Ex. 8. OE is the common perpendicular to AB and CD. For O is the middle point of the base AB of the isosceles triangle EAB, hence EO is perpendicular to AB. Again, E is the middle point of the base CD of the isosceles triangle OCD, hence OE is perpendicular to CD.

Ex. 9. The angle between any two faces of the tetrahedron is $\cos^{-1}(\frac{1}{3})$.

$$\text{For} \quad \cos COD = \frac{OG}{OD} = \frac{OG}{OC} = \tfrac{1}{3}.$$

The student will probably now have acquired considerable confidence in the use of his drawings, which may be further tested by applying certain of the above exercises, modified as required, to any tetrahedron. And if the principle on which these drawings have been made is borne in mind when the usual propositions are taken up, the advantage of having a definite way of constructing diagrams and a definite way of thinking and speaking of the lines there represented will be found to lessen considerably the difficulty of the subject.

PETER RAMSAY

Geometrical Construction for Refracted Ray.—

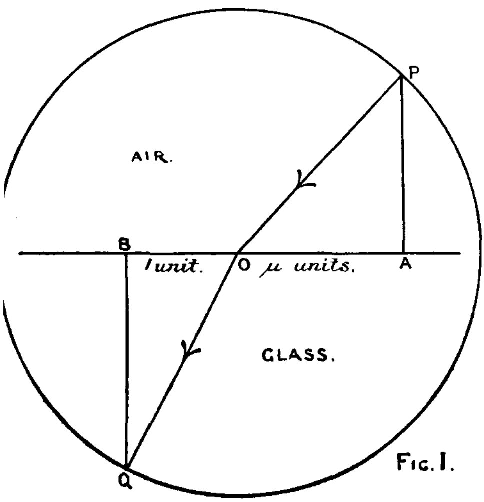

FIG. 1.

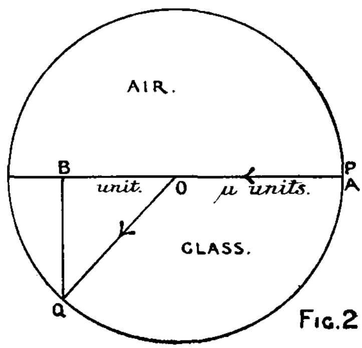

FIG. 2

The following variation seems an improvement on the usual method of tracing a refracted ray. In the usual method when an incident ray OP is given, *any* radius OP is taken and a circle described. PA is drawn perpendicular to the surface AB. OA is then sub-divided into the number of units expressed in the numerator of the fraction μ, and OB is measured backwards, the

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number of units expressed by the denominator. BQ is then drawn perpendicular to AB to meet the circle in Q. OQ is the refracted ray.

The objections to the method are (1) μ must be expressible as a ratio of two simple integers, (2) the trouble of sub-dividing OA.

The construction is simpler if we first make OA μ units in length and OB = 1 unit. Draw AP and BQ perpendicular to AB, AP cutting the incident ray in P. With centre O and radius OP describe a circle cutting BQ in Q. Then OQ gives the refracted ray.

Critical Angle.—To find the critical angle for a medium, make OA $= \mu$ units and OB = 1 unit.

Describe a circle with radius OA (Fig. 2), and complete the construction as before.

The critical angle can now be measured directly.

Pin method of finding refractive index.—Suppose the directions of the rays OP and OQ (Fig. 1) have been found experimentally, measure OB = 1 unit. Draw BQ perpendicular to BA cutting OQ in Q. With centre O and radius OQ describe a circle cutting OP in P. Draw perpendicular PA. Measure OA. This gives the refractive index at once without having a division operation, as in the other method.

WILLIAM MILLER

An Interesting Example in Curve Tracing.—It is proposed to trace the curves represented by the equation

$$y - ax - y^3 + x^3y^2 + x^2y^3 = 0$$

for the values of a (i) $a = 2$, (ii) $a = 1$.

Analysing the equation

$$y - 2x - y^3 + x^3y^2 + x^2y^3 = 0$$

by means of Newton's parallelogram, we obtain as a first approximation to the shape at the origin $y = 2x$, and as a second approximation $y = 2x + 8x^3$.

For a first approximation, when x and y are infinite, we have $y + x = 0$, and for a second $y = -x + \dfrac{y^3}{x^2y^2}$,

$$= -x - \frac{1}{x}.$$

Hence $y = -x$ is an asymptote.

When x is finite and y infinite, we have as a first approximation $x^2 = 1$. For a second $x^2 = 1 - \dfrac{x^3y^2}{y^3}$,

$$\text{i.e.}\quad x = \pm\sqrt{1 - \frac{x^3}{y}} = \pm\left(1 - \frac{x^3}{2y}\right).$$