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NOTE ABOUT LINDELÖF Σ-SPACES υX

J. KĄKOL and M. LÓPEZ-PELLICER ✉

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Dedicated to the Memory of Professor Klaus D. Bierstedt

Abstract

The paper deals with the following problem: characterize Tichonov spaces X whose realcompactification υX is a Lindelöf Σ-space. There are many situations (both in topology and functional analysis) where Lindelöf Σ (even K-analytic) spaces υX appear. For example, if E is a locally convex space in the class $\mathfrak{G}$ in sense of Cascales and Orihuela ($\mathfrak{G}$ includes among others (LM)-spaces and (DF)-spaces), then $\upsilon(E', \sigma(E', E))$ is K-analytic and E is web-bounded. This provides a general fact (due to Cascales–Kakol–Saxon): if $E \in \mathfrak{G}$, then $\sigma(E', E)$ is K-analytic if and only if $\sigma(E', E)$ is Lindelöf. We prove a corresponding result for spaces $C_p(X)$ of continuous real-valued maps on X endowed with the pointwise topology: υX is a Lindelöf Σ-space if and only if X is strongly web-bounding if and only if $C_p(X)$ is web-bounded. Hence the *weak** dual of $C_p(X)$ is a Lindelöf Σ-space if and only if $C_p(X)$ is web-bounded and has countable tightness. Applications are provided. For example, every $E \in \mathfrak{G}$ is covered by a family $\{A_\alpha : \alpha \in \Omega\}$ of bounded sets for some nonempty set $\Omega \subset \mathbb{N}^{\mathbb{N}}$.

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1. Introduction

There are many situations where one should decide if the realcompactification υX of a (Tichonov) space X is a Lindelöf Σ-space (or K-analytic); see [1, 7] for references. For separable X the space υX is Lindelöf if and only if every base in X is complete [2]. When exactly is υX a Lindelöf Σ-space? The following, see [1, 13], shows the link between Lindelöf Σ-spaces υX and envelopes Z of spaces $C_p(X)$.

PROPOSITION 1.1. *The space υX is a Lindelöf Σ-space if and only if there exists a Lindelöf Σ-space Z such that $C_p(X) \subset Z \subset \mathbb{R}^X$.*

A Tichonov space X is called a *Lindelöf Σ-space* if there is an upper semicontinuous compact-valued map from a nonempty subset $\Omega \subset \mathbb{N}^{\mathbb{N}}$ covering X; see [1, 12]. If the

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same holds for $\Omega = \mathbb{N}^{\mathbb{N}}$, then X is called *K-analytic*. The space X is *quasi-Suslin* if there exists a set-valued map T from $\mathbb{N}^{\mathbb{N}}$ into X covering X which is quasi-Suslin, that is, if $\alpha_n \to \alpha$ in $\mathbb{N}^{\mathbb{N}}$ and $x_n \in T(\alpha_n)$, then $(x_n)_n$ has a cluster point in $T(\alpha)$; see [15]. Note that a space is K-analytic if and only if it is Lindelöf and quasi-Suslin, and also the fact that K-analytic implies Lindelöf Σ.

Suppose we have a family $\{A_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ of (compact) subsets of X covering X such that $A_\alpha \subset A_\beta$ if $\alpha \leqslant \beta$; such a family is called a (compact) resolution on X. A space X which has a compact resolution is quasi-Suslin by [3, Proposition 1]. Then its realcompactification υX is K-analytic. Indeed, if $T : \alpha \mapsto T(\alpha)$ is a quasi-Suslin set-valued map on $\mathbb{N}^{\mathbb{N}}$, every $T(\alpha)$ is countably compact, so its closure $\overline{T(\alpha)}$ in υX is compact. Then $\alpha \mapsto \overline{T(\alpha)}$ is upper semicontinuous, so $Z := \bigcup \overline{T(\alpha)}$ is K-analytic. Since $X \subset Z \subset \upsilon X$, then $Z = \upsilon Z = \upsilon X$ is K-analytic. Thus every Lindelöf space with a compact resolution is K-analytic. A resolution $\{A_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ in a locally convex space is a *bounded resolution* if all A_α are bounded.

A locally convex space E belongs to the class $\mathfrak{G}$ if there is a resolution $\{A_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ of subsets in $(E', \sigma(E', E))$ such that each sequence in any A_α is equicontinuous; see [6]. All (LM)-spaces (hence metrizable locally convex spaces), dual metric spaces (hence (DF)-spaces), the space of distributions $D'(\Omega)$ and real analytic functions $A(\Omega)$ for open $\Omega \subset \mathbb{R}^{\mathbb{N}}$, belong to the class $\mathfrak{G}$; see [6, 10].

