STUDY OF INHOMOGENEITIES ON THE SURFACE OF MAGNETIC CP STARS

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ABSTRACT. The formulation and solution of the inverse problem to determine local Stokes parameters on the surface of a star , in particular magnetic chemically peculiar star, and hence to determine surface abundance distribution and magnetic field geometry is reviewed.

I will not speak here about those works in which the observed phase variations of the equivalent widths, mean radial velocities and the effective magnetic fields H_e are used to determine chemical inhomogeneities. Pioneer works by Deutch (1969) and Pyper (1969) are known to everybody. New formalism was also developed recently by Adelman et al., (1984).

Unfortunately, using these integrated over λ parameters leads to loss of a great deal of information, containing in spectra and therefore does not permit to get unique and detailed solution.

Using line profile variations Falk and Wehlaw (1974) have got more reliable result for Eu distribution over α^2CVn inspite of the fact that a crude simplified assumption of gaussian shape of local profiles was made. New method to determine surface chemical inhomogeneities and geometry of magnetic field is developed recently and described by Goncharsky et al., (1982) and partly had been reported at Liege in 1981. The outline of the essence of the new method is given below.

Spectral observations with the aid of circular and linear polarization analysers may give the information on Stokes parameters inside the spectral lines. The following expressions describe the observed Stokes parameters in a spectral line of a rotating star depending on and the phase of rotation ωt :

$$\tilde{R}_I(\lambda,\omega t)=\iint_{\cos\theta>0} R_I(M)\,\mathcal{U}(\theta)\,dM \qquad (1)$$

$$\tilde{R}_V(\lambda,\omega t)=\iint_{\cos\theta>0} R_V(M)\,\mathcal{U}(\theta)\,dM \qquad (2)$$

C. R. Cowley et al (eds.), Upper Main Sequence Stars with Anomalous Abundances, 125–132.

$$\tilde{R}_Q(\lambda,\omega t)=\iint_{\cos\theta>0} R_Q(M)\,u(\theta)\,dM \qquad (3)$$

$$R_U(\lambda,\omega t)=\iint_{\cos\theta>0} R_U(M)\,u(\theta)\,dM \qquad (4)$$

here $R_{I,V,Q,U}(M)$ are local Stokes parameters in the point M on the surface of a star, $u(\theta)$ - the limb darkening coefficient and dM - the elemental area around the point M.

The left sides of these equations are obtained from the observations. Solving the equations, one may determine $R_{I,V,Q,U}(M)$, that is to determine local abundances and surface magnetic field geometry.

Many physical parameters of a star are bound indirectly into these equations including even local model atmospheres because the local Stokes parameters result from radiation transfer in the atmosphere. Selfconsistent solution of the equations for all these unknowns is a very complicated problem and it is necessary therefore to introduce reasonable simplifications and try to solve it by parts. It is necessary first to clear up, which parameters may be neglected and when and what simplifications may be done.

Let us consider first the form of presentation of local profiles. To solve the equations (1 - 4) numerically, it is convinient to use analytical expressions obtained by Unno (1956) for Miln-Eddington approximation.

When there is no magnetic field (H=0) Unno's solution for the intensity profiles looks like as follows :

$$R_I(\lambda)=R(0)\cdot\eta(\lambda)/[1+\eta(\lambda)] \qquad (5)$$

where R( ) is a central depth of a line,

$$\eta(\lambda)=k(\lambda)/\varkappa=k(0)\cdot f(\lambda)/\varkappa=\eta(0)f(\lambda)$$

The shape of the profile is assumed to be a Foigt one, so depending on Lorenz and Doppler widths γ_L and γ_D

This expression for R_I( ) resembles the known empirical Minnaerts formulae

$$R(\lambda)=R(0)\,\frac{\tau(\lambda)}{R(0)+\tau(\lambda)} \qquad (6)$$

which nicely describes line profiles of different strengths in the Solar spectrum, if for R(0) the observed value is taken.

As the abundance changes over the surface of Ap-star, R(0) and η(0) are the functions of coordinates on stellar surface and so $R_I(M)$ contains two unknowns dependent on coordinates. But actually they are not completely independent. The dependance may be expressed by the analytical empirical relation as follows :

$$R(0)=R(0)_\infty\left(1-e^{-a\tau(0)}\right) \qquad (7)$$

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where $R(0)_\infty$ is the central depth of strongly saturated line when $\tau(0) \to \infty$, (Goncharsky et al., 1977, Pavlova and Khokhlova, 1983).

