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# A FIFTEENTH-CENTURY OTTOMAN SOLUTION TO THE LIAR PARADOX BY HATĪBZĀDE MUHYIDDĪN

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Abstract. This paper deals with a solution to the infamous liar paradox, usually known in the Arabic literature as <code>Maglaṭat al-ǧadr al-aṣamm</code>. The solution is raised by a fifteenth-century Ottoman treatise that is attributed, among others, to Ḥaṭībzāde Muḥyiddīn Efendī. The paper also compares it with the solution by the contemporary Persian philosopher, Ğalāl al-Dīn al-Dawānī. The short treatise devoted to the paradox is one of the few works by Ottomans on the subject and it comprehensively addresses the paradox in its two forms. An analysis of the solution offered by the treatise to the paradox, the paper aims to bring Ottoman discussions of the liar to the attention of contemporary scholarship and contribute to filling the obvious gap in the literature on the paradox in Islamic thought.

Résumé. Cet article traite d'une solution au tristement célèbre paradoxe du menteur, généralement connu dans la littérature arabe sous le nom de *Maġlaṭat al-ġaḍr al-aṣamm*. La solution est donnée dans un traité ottoman du xv<sup>e</sup> siècle attribué, entre autres, à Ḥaṭībzāde Muḥyiddīn Efendī. L'article la compare également à la solution du philosophe persan contemporain, Ğalāl al-Dīn al-Dawānī. Le court traité consacré au paradoxe est l'un des rares ouvrages des Ottomans sur le sujet et il aborde de manière exhaustive le paradoxe sous ses deux formes. En tant qu'analyse de la solution offerte par le traité au paradoxe, l'article fait connaître à l'érudition contemporaine les discussions ottomanes sur le menteur, et il contribue à combler le vide évident dans la littérature sur le paradoxe dans la pensée islamique.

A paradox is "an argument that begins with premises that appear to be clearly true, that proceeds according to inference rules that appear to be valid, but that ends in a contradiction." The liar paradox, accordingly, is a logical argument that arrives at a contradictory conclusion through reasoning about a "liar sentence," a statement that declares itself to be false or not to be true, such as "This sentence is false." It is either true or false according to the principle of bivalence. However, if it is true, then it must be false because the sentence says just that. If it is false, on the other hand, then it is true because it states that it is false. Then it is true if it is false and false if true. But, given that it is necessarily either true or false, then it is both true and false.

As such, the paradox threatens the very basic foundation of Aristotelian logic, namely the principle of non-contradiction, because it seems to conjoin two propositions that contradict each other and therefore cannot be true or false at the same time. For this reason, it has been treated as an important, but also entertaining or sometimes fatal, puzzle since ancient times.<sup>2</sup> In Islamic thought, too, it appeared here and there in various forms as a part of discussions associated with a wide array of fields, ranging from Islamic jurisprudence and theology to philosophy and logic. From the fourteenth century on, the perplexing examples of the liar came to be known as the "Paradox of the Irrational Root" (Maġlaṭat al-ǧaḍr al-aṣamm),<sup>3</sup> following the naming by the versatile Timurid scholar Sacd al-Dīn al-Taftāzānī (d. 1390), who emphasized its difficulty and insolubility by the remark that "This is

<sup>&</sup>lt;sup>1</sup> Charles Chihara, "The semantic paradoxes: A diagnostic investigation," *Philosophical review*, vol. 88, no. 4 (1979), p. 590; for a similar definition, see R. M. Sainsbury, *Paradoxes* (Cambridge, 1995), 1.

<sup>&</sup>lt;sup>2</sup> On the treatment of the paradox in ancient Greek thought and sometimes mortal threat it posed, see I. M. Bocheński, *Ancient formal logic* (Amsterdam, 1951), p. 101–102; Patrick Gray, "The liar paradox and the letter to Titus," *The Catholic biblical quarterly*, vol. 69, no. 2 (2007), p. 302–314; Mario Mignucci, "The liar paradox and the Stoics," in Katerina Ierodiakonou (ed.), *Topics in Stoic philosophy* (Oxford: Clarendon, 2004), p. 54–70.