In [10] we proved that the *weak** dual $(E', \sigma(E', E))$ of a locally convex space E in $\mathfrak{G}$ is quasi-Suslin. Hence (by the above argument) we have the following general property.

(∗) If a locally convex space E belongs to the class $\mathfrak{G}$ then $\upsilon(E', \sigma(E', E))$ is K-analytic.

This provides an alternative approach to the result from [4] stating that for any locally convex space E in the class $\mathfrak{G}$, the space $(E', \sigma(E', E))$ is K-analytic if and only if $(E', \sigma(E', E))$ is realcompact if and only if $(E, \sigma(E, E'))$ has countable tightness. Consequently, the *weak** dual of an (LF)-space is K-analytic.

In this note, motivated by these facts about the class $\mathfrak{G}$, we prove the following theorem.

Theorem 1.2. *For a Tichonov space X the following are equivalent.*

(i) *υX is a Lindelöf Σ-space.*
(ii) *X is strongly web-bounding.*
(iii) *$C_p(X)$ is web-bounded.*
(iv) *$L_p(X)$, the weak* dual of $C_p(X)$, is web-bounded.*
(v) *$C_p(X)$ is a dense subspace of a locally convex space which is a Lindelöf Σ-space.*
(vi) *$L_p(\upsilon X)$ is a Lindelöf Σ-space.*

At the first glance, Theorem 1.2 looks somewhat technical but it covers many concrete cases. Theorem 1.2 contains the equivalence between (i) and (ii) in [8, Theorem 10] and can be applied to describe the following general property for any locally convex space E in the class $\mathfrak{G}$.

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Corollary 1.3. *If E is a locally convex space in the class $\mathfrak{G}$, then E is web-bounded. Consequently, E is covered by a family of bounded sets $\{A_\alpha : \alpha \in \Omega\}$ for some nonempty $\Omega \subset \mathbb{N}^{\mathbb{N}}$.*

On the other hand, for any uncountable discrete space X the space $C_p(X) = \mathbb{R}^X$ is not in the class $\mathfrak{G}$ by [5] and does not admit a bounded resolution by [11, Corollary 1] as a nonmetrizable Baire locally convex space. In fact a Baire locally convex space is web-bounded if and only if it is metrizable; see [8] or [11, Theorem 1].

A space (respectively a locally convex space) E is *strongly web-bounding* [14] (respectively *web-bounded* [3]) if there is a family $\{A_\alpha : \alpha \in \Omega\}$ of sets covering E (for some nonempty $\Omega \subset \mathbb{N}^{\mathbb{N}}$) such that if $\alpha = (n_k) \in \Omega$ and $x_k \in C_{n_1,n_2,\ldots,n_k} := \bigcup\{A_\beta : \beta = (m_k) \in \Omega, m_j = n_j, j = 1, \ldots, k\}$, then $(x_k)_k$ is functionally bounded (respectively bounded). Clearly if E is web-bounded, then the sets A_α are bounded. It is easy to see that a locally convex space E with a bounded resolution is web-bounded. A cosmic space X is σ-compact if and only if $C_p(X)$ has a bounded resolution [9]. A web-bounded space $C_p(X)$ is angelic. Indeed, by Theorem 1.2 the space υX is a Lindelöf Σ-space. Then $C_p(\upsilon X)$ is angelic [14] and $C_p(X)$ is angelic by [7, Note 4].

2. Proof of theorem and corollaries

We shall need the following result of Nagami [12] (see also [1, Proposition IV.9.2]) and Proposition 2.2.

Proposition 2.1. *A space X is a Lindelöf Σ-space if and only if there exists a compactification bX of X and a countable family $\mathcal{F}$ of compact sets in bX such that if $x \in X$ and $y \in bX \setminus X$ there exists $B \in \mathcal{F}$ for which $x \in B$ and $y \notin B$.*

Proposition 2.2. *For a locally convex space E the following are equivalent.*

(a) *E is web-bounded.*
(b) *$(E, \sigma(E, E'))$ is embedded in a locally convex Lindelöf Σ-space $(W, \sigma(W, E'))$, where $E \subset W \subset (E')^*$.*
(c) *$(E', \sigma(E', E))$ is web-bounded.*
(d) *$(E', \sigma(E', E))$ is embedded in a locally convex Lindelöf Σ-space $(Z, \sigma(Z, E))$, where $E' \subset Z \subset E^*$ and E^* denotes the algebraic dual of E.*

Proof. Indeed, (a) implies (d): assume that E is web-bounded and $\{A_\alpha : \alpha \in \Sigma\}$ is a covering of E such that if $\alpha = (n_k) \in \Sigma$ and $x_k \in C_{n_1 n_2 \cdots n_k}$, then $(x_k)_k$ is bounded. Clearly for each $\alpha \in \Sigma$ and each $x' \in E'$ there exists $k \in \mathbb{N}$ such that $x'(C_{n_1 n_2 \cdots n_k}) \subset [-k, k]$.