The parameter „a" as well as the parameters γ_L and γ_D in the Foight absorption coefficient profile $f(\lambda)$ may be considered in the first approximation as coordinate independent. Any "exact" profiles may be well represented by these analytical formula. The parameters of the representation may be easily determined in the way described below : the approximated profiles being computed using the formulae (5) for a grid of parameters $\tau(0)$, γ_L and γ_D, the parameter $R(0)$ being taken from the given theoretically computed "exact" profiles for a given model atmosphere for a set of abundances of an element. The r.m.s. deviation σ of an approximated $R(\lambda)$ from the exact one is computed along all the line and dependance of σ on $\lg\tau(0)$ is plotted for different γ_D (see Figure 1). One can see that σ has a minima at certain values of $\tau(0)$ and γ_D. On the field of parameters $\lg\tau(0)$ and γ_D one can easily determine the location of those values which provide a representation of all "exact" profiles by the approximation formula with the r.m.s. deviation σ =1%, 2%, 3% and so on (see Figure 2). When γ_D is fixed on Figure 2 (γ_D =0.04 for example), one obtains the set of values of $\tau(0)$ which provide σ being 1%, 2% and so on.

Let us now plot $R(0)$ as a function of $\tau(0)$ thus obtained, which must be described by formula (7). Parameter "a" may be found by fitting a computed dependance $R(0)$ on $\tau(0)$ using equation (6) to the values found above (Figure 3).

So introducing formula (7) we use in fact not a Miln-Eddington solution for local profiles but much better approximation to the "exact" theoretical profiles. The only problem is to choose properly the model atmosphere, but that is the problem not only for this case.

Inserting expressions (6) and (7) into the equations (1 -4) and solving the equations in some way, one should obtain local Stokes parameters and hence, the distribution of elements and magnetic field geometry.

When the magnetic field is weak ($H_e \simeq 0$) or lande-factor of a line under consideration is small ($z \simeq 0$), the only one equation (1) should be solved to obtain a map of a chemical element. The Tikhonov method of numerical solution of equation (1) for mapping chemical elements was used in the work by Goncharsky et al., (1977, 1982). The influence of the magnetic field on the line profiles was not taken into account that time.

Up to now five stars shown in the Table have been investigated by this method.

For the stars ε UMa and θ Aur high precision line profiles were obtained with a reticon detector by Rice and Wehlau and a very good agreement between the observed and computed line profiles has been achieved. The paper on θ Aur is presented at this Colloquium.

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TABLE.

Star	Element		S/N	Reference
α^2CVn	Eu	0.02	Ph	Goncharsky et al., (1983)
	Ti, Cr, Fe	0.02	Ph	Khokhlova and Pavlova (1984)
CU Vir	Si	0.02	Ph	Goncharsky et al., (1983)
χ Ser	Sr	0.02	Ph	Goncharsky et al., (1983)
εU Ma	Cr, Fe	0.01	$\simeq$200	Wehlau et al., (1982)
θAur	Si, Cr, Fe	0.01	$\simeq$500	Wehlau et al., (1985)

In the process of solution the minimization of r.m.s. deviation σ of the computed profiles from the observed ones is performed. The values of σ obtained are shown in the 3-d column of the Table.

The assumption is made that the model atmosphere does not depend on the coordinates on the surface of a star. Principal limitations of the method should be noted : 1) the map may be obtained only for a strip + 45° along the "subsolar" line. 2) The longitude of a "spot" may be determined much better than the latitude.*/

Numerical modelling of profiles $\widetilde{R}_{I,V,Q,U}(\lambda)$ made by Piskunov and Khokhlova (1983, 1984) showed the distortion of the intencity profiles by the magnetic field (see Figure 4).

Similar results were obtained earlier by Borra (1977), but he paied attention to the shift of a line as a whole and found it to be rather small. When rotational broadening becomes larger than the Zeeman splitting, line asymmetry does not depend on rotational velocity.

The distortion of profiles may be explained by the fact that the magnetic intensification of lines depending on the angle γ between the magnetic field vector H and the line of sight, reaches its maxima according to Unno (1956) at $\gamma \simeq 55°$. When a star is rotating the regions of enhanced magnetic intensification move along stellar surface and that makes difference between them and spots of real enhanced abundances. This distortion of profiles appears as noises when solving the inverse problem for chemical elements distribution, preventing the convolution of iterative process.

