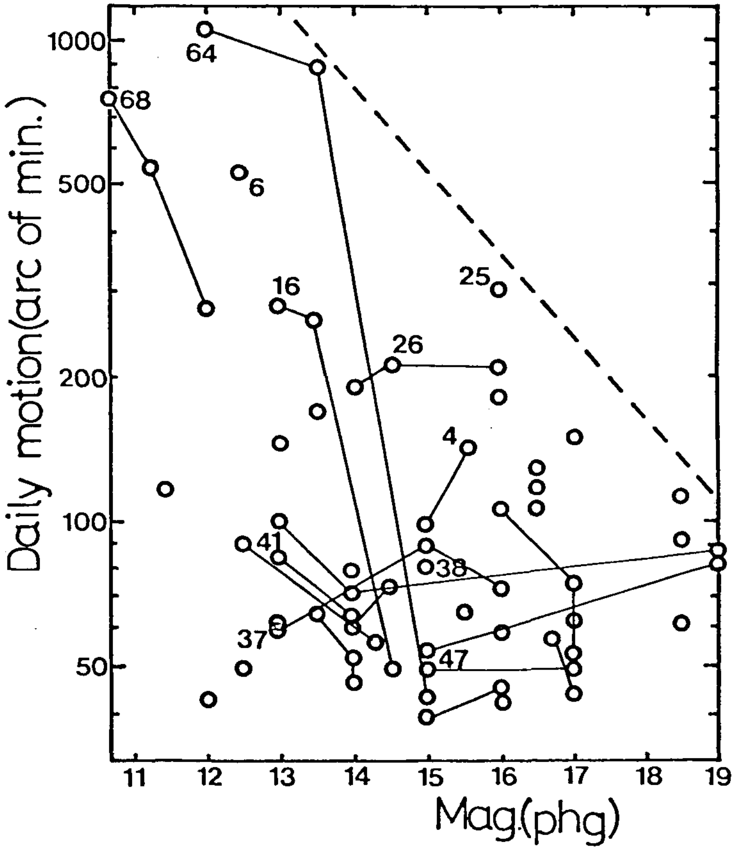

Fig.3. Observed daily motions of "fast-moving objects" near their discovery(1972-81) plotted against apparent magnitude. Attached numbers are inclinations. The circles connected by straight lines belong to the same object.

image on a photographic plate is proportional to 1/v (v: mean relative speed of high-inclination objects measured in the unit of low-inclination ones'). Unobservable number N_u due to large relative speed and still observable number N_v are given respectively as follows,

$$N_u \propto (\sqrt{v}\, D_L)^{1-b} - D_L^{1-b},$$

$$N_v \propto D_{max}^{1-b} - (\sqrt{v}\, D_L)^{1-b},$$

in which D_L is the diameter corresponding to the limiting magnitude for low-inclination bodies and D_{max} is the diameter of the brightest bodies. Simple calculation brings about a result that the ratio of N_u to N_v is roughly equal to v - 1. This expression holds exactly for the case b = 3.0.

Mean observed daily motions deduced from the data of Fig.3 for high- and low-inclination Apollos are 435' and 91' respectively; thus v = 4.8. Then the number of high-inclination Apollos corrected for this selection effect will be 7.6 or so. In this case, the fraction of asteroidal survivors is 4.3 %. Even if ambiguity of the adopted constants involved in our analysis is taken into account, the resultant fraction will be raised by a factor of two at most, whereas it is very probable that the fraction may be lowered, say, by one order of magnitude. In conclusion, the fraction of the cometary nuclei which survive as sizable asteroid-like bodies is less than 2-8 %. This result coincides well with the estimate by Rickman and Froeschlé(1980), though quite different data are analysed.

At present the origin of SP comets is understood, at least qualitatively, as a result of orbital evolution of NP or LP comets(Everhart, 1972). If the existence of the comet Encke is a probable event among SP comets and the fraction obtained above is also applicable to low-inclination comets, the number of Apollo objects deduced from Encke-type comets will have to be reduced by the same fraction. As far as the problem of Apollo objects is considered in the framework of orbital evolution of LP comets, this seems to be an inevitable outcome. The origin of AA objects is obviously still an unsettled problem.

REFERENCES

Cowan,J.J. and A'Hearn,M.F.: 1979, The Moon and the Planets, 21, 155.
Everhart,E.: 1972, Astrophys. Letter, 10, 131.
Kresák,L.: 1980, The Moon and the Planets, 22, 83.
Levin,B.J. and Simonenko,A.N.: 1981, Icarus, 47, 487.
Marsden,B.G., Sekanina,Z. and Yeomans,D.K.: 1973, Astron. J., 78, 211.
Marsden,B.G., Sekanina,Z. and Everhart,E.: 1978, Astron. J., 83, 64.
Marsden,B.G.: Catalogue of Cometary Orbits, 3rd Ed.(1979), Central Bureau of Astronomical Telegram, IAU.
Nakamura,T.: 1981, Icarus, 45, 529.
Öpik,E.J.: 1963, Advances Astron. Astrophys., 2, 219.
Rahe,J.: Landoldt-Bornstein, New Series, Vol.2, Subvol.a, Methods, Contants, Solar System(Ed. by K. Schaifers and H.H. Voight), Chap.3, Springer-Verlag, Berlin(1981).
Rickman,H. and Froeschlé,C.: 1980, The Moon and the Planets, 22, 125.
Weissman,P.R.: 1980, Astron. Astrophys., 85, 191.
Wetherill,G.W.: 1979, Icarus, 37, 96.
Whipple,F.L.: Comets, in Cosmic Dust(Ed. by J.A.M. McDonnell), Chap.1, John Wiley & Sons, New York(1978).