Set

$$Z := \{x' \in E' : \forall \alpha = (n_i) \in \Sigma, \exists k \in \mathbb{N}, x'(C_{n_1 n_2 \cdots n_k}) \subset [-k, k]\}.$$

Since $(C_{n_1,n_2,\ldots,n_k})_k$ is decreasing, then Z is a vector subspace of E^* and $E' \subset Z \subset E^* \subset \mathbb{R}^E$. Using a proof similar to that used in the next page for showing that (ii) implies (i), we find that $(Z, \sigma(Z, E))$ is a Lindelöf Σ-space. The fact that (d) implies c) is obvious. The fact that (c) implies (b) is proved as follows: by hypothesis $(E', \sigma(E', E))$ is web-bounded and if we apply to this space the fact that (a) implies

(d) we find that $(E, \sigma(E, E'))$ is embedded in a Lindelöf Σ locally convex space $(W, \sigma(W, E'))$. The proof that (b) implies (a) is trivial. The proposition has thus been proved. □

We are ready to prove Theorem 1.2; the proof is motivated by [1, Proposition IV.9.3].

Proof. (ii) ⇒ (i). Assume X is strongly web-bounding and that $\{A_\alpha : \alpha \in \Sigma\}$ is a covering of X verifying the web-bounding condition. Then for each $f \in C(X)$ and each $\alpha = (n_k) \in \Sigma$ there exists $k \in \mathbb{N}$ such that $f(C_{n_1 n_2 \cdots n_k}) \subset [-k, k]$. Then the set

$$Z := \{f \in \mathbb{R}^X : \forall \alpha = (n_i) \in \Sigma, \exists k \in \mathbb{N}, f(C_{n_1 n_2 \cdots n_k}) \subset [-k, k]\} \tag{2.1}$$

satisfies the following condition:

$$C_p(X) \subset Z \subset \mathbb{R}^X. \tag{2.2}$$

Endow Z with the topology induced by $\mathbb{R}^X$. Let $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ be the natural compactification of $\mathbb{R}$. Then $\overline{\mathbb{R}}^X$ is a compactification of Z. For each $\alpha = (n_i) \in \Sigma$ and $k \in \mathbb{N}$ let $F_{\alpha|k} = F_{n_1 n_2 \cdots n_k}$ be the closure in $\overline{\mathbb{R}}^X$ of the set

$$\{f \in \mathbb{R}^X : f(C_{n_1 n_2 \cdots n_k}) \subset [-k, k]\}.$$

Now $S := \{F_{\alpha|k}, \alpha \in \Sigma, k \in \mathbb{N}\}$ is a countable family of compact subsets of $\overline{R}^X$. Clearly

$$\overline{\mathbb{R}}^X \setminus Z = (\overline{\mathbb{R}}^X \setminus \mathbb{R}^X) \cup (\mathbb{R}^X \setminus Z).$$

Take $g \in \overline{Z}^{\overline{\mathbb{R}}^X} \setminus Z$. If $g \in \overline{\mathbb{R}}^X \setminus \mathbb{R}^X$, then there exists $a \in X$ such that $g(a) \in \{-\infty, +\infty\}$. There exists $\alpha = (n_i) \in \Sigma$ such that $a \in A_\alpha$. Then from $g(C_{n_1 n_2 \cdots n_k}) \cap \{-\infty, +\infty\} \neq \emptyset$ it follows that $g \notin F_{\alpha|k}$ for each $k \in \mathbb{N}$.

If $g \in \mathbb{R}^X \setminus Z$, then there exists $\alpha = (n_i) \in \Sigma$ such that $g(C_{n_1 n_2 \cdots n_k}) \not\subset [-k, k]$ for each $k \in \mathbb{N}$. Also $g \notin F_{\alpha|k}$ for each $k \in \mathbb{N}$. Therefore, if $f \in Z$ and $g \in \overline{\mathbb{R}}^X \setminus Z$, then there exists $\alpha = (n_i) \in \Sigma$ such that $g \notin F_{\alpha|k}$ for each $k \in \mathbb{N}$, and from the definition of Z it follows that for this α there exists a $k \in \mathbb{N}$ such that $f \in F_{\alpha|k}$. Therefore by Proposition 2.1 it follows that Z is a Lindelöf Σ-space. Finally we apply Proposition 1.1 to show that υX is a Lindelöf Σ-space.