<sup>&</sup>lt;sup>3</sup> As stated by the anonymous reviewer of the *ASP*, the term "maġlaṭa" could also be translated as "fallacy," but I consider "paradox" more appropriate here because I assume between two terms such a relationship that the latter may involve the former. I mean that paradoxical arguments usually include an implicit logical fallacy, and because of that, it becomes paradoxical. In this case, one has to spot the fallacious element in the argument to solve the paradox. The maġlaṭa here is a paradox and I will show in the following what a fallacy it involves according to the treatise in question. Besides, in manuscripts and library catalogues, the term al-ǧaḍr al-aṣamm occurs in a number of misspelled forms, such as ǧaḍr al-aṣamm and the like, very probably because it has never been widespread and well-known among scribes.

a paradox, about the solution of which the most intelligent and most brilliant people have fallen into perplexity. Therefore, I called it the 'Paradox of the Irrational Root.'"<sup>4</sup> The term "irrational root" was used by Muslim mathematicians to refer to the root numbers that are impossible to be expressed as fractions of two integers and which are said to have compelled Greek mathematicians to focus their attention on geometry rather than arithmetic.<sup>5</sup> It is intriguingly noted in Muslim literature that no one has access to the very nature of such numbers, except for God. The prayer "How exalted is God, who knows the essence of irrational roots!"<sup>6</sup> is often mentioned as a telling expression of the difficulty of the issue.

Skeptical of the possibility of a solution, Taftāzānī recommended admitting our weakness and therefore giving up trying to solve it. Yet, fortunately, later Muslim scholars would not follow his recommendation, and the following centuries would witness many attempts to offer a solution to the infamous paradox. However, contemporary scholarship on these solutions in Islamic thought has not emerged until very recently. To the best of my knowledge, the first modern study on the subject is Miller's short paper of 1989, which does not seem to have been sufficient to arouse notable interest in the subject given that following decades register no further study on the subject. Finally, two articles co-authored by Alwishah and Sanson break the silence to be followed

<sup>&</sup>lt;sup>4</sup> Taftāzānī, Šarḥ al-Maqāṣid, ed. ʿAbd al-Raḥmān ʿUmayra, 5 vol. (Beirut, 1419/1998), IV, p. 287. For possible reasons why Taftāzānī called the paradox the way he did, see Muḥammad ʿIsmatAllah al-Buḥārī, "Al-muntaḥab min šarḥ risālat al-Muġālaṭāt," in Āḥad Farāmarz Qaramalikī (ed.), Davāzdah risāla dar pārāduks-i durūġgū (Tehran, 1386/2007), p. 315–316; David Sanson & Ahmed Alwishah, "Al-Taftāzānī on the liar paradox," Oxford studies in medieval philosophy, 4 (2016), p. 121–122.

<sup>&</sup>lt;sup>5</sup> Morris Kline, *Mathematical thought from ancient to modern times*, 3 vol. (Oxford, 1972), I, p. 173–176.

<sup>&</sup>lt;sup>6</sup> While the tenth-century Muslim encyclopedist Muḥammad al-Ḥwārizmī attributes this prayer to Hindu brahmins (Mafātīḥ al-culūm, ed. Ibrāhīm al-Abyārī [Beirut, 1409/1949], p. 221), cUmar al-Baġdādī, who glossed Ḥayālī's commentary on Al-qaṣīda al-nūniyya, ascribes it to the Prophet's wife, cĀiša: Aḥmad Šawqī, Ḥall-i muġālaṭa-i ġaḍr-i aṣamm (Istanbul: Millet Yazma Eser Kütüphanesi, MS Ali Emirî Arabî 1281), fol. 16v.

<sup>&</sup>lt;sup>7</sup> Taftāzānī, Šarh al-Magāsid, IV, p. 287.

<sup>&</sup>lt;sup>8</sup> Larry B. Miller, "A brief history of the liar paradox," in Ruth Link-Salinger (ed.), *Studies in philosophy and religious thought* (New York, 1989), p. 173–182.

