DRIVING STRESSES IN THE ANTARCTIC ICE SHEET

by

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ABSTRACT

Radio echo-soundings obtained during joint programmes of the US National Science Foundation (NSF), the Scott Polar Research Institute (SPRI), and the Technical University of Denmark (TUD) have provided sufficient data of good quality to map driving stress over approximately one half of the Antarctic ice sheet. Computerized averaging of data over 1 degree of latitude squares has largely eliminated local effects of longitudinal stress variations on surface slope. Methods of data handling are outlined. Variations in driving stresses between different regions are discussed in relation to the balancing restraining forces. These are primarily related to ice velocity and thickness, bedrock topography, the presence of basal water and possibly to the development of easy-glide fabric in the ice mass, to the temperature of basal ice layers, and to the excess hydrostatic pressure in areas where ice rests on bedrock well below sea-level.

SOURCES OF DATA AND METHOD OF COMPUTATION

Sources of data

Three sources of data are used in this study. (1) Data from radio echo-sounding (RES) studies, controlled by inertial navigation and carried out by SPRI since 1971 under the joint NSF-SPRI-TUD programme. This provided the bulk of the data used. A 50 km grid of flight lines was flown over most of the area, with good bedrock coverage. (2) Selected data obtained by seismic sounding on over-snow traverses. (3) Data from Tropical Wind, Energy Conversion and Reference Level Experiment (TWERLE) balloons (Levanon and others 1977) on surface elevations which gave additional coverage, enabling estimates of surface slope to be made in areas where RES data are limited.

Method of computation

The term "driving stress" rather than basal shear stress has been used to indicate that we are mapping the mean driving stresses which are in balance with variable basal shear stresses and longitudinal stress gradients. Robin (1967) and Budd (1968) suggest that longitudinal stress gradients are unimportant when smoothed over great distances, i.e. above 10 or 20 times the ice thickness. The area under consideration was divided into an overlapping set of rectangles, each being 1 degree of latitude square (111 km x 111 km) with 25 km displacement between the centres of successive rectangles. The surface elevation data within each rectangle were used to define a plane, using a multiple linear regression technique. The maximum slope and direction of dip were then calculated from the equation of the plane. The mean of the thickness measurements within the rectangle was taken, or, if none was available for

that rectangle, the last computed average thickness was used. The driving stress was then calculated according to the equation

$$\tau = \rho.g.h. \sin(\alpha), \qquad (1)$$

where τ is the driving stress, ρ the density of ice, g the acceleration due to gravity, h the thickness of the ice, and α the surface slope. Contour maps of the driving stress were then drawn, using a simple contour plotting program, followed by minor smoothing (Figs.1 and 2). Arrows indicating the direction of maximum slope were also derived by the program, allowing the compilation of flow-line maps, thus providing a check on the consistency of the surface slope. A consequence of the grid size is that the plotted data stop 50 km from the coast. Furthermore, some detail is lost for the West Antarctic ice streams.

Errors

Surface elevations of the ice sheet are the main source of error. In certain areas, notably northern Victoria Land, frequent changes of aircraft altitude caused significant errors. Results from these areas have not been included in Figures 1 and 2. The error in the measurement of the surface elevation is ±30 m (Drewry 1975) for RES measurements, and about ±60 m for TWERLE data (Levanon and others 1977). For an error of +30 m on one side of a grid square, and -30 m on the other, the error would be ±5 kPa (0.05 bar) 1 000 m^{-1} of ice thickness. Irregularities in the plotted values and the flow direction suggest that the error is usually less than 10 kPa (0.1 bar). Contours in Figures 1 and 2 therefore indicate real variations of driving stress.