(i) ⇒ (ii). For the converse implication, assume that υX is a Lindelöf Σ-space. Then there exists $\Sigma \subset \mathbb{N}^{\mathbb{N}}$ and an upper semicontinuous map T from Σ into compact sets in υX covering υX. Set $A_\alpha := T(\alpha) \cap X$ for $\alpha = (n_k) \in \Sigma$. Take a sequence $x_k \in C_{n_1, n_2, \ldots, n_k}$. Then there exists a sequence $(\alpha_k)_k$ in Σ which converges to α such that $x_k \in T(\alpha_k)$ for each $k \in \mathbb{N}$. Since T is upper semicontinuous, then the set $\{x_k : k \in \mathbb{N}\}$ is countably compact; hence it is functionally bounded in υX, and then also in X.

(iii) ⇒ (i). Replace X in (2.1) and (2.2) by $C_p(X)$. If $C_p(X)$ is web-bounded, one gets (analogously as above) that there exists a Lindelöf Σ-space Z such that $L_p(X) \subset Z \subset \mathbb{R}^{C_p(X)}$. Since $X \subset L_p(X)$, then $X \subset Z \subset \mathbb{R}^{C_p(X)}$.

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Now note that the space υX is a Lindelöf Σ-space. Indeed, if Y is the closure of X in Z, then Y is Lindelöf Σ. Since every real-valued function on X can be continuously extended to $\mathbb{R}^{C_p(X)}$, then $\upsilon X = \upsilon Y = Y$ is Lindelöf Σ.

(i) $\Rightarrow$ (iii). If υX is Lindelöf Σ, then by Proposition 1.1 there exists a Lindelöf Σ-space Z such that $C_p(X) \subset Z \subset \mathbb{R}^X$ and then $C_p(X)$ is web-bounded.

To prove the equivalence of (iii) and (v) we apply Proposition 2.2.

(iii) $\Leftrightarrow$ (iv). Since $L_p(X)' = C_p(X)$ we again apply Proposition 2.2.

(i) $\Rightarrow$ (vi). This implication follows from [1, Proposition 0.5.13].

(vi) $\Rightarrow$ (i). This implication follows from the fact that υX is closed in $L_p(\upsilon X)$. □

Corollary 2.3. *The following conditions are equivalent.*

(i) *$C_p(X)$ is web-bounded and X is Lindelöf.*
(ii) *$L_p(X)$ is a Lindelöf Σ-space.*
(iii) *$C_p(X)$ is a web-bounded space with countable tightness.*

Proof. (i) $\Rightarrow$ (ii). Assume $C_p(X)$ is web-bounded and X Lindelöf. By Theorem 1.2 the space $X = \upsilon X$ is a Lindelöf Σ-space, and then by [1, Proposition 0.5.13] the space $L_p(X)$ is a Lindelöf Σ-space.

(ii) $\Rightarrow$ (i). If $L_p(X)$ is a Lindelöf Σ-space, then $X \subset L_p(X)$ as a closed subspace [1, Proposition 0.5.9] is also a Lindelöf Σ-space. By Theorem 1.2 the space $C_p(X)$ is web-bounded.

(iii) $\Rightarrow$ (i). This implication follows from [1, Theorem II.1.1].

(i) $\Rightarrow$ (iii). By (ii) the space $X \subset L_p(X)$ is Lindelöf Σ. Since countable products of Lindelöf Σ-spaces are Lindelöf Σ, we apply [1, Theorem II.1.1] to show that $C_p(X)$ has countable tightness. □

Example 2.4. There is a web-bounded space $C_p(X)$ not having countable tightness.