<sup>&</sup>lt;sup>9</sup> Ahmed Alwishah & David Sanson, "The early Arabic liar: The liar paradox in the Islamic world from the mid-ninth to the mid-thirteenth centuries CE," *Vivarium*, 47 (2009), p. 97–127; Sanson & Alwishah, "Al-Taftāzānī on the liar paradox," p. 100–

by those of Rezahkany, El-Rouayheb, and Zarepour. When it comes to the secondary literature on the issue in the modern Islamic world, the most outstanding is a series of articles by Qaramalikī that appeared in the 1990s and then were compiled by the author himself in a volume titled *Davāzdah risāla dar pārāduks-i durūģgū*, which consists of the annotated editions of twelve treatises on the paradox by scholars of the fifteenth- and sixteenth-century Iran. Ottoman scholars' contribution to the discussions, however, has been so far given little academic interest. Through an analysis of a rare Ottoman treatise on the subject, this paper aims to serve to fill this yawning gap in the literature by bringing Ottoman discussions of the liar to the attention of contemporary scholarship.

The treatise in question is often titled as the "Solution to the Paradox Called the Irrational Root" (Ḥall al-maġlaṭa al-musammāt bi-al-ǧaḍr al-aṣamm), and it is anonymous in a number of the manuscript copies while in others attributed to either Ḥocazāde Muṣliḥuddīn (d. 1488)<sup>12</sup> or Ḥaṭībzāde Muḥyiddīn (d. 1496),<sup>13</sup> both of whom were pre-eminent scholars in the reign of the Ottoman sultans Meḥmed II (r. 1444–1446 and 1451–1481) and his son Bāyezīd II (r. 1481–1512). As I have discussed the problem of the identity of the author in another study,<sup>14</sup> I

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Hassan John Rezakhany, "Jalāl ad-Dīn ad-Dawānī's solution to the liar paradox and its reception in Qāḍī Mubārak and Mullā Mubīn," Journal of South Asian intellectual history, 1 (2018), p. 183–220; Khaled El-Rouayheb, "The liar paradox in fifteenth-century Shiraz: The exchange between Ṣadr al-Dīn al-Dashtakī and Jalāl al-Dīn al-Dawānī," British journal for the history of philosophy, vol. 28, no. 2 (2020), p. 251–275; Mohammad Saleh Zarepour, "Abharī's solution to the liar paradox: A logical analysis," History and philosophy of logic, vol. 42, no. 1 (2021), p. 1–16.

<sup>&</sup>lt;sup>11</sup> Āḥad Farāmarz Qaramalikī (ed.), Davāzdah risāla dar pārāduks-i durūġgū (Tehran, 1386/2007); for the information of the articles, see p. 112–113.

<sup>&</sup>lt;sup>12</sup> On him, see Taşköprülüzâde Ahmed Efendi, Al-šaqā iq al-nu māniyya fī ulemā al-Dawla al-"Utmāniyya Osmanlı âlimleri, ed. Muhammet Hekimoğlu (Istanbul, 2019), p. 214–238; Bursalı Mehmed Tahir, Osmanlı müellifleri, ed. M. A. Yekta Saraç (Ankara, 2016), p. 307; Saffet Köse, "Hocazâde Muslihuddin Efendi," in Türkiye Diyanet Vakfı İslam ansiklopedisi, 45 vol. (Istanbul, 1998), XVIII, p. 209.

<sup>&</sup>lt;sup>13</sup> See İlyas Üzüm, "Hatibzâde Muhyiddin Efendi," Türkiye Diyanet Vakfı İslam ansiklopedisi, 45 vol. (İstanbul: TDV, 1997), XVI, p. 463–464; Taşköprülüzâde, Al-šaqā 'iq, p. 250–255; Bursalı, Osmanlı müellifleri, I, p. 307; Karl Brockelmann, History of the written Arabic tradition, trans. Joep Lameer, 2 vol. (Leiden, 2016), I, p. 301, 588; II, p. 257; Suppl. I, p. 668, 880, 964; Suppl. II, p. 332.

<sup>&</sup>lt;sup>14</sup> Yusuf Daşdemir, "A fifteenth-century Ottoman treatise on the liar paradox: A case for Ḥaṭībzāde's authorship," in Stephen Read & Barbara Bartocci (eds.), *Theories of Paradox in the Middle Ages* (London: College Publications, *forthcoming*), which includes also a critical edition and translation of the treatise. Throughout the paper, I

will not deal with the same problem here, taking for granted the conclusion that the attribution of the treatise to Ḥaṭībzāde seems to be more probable. Generally known for his glosses on the madrasa handbooks of the time, Ḥaṭībzāde seems to have written the treatise as an appendix to his glosses on the problem of good and evil, dedicated to the Sultan Mehmed II.<sup>15</sup>

The paper consists of four sections. The first handles Ḥaṭībzāde's treatment of the solution raised by Taftāzānī in his Šarḥ al-Maqāṣid whereas the second and the third examine his solutions to the two forms of the paradox, respectively. The fourth section makes a comparison between Ḥaṭībzāde's solution and that of his Persian contemporary, Ğalāl al-Dīn al-Dawānī (d. 1502), discussing possible interaction between these two philosophers.