ANALYSIS AND DISCUSSION

A number of patterns may be identified from Figures 1 and 2. Reference to surface and bedrock contour maps in Drewry and others (1981), Jankowski and Drewry (1981), and Drewry and Steed (in press), will assist. These patterns provide more detailed evidence for testing theoretical concepts than earlier studies (Budd and others 1971, Rose 1978). The most important trend is the increase in driving stresses from the ice divides (<20 kPa (0.2 bar)) to the ice-sheet margins (>130 kPa (1.3 bar)) except where these are bordered by major ice shelves. This trend can be seen in both East and West Antarctica, although there are two important differences between these areas; firstly, the rate of increase with distance in West Antarctica is notably greater than in East Antarctica, and, secondly, the highest values in West Antarctica are consistently less than those in East Antarctica. The pattern of driving stresses
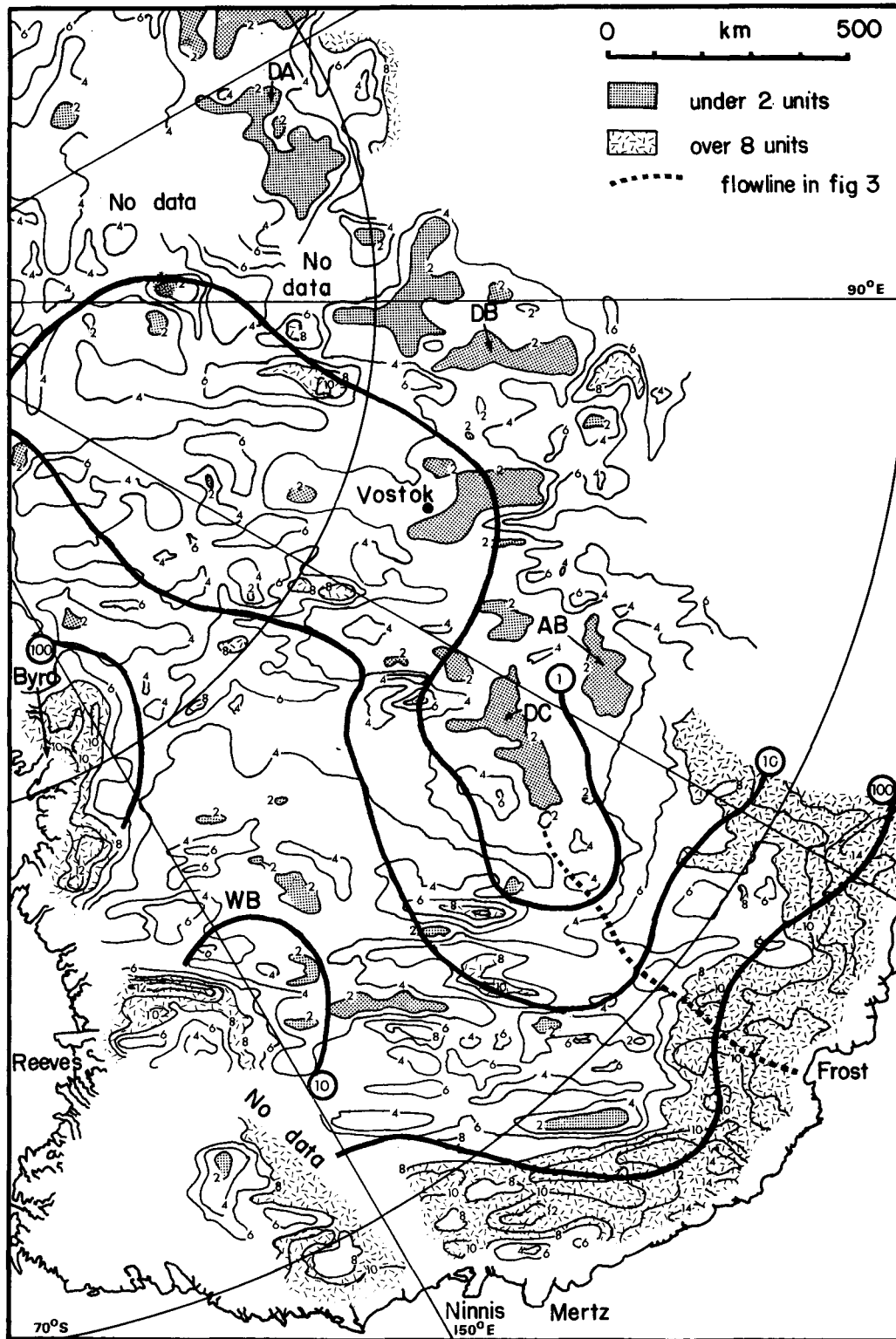
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Fig.1. Driving stresses (thin contours) and mean shear strain-rates (bold contours) in East Antarctica. Units at 10 kPa (0.1 bar) and 1.0×10^{-4} a^{-1} respectively. Driving stresses under 20 kPa (0.2 bar) are stippled and those over 80 kPa (0.8 bar) are hatched. DA: Dome A, DB: Dome B, DC: Dome C, WB: Wilkes basin, AB: "Aurora" basin.
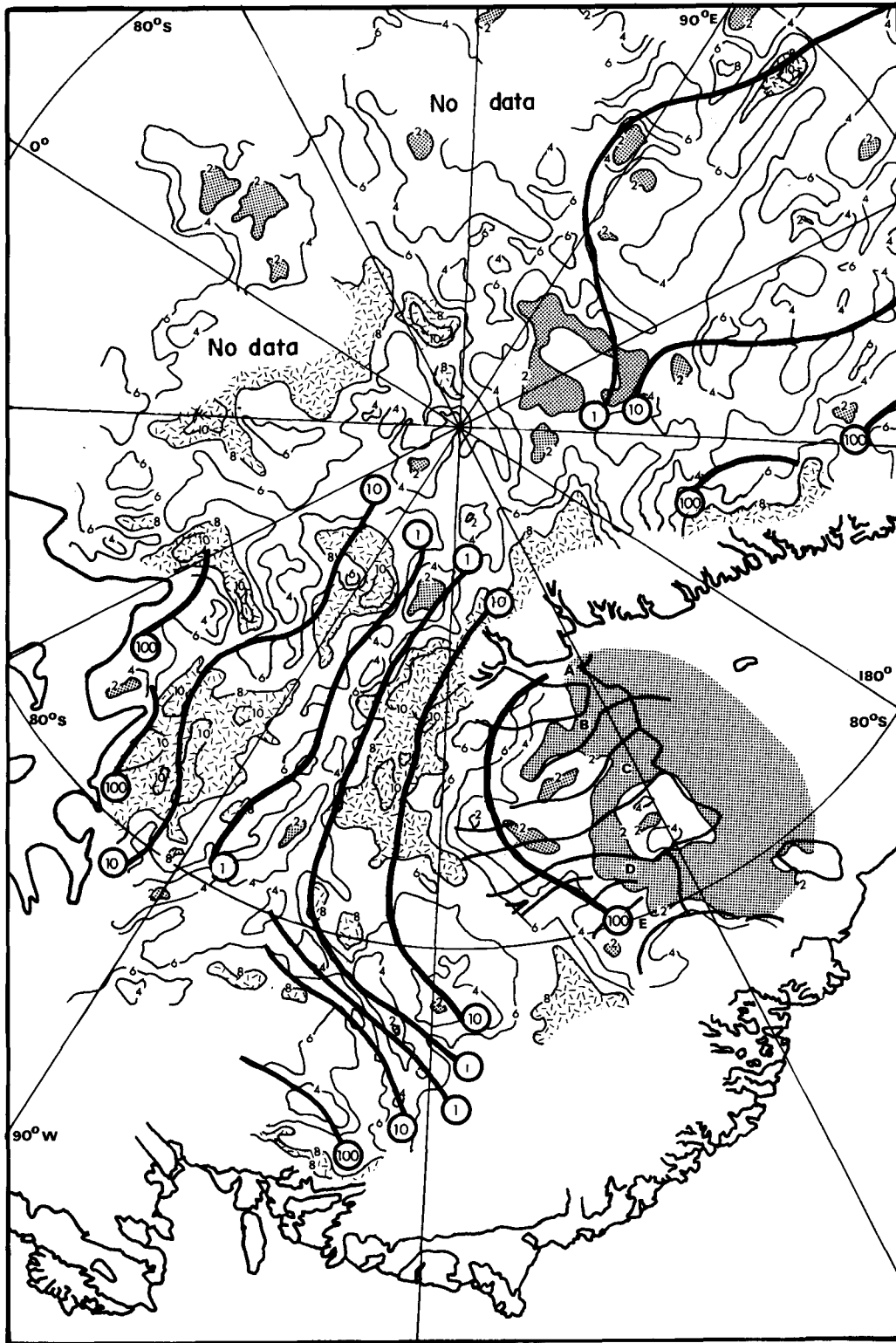
Fig.2. Driving stresses and mean shear strain-rates in West Antarctica. Key as for Figure 1. A,B,C,D, and E are ice streams flowing into the Ross Ice Shelf.
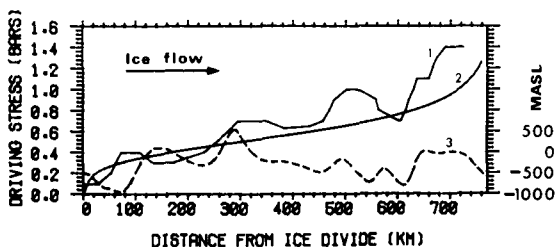
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Fig.3. Calculated (1) and theoretical (2) (Vialov 1958) profiles for driving stress between Dome C and Frost Glacier (see Fig.1). Bedrock elevations (m a.s.l.) (3) are also shown.