Proof. Take a quasi-Suslin space X which is not K-analytic; see [3, 4, 15] for such examples. Then X is not Lindelöf and $C_p(X)$ does not have countable tightness [1, Theorem II.1.1]. The space X is strongly web-bounding. Indeed, X as quasi-Suslin admits a resolution $\{A_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ of relatively countably compact sets (hence functionally bounded). If $x_k \in C_{n_1 n_2 \cdots n_k}$ for each $k \in \mathbb{N}$ there exists $\beta_k = (m_n^k)_n \in \mathbb{N}^{\mathbb{N}}$ such that $x_k \in A_{\beta_k}$, $n_j = m_j^k$ for $j = 1, 2, \ldots, k$. Let $a_n = \max\{m_n^k : k \in \mathbb{N}\}$, for $n \in \mathbb{N}$ and $\gamma = (a_n)$. Since $\gamma \geqslant \beta_k$ for every $k \in \mathbb{N}$, then $A_{\beta_k} \subset A_\gamma$, so $x_k \in A_\gamma$ for all $k \in \mathbb{N}$. Then $C_p(X)$ is web-bounded by Theorem 1.2. □

The following corollary is a version of Corollary 2.3.

Corollary 2.5. *Let E be a barrelled space. Then E is web-bounded and $(E, \sigma(E, E'))$ has countable tightness if and only if $(E', \sigma(E', E))$ is a Lindelöf Σ-space.*

Proof. Assume $(E, \sigma(E, E'))$ has countable tightness. We show that the space $F := (E', \sigma(E', E))$ is realcompact. By the Corson criterion, see [15, p. 137], it is enough to show that every linear functional f on E which is $\sigma(E, E')$-continuous

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on each $\sigma(E, E')$-closed separable vector subspace is continuous. Note that the kernel $K := f^{-1}(0)$ is closed in E. Indeed, if $y \in \overline{K}$, there is a countable $D \subset K$ with $y \in \overline{D}$ (the closure in $\sigma(E, E')$). By assumption, $f|\overline{\text{lin}(D)}$ is $\sigma(E, E')$-continuous; hence $f(y) \in \overline{f(\text{lin}(D))} \subset \overline{f(K)} = \{0\}$, so $y \in K$ and $f \in E'$. Let E be web-bounded. By Proposition 2.2 the space $(E', \sigma(E', E))$ is also web-bounded. Hence E' is covered by a family $\{A_\alpha : \alpha \in \Omega\}$ of sets such that each sequence $x'_k \in C_{n_1,\ldots,n_k}$ is $\sigma(E', E)$-bounded. By assumption each $(x'_n)_n$ is equicontinuous, so $\sigma(E', E)$-relatively compact. Hence F is strongly web-bounding and $F = \upsilon(E', \sigma(E', E))$ is a Lindelöf Σ-space by Theorem 1.2. Now assume that F is a Lindelöf Σ-space. Then E is web-bounded by Theorem 1.2. We again apply [1, Theorem II.1.1] to deduce the countable tightness of $C_p(F)$. Hence $(E, \sigma(E, E')) \subset C_p(F)$ has countable tightness. □

The next corollary follows from Theorem 1.2 and Proposition 1.1 and supplements Proposition 1.1.

Corollary 2.6. *The space $C_p(X)$ is web-bounded if and only if there is a Lindelöf Σ-space Z such that $C_p(X) \subset Z \subset \mathbb{R}^X$.*

Every (LF)-space, that is, the inductive limit of a sequence of metrizable and complete locally convex spaces, is a quasi-(LB)-space, that is, has a resolution consisting of Banach discs, and the strong dual of an (LF)-space is also a quasi-(LB)-space; see [16]. Clearly every locally complete locally convex space with a bounded resolution is a quasi-(LB)-space, and every locally convex space that has a fundamental sequence $(S_n)_n$ of bounded sets has a bounded resolution: set $A_\alpha := S_{n_1}$ for $\alpha = (n_k) \in \mathbb{N}^{\mathbb{N}}$. We do not know if any locally convex space in the class $\mathfrak{G}$ has a bounded resolution; nevertheless Theorem 1.2 yields Corollary 1.3 listed in the Introduction. We provide a simple proof of this.

Proof. We see that $F := (E', \sigma(E', E))$ is quasi-Suslin by [10]. Hence F is strongly web-bounding. By Theorem 1.2 the space $C_p(F)$ is web-bounded. Hence $(E, \sigma(E, E')) \subset C_p(F)$ is web-bounded. □

The following question is motivated by the property labelled (∗) in the Introduction and Corollary 1.3.

Problem 2.7. Let E be a web-bounded locally convex space. Is $\upsilon(E', \sigma(E', E))$ K-analytic?

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J. KĄKOL, Faculty of Mathematics and Informatics, A Mickiewicz University, 61-614 Poznań, Poland
e-mail: kakol@amu.edu.pl

M. LÓPEZ-PELLICER, Depto. de Matemática Aplicada and IUMPA, Universitat Politècnica de València, E-46022 Valencia, Spain
e-mail: mlopezpe@mat.upv.es

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