### 1. HATĪBZĀDE ON TAFTĀZĀNĪ'S SOLUTION

As is evident from its very first sentence, namely "To refute rational goodness and evilness, it could be argued […]," <sup>16</sup> the treatise of the *Solution* situates the paradox in the context of theological discussions of good and evil (al-husn wa-al-qubh), as did first the fourteenth-century theologians, 'Adud al-Dīn al-Īǧī (d. 1355)<sup>17</sup> and Taftāzānī. After this opening

will refer to the treatise (hereafter the *Solution* in short) by the paragraph numbers of that edition. The treatise was also edited in Mehmet Aydın, "İslam düşüncesinde yalancı paradoksu ve Hocazade Bursevi'nin 'Hallu mağlatati'l-müsemmâti bi'l-cezri'l-esamm' adlı risalesi," *Dokuz Eylül Üniversitesi Ilahiyat Fakültesi dergisi*, vol. 40, no. 2 (2014), p. 167–213. Based on this edition, it was translated into Turkish in Hatipzâde/Hocazâde, "Cezrü'l-esamm yanıltmacasının çözümü," trans. Mustafa Bilal Öztürk, in Recep Alpyağıl (ed.), *Din felsefesi açısından Eş'ari gelen-ek-i / Klasik ve çağdaş metinler seçkisi* 5 (Istanbul, 2020), p. 930–934.

<sup>15</sup> Ḥaṭībzāde, Ḥāšiya li-ḥall al-muqaddimāt al-arba<sup>c</sup> min al-Talwīḥ (Istanbul, Süleymaniye Yazma Eser Kütüphanesi, MS Şehid Ali Paşa 2830), fols. 1v-15r.

16 Contemporary with Ḥaṭībzāde, the Ottoman scholar Mollā Ḥayālī also refers to the paradox in a theological text and in the context of the husn-qubh discussions: Šarh al-ʿAllāma al-Ḥayālī ʿalā al-Nūniyya, ed. ʿAbd al-Naṣīr N. al-Ḥindī (Cairo, 2008), p. 234–236. Another Ottoman scholar from the sixteenth century, Muṣlihuddīn Lārī, however, deals with the paradox in a completely logical context, with no reference to its theological conclusions: [Maġlaṭat iġtimā ʿal-naqīḍayn] (Istanbul: Süleymaniye Yazma Eser Kütüphanesi, MS Damad İbrahim Paṣa 791), fols. 92v-96r. We also see the debate on the liar paradox between Ḥaṭībzāde's Persian contemporaries Ğalāl al-Dīn al-Dawānī and Ṣadr al-Dīn al-Daštakī proceed from their respective glosses upon a widely-read commentary on Naṣīr al-Dīn al-Ṭūsī's philosophical theology handbook, Taǧrīd al-i ʿtiqād, by ʿAlī Ḥuṣcu. On this, see El-Rouayheb, "The liar paradox," p. 251.

 $^{17}$  Īǧī dismissively mentions the liar sentences as a weak argument by the Aš $^{\rm c}$ arites

sentence, Ḥaṭībzāde gives an account of the paradox, paraphrasing from Taftāzānī's Šarḥ al-Maqāṣid and pointing out two different forms of the paradox, the first of which originates from the following sentence (Solution,  $\S$  1):

(1) "The sentence I am uttering now is not true."

This sentence is paradoxical, Ḥaṭībzāde says, since its truth implies its falsity and *vice versa*. As it depends on the negative, "strengthened" version of the liar sentence, I will call this form of the paradox the "Strengthened Liar Paradox" (SLP).