Fig.4(a). Profiles of driving stresses down outlet glaciers in Figure 1.

Fig.4(b). Profiles of driving stresses down ice streams B,C,D, and E of Figure 2.

along one flow line in Wilkes Land is shown in Figure 3 and is compared with that for a theoretical profile of the ice surface on flat bedrock with deformation by internal shear, which gives the surface profile

$$(h/H)^{2 \cdot 6} + (x/L)^{1 \cdot 3} = 1 \qquad (2)$$

(Vialov 1958, Paterson 1981), where h is the ice thickness at any point, H the maximum ice thickness, x the distance from the ice divide, and L the distance from the ice divide to the ice margin. It is clear that bedrock irregularities of a few tens of kilometres in extent produce long wavelength variations in driving stress.

At a smaller scale, it is possible to associate characteristic values with certain glaciological circumstances. Thus, low driving stresses (typically less than 20 kPa (0.2 bar)) are often found in the following five situations: (1) at Domes A, B, and C (Fig.1), and around ice divides (especially Fig.2) (although ice velocities are at their lowest here, and are expected to fall to zero if the ice divide has no slope, smoothing has eliminated zero values), (2) in subglacial basins; this can be seen in the Wilkes basin and the tentatively named "Aurora" subglacial basin (69°S, 137°E), (3) where subglacial "lakes" (Oswald and Robin 1973) of large extent are present, (4) at the seaward ends of the West Antarctic ice streams (Fig.4(b)), and (5) in small areas where effects of longitudinal stress gradients have not been entirely eliminated as discussed below.

High driving stresses of limited extent (>100 kPa (1 bar)) are found mainly in two situations: firstly, in crescentic zones around the heads of outlet glaciers, that is, in areas of great convergence of flow. This can be seen around Frost and Byrd glaciers and inland of the West Antarctic ice streams; here the proximity of the catchment areas has caused the coalescence of these crescentic high zones. Secondly, we find isolated patches with a high stress. The occurrence of alternate bands of high and low stress oriented across the flow-line direction between 130°E and 150°E north of 75°S (Fig.1), and in Figure 3, suggests that longitudinal stress gradients caused by bedrock irregularities have not been fully smoothed out by the averaging process, and that such stresses can be transmitted over distances of 50 to 100 km, a somewhat greater distance than suggested earlier.

Single RES flights down flow lines of individual outlet glaciers have also been used to show the patterns of driving stresses. Data have been averaged using a 50 km window on these lines. The resulting profiles down glaciers and ice streams can be divided into two types. Most outlet glaciers show increasing driving stresses to 150 km or so from the grounding line. At this point, there is a further increase to a peak around 50 km from the grounding line before it falls by about 50 kPa (0.5 bar) over the next 25 km (Fig.4(a)). Glaciers showing this pattern include Reeves, Byrd, Ninnis, and Frost. This is very similar
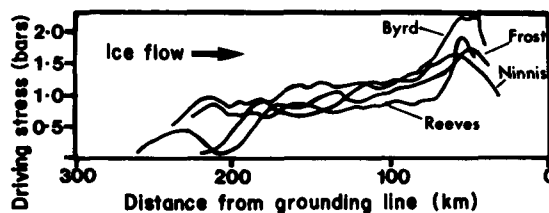
to that found by Budd and Young (1979) along the International Antarctic Glaciological Project (IAGP) flow line in Wilkes Land. Ice entering the West Antarctic ice streams shows increasing driving stresses with distance until some 400 to 550 km from the grounding line, after which the value falls more gradually than in East Antarctica from a peak of around 100 kPa (1.0 bar) to 20 kPa (0.2 bar) around 150 km (Fig.4(b)).