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We tried to solve the inverse problem for a set of profiles presented on Figure 4 and got a homogenuous distribution of an element over the surface of a star with the r.m.s. deviation of profiles σ=0.04 (which is four times larger than that obtained for ε UMa and θ Aur). The influence of the magnetic field may be in principle considerably reduced when the lines with small Z are used for the analysis. It is also of no importance if the stars with weak magnetic field are investigated. For five stars mentioned in the Table only for α^2 CVn the effect may be noticeable, but seems to be also not very important.

The influence of chemical inhomogeneities on polarization profiles was studied by Piskunov and Khokhlova. One may see on Figure 5 that chemical spots considerably distort polarization profiles. It is clear that this effect should not be neglected when a magnetic field is being measured.

An algorithm of the solution of the inverse problem to determine the magnetic field geometry using circular polarization profiles, that is a solution of the equation (2), was developed by Piskunov (1985) for a particular case of shifted dipole field configuration. The distribution of an element is supposed to be known, that is determined independently. This code was used to determine the geometry of the magnetic field of α^2 CVn star on the base of circular polarization profiles of metallic lines observed photographically at the 6-m telescope (Glagolevsky et al., 1985). When inhomogenuous distribution of the elements determined by Khokhlova and Pavlova had been taken into account, centered dipole geometry was obtained from four lines of TiII and FeII and the uniqueness of the solution was confirmed by using different initial approximations and by the coinsidence of the results obtained from all the four lines of two elements.

It should be noted, that measurements of polarization in hydrogen lines cannot give the possibility to determine the geometry of a magnetic field by the inverse problem solution because the intrinsic widths of the lines are large as compared with the rotational broadening and so rotational shifts do not work.

The results of mapping the elements distribution over the surfaces of stars mentioned above in the Table show that the inhomogeneities exist on the surfaces of CP stars with strong magnetic field (α^2 CVn) as well as with very weak (if any) magnetic field (ε UMa, χ Ser). In some cases the distribution of elements is very complex (θ Aur) and cannot be connected with the magnetic field directly in a simple way.

Further investigations of the magnetic field geometry and distribution of elements using inverse problem solution method and precise observations of Stokes parameters in spectral lines are needed.

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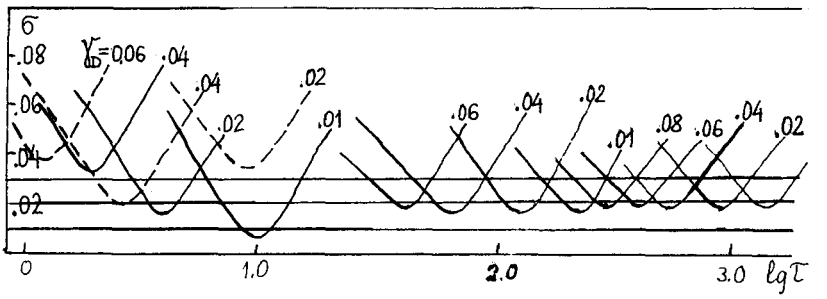

Figure 1. σ plotted versus $\lg\tau$ for different γ_D and abundances.

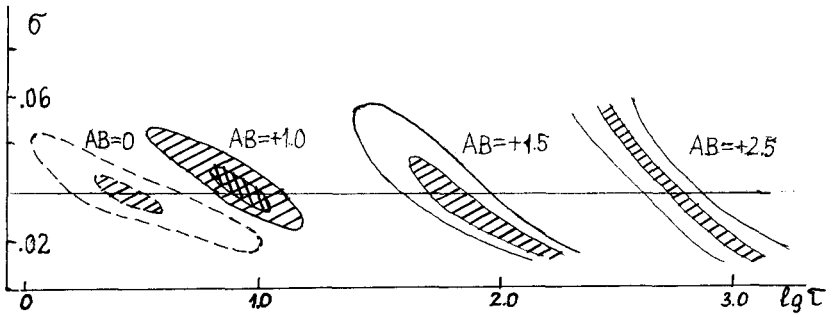

Figure 2. The area of given σ in the field of parameters γ_D-$\lg\tau$ for different abundances AB.