The second form of the paradox, which was very probably first suggested by Taftāzānī, <sup>19</sup> includes two sentences by one who says one day

(2) "The sentence I will utter tomorrow is not true," (yesterday-sentence)

and the next day voices only the sentence:

(3) "The sentence I uttered yesterday is true." (tomorrow-sentence)

I will call this version the "Circular Liar Paradox" (CLP) because the truth of each sentence here implies its falsity, thereby forming a circle through the other. To make it a bit clearer, suppose (2) above to be true, then (3) will be false; but, in this case, (2) must be false as well since (3) says it is true, which leads to a contradiction because (2) was supposed to be true. Conversely, if you take (2) to be false, then (3) will be true, which implies that (2) is true, too. But this also leads to a flat contradiction because (2) was taken to be false. In both cases, therefore, we are faced with a contradiction, which is why this case is also paradoxical. Both Taftāzānī and Ḥaṭībzāde stipulate that the yesterday-sentence should be taken as an external proposition ( $h\bar{a}rigiyya$ ), a crucial point I will discuss in section 4 below.

for their position that good and evil can only be known through divine revelation: Al-Sayyid al-Šarīf al-Ğurǧānī, Šarḥ al-Mawāqif, ed. Maḥmūd °Umar al-Dimyāṭī, 8 vol. (Beirut, 1419/1998), VIII, p. 209–210. Unlike Taftāzānī, Īǧī dwells solely on the theological problems related to the sentence "I will lie tomorrow" and does not deal with the paradoxical cases of the liar sentence.

- <sup>18</sup> For the reasons why the negative formula of the liar sentence is called "strengthened" and its advantages when compared with the simple one, see, for example, Laurence Goldstein, "This statement is not true," and true, "Analysis, 52 (1992), p. 1–5; Adam Rieger, "The liar, the strengthen liar, and bivalence," *Erkenntnis*, vol. 52, no. 2 (2001), p. 195–203.
- <sup>19</sup> Taftāzānī, Šarḥ al-Maqāṣid, IV, p. 287. See also Sanson & Alwishah, "Al-Taftāzānī on the liar paradox," p. 109.

On the following lines of the treatise, Ḥaṭībzāde quotes a long passage from  $\check{S}arh$  al- $Maq\bar{a}sid$ , in which, after some preliminary remarks, such as the name Taftāzānī gives to the paradox and his pessimistic stance on its solution, we find a solution put forward by him. As examined in detail by Sanson and Alwishah, this solution relies on the distinction between truth and falsity as concomitant features ( $law\bar{a}zim$ ) of propositions on the one hand and as their predicates ( $mahm\bar{u}l$ ) on the other:

[t1] Truth and falsity, as well as a state ( $\hbar\bar{a}l$ ) of the judgment, that is, the affirmative or negative nexus [between the subject and the predicate], as is a concomitant feature of every proposition, might be itself a judgment ( $\hbar ukm$ ), i.e., what is judged ( $ma\hbar k\bar{u}m\ bih$ ), namely, a predicate of the thing by derivation ( $bi\text{-}al\text{-}i\check{s}tiq\bar{a}q$ ),  $^{20}$  as in our statement "This is true" and "That is false." The two [i.e., truth and falsity] do not contradict each other unless they are taken as two states of the same judgment or as two judgments on the same subject. On the contrary, when either is taken to be a state of a judgment and the other a judgment, [they do not contradict each other] because their respective subjects would be different [in that case], either explicitly as in our statement "The heavens are beneath us' is true or false," or implicitly as is the case in the individual proposition that lies at the centre of the paradox. [Solution, § 2]<sup>21</sup>

Based on the distinction he drew between the two different functions of truth and falsity, namely as a predicate of a subject and as an inseparable feature of a judgment or proposition, Taftāzānī argues that there occurs no contradiction when, for example, truth functions as a concomitant of the proposition while falsity functions as its predicate. To proceed through Taftāzānī's example, "The heavens are beneath us" is a proposition and therefore necessarily true or false, according to the principle of bivalence. In other words, it necessarily has the property of truth or falsity because inevitably, the nexus between its subject and predicate either corresponds to the reality, and then it is true, or it does not, and then it is false. However, let us take P to stand for this proposition and make it the subject of a second-order proposition, "P is true" or "P is false." As evident, in these propositions truth and falsity are predicates. According to Taftāzānī's solution, the truth that is the predicate of "P is true" does not contradict the falsity that is a concomitant feature of the proposition "The heavens are beneath us."