The most important parameters influencing the patterns of driving stress appear to be ice velocity and thickness (Nye 1957, Weertman 1957, Budd and Radok 1971, Budd and Jenssen 1975, Budd and Smith 1981, Young 1981). Mass flux must increase from zero at centres of outflow to high values near the coast; highest values will be found where ice is channelled into ice streams and outlet glaciers. In the absence of appreciable sliding, and if longitudinal stress gradients have been effectively smoothed out, we expect surface velocity to be proportional to some power of the driving stresses and to thickness: thus

$$V = k.h.\tau^n$$

(Budd and Radok 1971), and hence

$$V/h = k.\tau^n, \qquad (3)$$

where V is the ice velocity, and k and n are constants. In addition to driving stresses, Figures 1 and 2 show contours of V/h derived from balance velocities and thickness, a ratio that is proportional to the mean shear strain-rate through the ice thickness. Balance velocities were calculated from the flow lines previously obtained (see above) and from accumulation and thickness data being compiled for the Antarctic glaciological and geophysical folio (Drewry in preparation). The pattern of increase of V/h from the ice divides to the continental margins is broadly similar to some power of the driving stress.

However, a zone of low values of V/h in the Wilkes basin is apparent. The 1.0×10^{-4} a^{-1} contour encircles the low driving stresses of the ice divides, where velocities are also at their lowest, while the 100×10^{-4} a^{-1} contour is generally associated with driving stresses of over 80 kPa (0.8 bar) and with the zones of convergence of flow and high stresses around the heads of outlet glaciers. Values of n and k in Equation (3) of 3.45 and 0.02225 (Budd and Smith 1981) and of 2.50 and 0.08555 (Budd and Jenssen 1975) have been suggested for polar ice. Plotting balance

velocities calculated from data from Wilkes Land
against driving stresses allows an extension of the
relationship down to values of 1.0×10^{-4} a^{-1} for
V/h, which gives n = 2.51 and k = 0.0085 (Fig.5).
Although the scatter for these data is greater than
for other sources (the coefficient of correlation
R = 0.810, c.f. R = 0.936 for Budd and Smith 1981),
the data seem sufficiently consistent to draw the
following conclusions. (1) Although our data in
Figure 5 refer to balance velocities, like Budd and
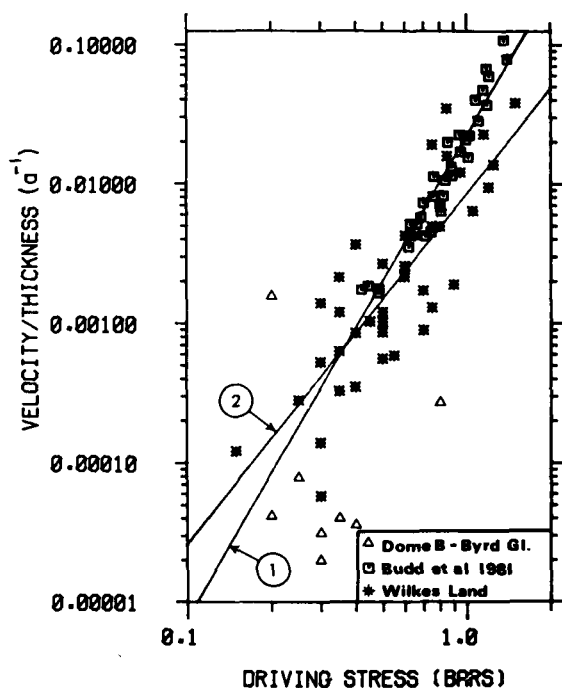Smith (1981) we find a closer relationship between

Fig.5. Variations of mean shear strain-rate V/h with
driving stress. Data are from Budd and Smith (1981)
(regression line 1), Wilkes Land (regression
line 2), and also show points from a Dome C-Byrd
Glacier profile.