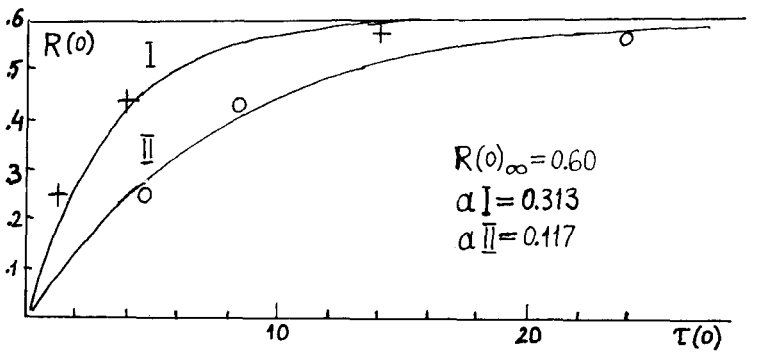

Figure 3. Fitting the analytical expression (7) (solid line) to the values found with the expressions (5), (6) and (7) (crosses and circles).

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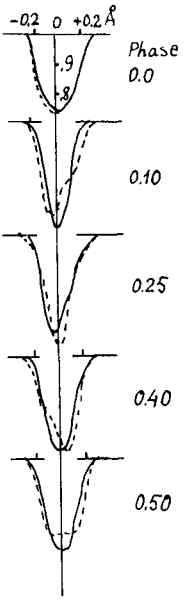

Figure 4.
Phase variations of the computed profiles of the line λ 4520 FeII in the spectra of rotating magnetic star with Hp=10 KGs, V sin i=20 km/s, β=i=45° (solid line) and β=i=90° (broken line)

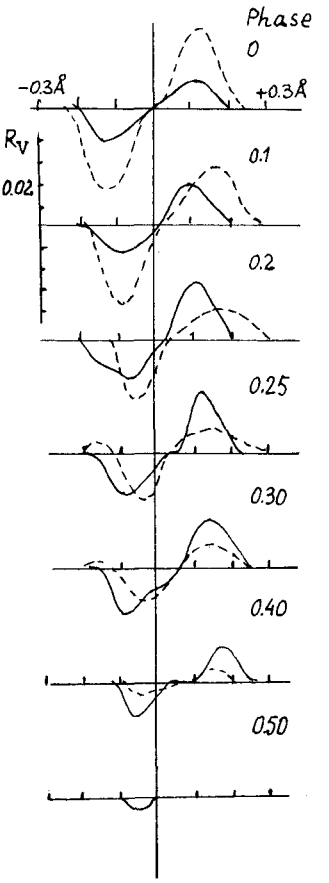

Figure 5.
Computed profiles of circular polarization of a centered dipole model Hp=10 KGs, β =i=45°, V_{eq}=10 km/s with a chemical spot situated on the magnetic pole (broken line) and a spot situated at the magnetic equator (solid line).

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References

Adelman S., Hensberge H., van Rensbergen W., Astron. Astrophys.Suppl.Ser. 1984,v.57, N 1, p.121
Borra E.F. and Landstreet J.D. Astrophys.J. 1977, v."1",p.142
Deutsch A.J. Astrophys.J. 1970,v.159,p.985
Glagolewsky Ju.V.,Piskunov N.E.,Khokhlova V.L. Soviet Astronomy Letters 1985,(Russ.) v.11,N 5,p.371
Goncharsky A.V.,Stepanov V.V.,Khokhlova V.L. and Yagola A.G. Soviet Astronomy Letters,1977,v.3,N 3,p.147
Goncharsky A.V.,Stepanov V.V.,Khokhlova V.L.,Yagola A.G. Soviet Astronomy 1982 (Russ.) v.59,p.1146
Goncharsky A.V.,Rjabchikova T.A.,Stepanov V.V.,Khokhlova V.L. and Yagola A.G. Soviet Astronomy 1983,v.27(1),p.49
Falk A.F., Wehlau W.M. Astrophys.J.1974,v.192,p.409
Khokhlova V.L.,Pavlova V.M.1984,Sovjet Astr.Lett.v.10(3),p.158
Khokhlova V.L. Soviet Scientific Reviews,Section E,Astro-Astrophysics and Space physics reviews ed.R.A.Syunyaev, 1985,p.99
Pavlova V.M.,Khokhlova V.L. Nauchnye informatzii Astrofis.ser. 1980,N 43,p.65
Pyper D.M. Astrophys.J.Suppl.Ser.1969,v.18,p.374
Piskunov N.E.,Khokhlova V.L.Soviet Astron.Lett.1984,v.10(3), p.187.
Piskunov N.E.,Khokhlova V.L.1983,Soviet Astron.Lett.v.9(6), p.346
Piskunov N.E. 1985,Soviet Astron.Lett.v.11(1),p.346
Wehlau W.M.,Rice J.,Piskunov N.E.,Khokhlova V.L. 1982, v.8(1), p.15

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