Before moving on to Ḥaṭībzāde's counter-argument against this solution, let me clarify a point to avoid possible misunderstandings.

 $<sup>^{20}</sup>$  Taftāzānī here means that "truth" and "falsity" cannot be predicated of sentences, but "true" and "false" that are derived from them can.

<sup>&</sup>lt;sup>21</sup> Cf. Taftāzānī, Šarḥ al-Maqāṣid, IV, p. 287.

Taftāzānī raised this solution in his work, but this does not necessarily imply his subscription to it. He seems to have put it forward as a possible argument for the Mu<sup>c</sup>tazilite position that good and evil are comprehensible by human reason. This is supported by his remarks in the last sentence Hatībzāde quoted (Solution, § 2): "Perhaps the respondent (al- $mu\check{g}\bar{\imath}b$ ) can deny ( $yamna^c$ ) the contradiction between the truth and falsity, which imply each other, on the ground that one of them is related to the judgment in that individual proposition while the other to its subject". 22 "The respondent" here is probably a Muctazilite interlocutor in a hypothetical debate on the problem of good and evil. Also, Taftāzānī makes clear his stance on the paradox in the following lines, saying "However, the correct attitude in my opinion towards this [paradoxical] proposition is to give up on a solution and to admit our incapability to solve this conundrum,"23 which is quite understandable given that he was an Ašcarite theologian who embraced the idea that good and evil are not essential properties of things and could only be known through divine revelation. It is even possible to argue here that by coming up with such a weak solution to show the insolubility of the paradox, he might have committed a straw-man fallacy given that, as far as I know, no one ever supported such a solution.

Like his contemporaries, Ṣadr al-Dīn al-Daštakī (d. 1498) and Dawānī,<sup>24</sup> Ḥaṭībzāde is aware of the weakness of the solution and raises immediately this possible objection to it:

[t2] One can object to it by [saying] that the truth of the proposition whose predicate is either truth or falsity, although it does not contradict the truth and falsity taken as a state of the proposition, implies its contradictory, which is truth and falsity, and their combination requires the combination of truth and falsity that are states of the judgment. [Solution, § 3]

In this enigmatic objection, which may not be original to Ḥaṭībzāde, he first grants that there may not occur a contradiction between truth

<sup>&</sup>lt;sup>22</sup> Cf. Taftāzānī, Šarh al-Magāsid, IV, p. 287.

<sup>&</sup>lt;sup>23</sup> Taftāzānī, Šarḥ al-Maqāṣid, IV, p. 287. Ḥaṭībzāde does not quote this sentence, probably to avoid the impression that by proposing a solution he goes against such a towering authority as Taftāzānī.

<sup>&</sup>lt;sup>24</sup> Daštakī and Dawānī do not address Taftāzānī's solution in detail; nor do they refer to any argument raised against it. For Daštakī's dismissal of it, see "al-Muntaḥab min ḥāšiya <sup>c</sup>alā al-šarḥ al-ğadīd li-al-*Tağrīd*," in Qaramalikī, *Davāzdah risāla*, p. 5; "Risāla fī ḥall šubhat ğadhr al-aṣamm," in Qaramalikī, *Davāzdah risāla*, p. 29. For Dawānī's stance on it, see "Nihāyat al-kalām fī ḥall šubhat ğadhr al-aṣamm," in Qaramalikī, *Davāzdah risāla*, p. 106.

and falsity that are states of the proposition, on the one hand, and truth or falsity that are the predicates, on the other. The reason for this is obvious: a contradiction occurs between two propositions with the same subject and predicate, but different in quality and quantity, if they are quantified at all. However, this is not the case in the two propositions discussed here. To use Hatībzāde's examples, let us take the propositions "Zayd is standing" and "Zayd is standing' is false" and suppose the former to be true. Although the latter patently declares the former to be false, these two do not contradict each other because they are not common in the subjects and predicates. However, according to Hatībzāde (Solution, § 3), there is an indirect contradiction here, though. If the former is true, then Zayd is standing, but if the latter is true, then he is not standing, these two situations excluding each other. Then, Taftāzānī's solution is right that there is no direct contradiction between the two propositions, but they together obviously lead to a contradiction and therefore cannot hold at the same time.