V/h and driving stress than between V and driving
stress. From this we conclude that the major part of
surface motion is due to internal shear within the
ice mass, not to sliding. (2) The agreement shown
in Figure 5 between our steady-state values of V/h
and with the observed relationship for Equation (3)
of Budd and Smith (1981) indicates that balance
velocities are correct to a first approximation.
(3) The above agreement also indicates that driving
stresses are not closely related to ice temperatures
over much of the ice sheet. If the temperature of
basal layers were all at pressure-melting point, this
could explain the observation. However, it seems
clear from studies of temperature-depth relationships
that many parts of the Antarctic ice sheet with basal
temperatures below pressure-melting point still fall
on the V/h-driving stress plot. This could be due to
the development of easy-glide fabric in the ice com-
pensating for the colder temperatures that would
otherwise require greater driving stresses.

It should be noted that points for the flow line
between Dome B and Byrd Glacier on Figure 5 show low
values of V/h corresponding to higher values of driv-
ing stress than those predicted by the regression
lines. Values of driving stress on Figure 1, near
Dome A, suggest that the effect may be still more
marked there. In these regions, basal as well as
surface temperatures will be low at many points and
require high driving stress. Also, near centres of

outflow, crystal fabrics favouring easy glide may not
be fully developed.

Another obvious factor related to driving stress
is the effect of basal water. On the macroscopic
scale, this is readily shown at the sub-ice lake near
Vostok described in Robin and others (1977). This
lake is estimated to be ~150 km x 50 km, with a few
grounding points of limited extent only. We see in
Figure 1 that values of driving stress fall to under
0.2 bar over the region of the lake (77.5°S, 104°E).
Other sub-ice lakes described in that paper are small
in extent compared to the smoothing used for analysis,
so while they may cause some of the abnormally low
values shown in individual squares, they will not
generally have a major effect on driving stress.

Sub-ice water may also have an important effect
under major ice streams and outlet glaciers, where
low driving stresses are explained by the presence of
subglacial water of considerable extent. Owing to its
depth below sea-level, this is at an even greater
hydrostatic pressure than elsewhere, and so reduces
bottom drag by carrying a high proportion of the
weight of the overlying ice.

On both the macroscopic and smaller scales,
bedrock roughness generally appears to have little
effect on driving stress, presumably because most
deformation in the ice mass is taking place above
the level of bedrock relief (Robin and Millar 1982).
On the smaller scale, longitudinal and basal shear-
stress gradients in the flow direction do have a
large effect on local surface slopes, although not on
ice velocity. There are, however, certain areas, such
as the northern parts of the Wilkes and "Aurora" sub-
glacial basins where the driving stress appears
abnormally low. While the great ice thickness in
these basins will cause some lowering of velocity for
the same steady-state mass flux, some additional
effect, such as a smooth bedrock, may be necessary to
explain the low driving stresses observed since there
are no obvious signs of large subglacial lakes.

The geometry of basins leading to major outlet
glaciers shows a common pattern of distribution of
stress. The region of highest stress lies on a rough
arc centred on the head of the outlet glacier. This
is shown most clearly for the largest glaciers, such
as Byrd and Frost, but our grid size is too large to
see the effect on smaller glaciers, although contour
maps suggest that it is present. However it does
appear that the larger the outlet glacier, (that is,
the larger the mass flux involved), the larger is the
radius of the arc of highest stress. This suggests
that this radius will vary in response to changes of
mass flux through an individual outlet glacier. Such
changes should be taken into account when modelling
the flow and possible instabilities of polar ice
sheets.

The observed gradual fall of driving stress from
its maximum value to the grounding line on ice
streams shown in Figure 4(b) should also be noted.
The peak driving stress is again similar to that
reached before a surge of a temperate glacier, but
this could be a stable feature in the ice streams,
as it is repeated in four cases. This may indicate
that the sliding of cold polar glaciers with the
base at melting point differs from the behaviour of
temperate glaciers approaching a critical sliding
velocity.

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