Having refuted the solution couched in *Šarḥ al-Maqāṣid*, Ḥaṭībzāde reveals his disagreement with Taftāzānī's skepticism of the possibility of a solution, confidently stating that both forms of the paradox are soluble and sets out to elucidate his ways of solution beginning with the Strengthened Liar Paradox.

#### 2. HATĪBZĀDE ON THE STRENGTHENED LIAR PARADOX (SLP)

Ḥaṭībzāde's Muslim predecessors, who had something to say about the paradox, usually preferred the positive or simple formula of the liar sentences, such as "What I am saying now is false". However, Ḥaṭībzāde, probably following the lead of Taftāzānī, makes use of the negative or "strengthened" formula of the sentences, like "What I am saying now is not true". Although neither Ḥaṭībzāde nor any other medieval Arabic logician displays any sign of awareness of the difference between these two ways of formulating the liar. <sup>25</sup> his usage will be followed in this paper.

According to  $\mu$ aț $\bar{\nu}$ bz $\bar{\nu}$ de, the solution of SLP depends on three premises:

[t3] I say: Both forms of the paradox are possible to solve. As for [the solution of] the first, it is based on some premises: [I] First, truth and falsity, as well as two states of judgment, may be two predicates of it [i.e., the judgment] when it is meant by them to affirm the presence of that state in

 $<sup>^{25}</sup>$  Ḥaṭībzāde uses once in the treatise the positive, simple form of the liar sentence as well (Solution,  $\S$  7); therefore, he seems to be unaware of the difference.

the subject  $(ma^c r \bar{u} \dot{q})$ , which is the judgment. [II] Second, this state is essentially different from the judgment since it amounts to the correspondence of the judgment to the facts and the lack thereof. Both indubitably differ from it [i.e., the judgment] in essence. [III] Third, the judgment [consisting] of a state of the thing that is essentially distinct from it [i.e., the thing] is [also] essentially distinct from it [i.e., the thing], which is obvious. [Solution, § 4]

The first premise to the effect that "true" and "false" can either figure in propositions as predicates or function as their tacit properties seems to be taken from Taftāzānī almost verbatim (see t1) and is relatively uncontroversial. It is perfectly reasonable to predicate "true" and "false" of a proposition if it is taken as their subject. The second premise sounds also quite plausible on the ground that a property should be different from its subject if this attribution is meant to be informative, that is, not tautological. Looking for the truth of the proposition in its correspondence to the extramental reality, on the other hand, is a theory of truth largely agreed upon in the Arabic tradition of logic 27 that developed mainly along Aristotelian lines. So far so good, but when it comes to the third, the backbone of Ḥaṭībzāde's argument, it seems rather disputable, not to mention that it is puzzling and in dire need of explanation, although he claims that it is obvious.

To disentangle Ḥaṭībzāde's third premise to some extent, let me make use of the example above, "Zayd is standing." Truth and falsity are two properties of this proposition, which are essentially different from it. If we predicate one of these properties of it, "true" for example, we get the second-order proposition of "Zayd is standing' is true." This new proposition is also essentially different from the former, which functions as the subject here. Though formulated in a somewhat obscure way, what Hatībzāde means is now clearer: the subject of a proposition should be distinct from the proposition itself. To put it another way, the proposition cannot be identical to, or included in, what the subject-term refers to; it cannot be self-referential. Indeed, the example above satisfies this condition and therefore leads to no paradox. However, as Hatībzāde noted, when it comes to such ambiguous propositions as "This is true" or "This is false," the situation gets a little more complicated since the pronoun "this" may refer to something other than this proposition, in which case no problem occurs. But if it refers to the proposition itself, in other words,

<sup>&</sup>lt;sup>26</sup> Taşköprülüzâde, "Qawācid al-ḥamliyyāt fī taḥqīq mabāḥit al-kulliyyāt," ed. & trans. into Turkish M. Zahid Tiryaki, in Kübra Şenel et al. (eds.), Felsefe Risaleleri (Istanbul, 2016), p. 156–161.

 $<sup>^{27}</sup>$  For some exceptions to this general convention, see Taftāzānī, Al-muṭawwal  $^cal\bar{a}$  al-Talhīṣ (Istanbul, 1286), p. 36–38.

if the proposition is taken as self-referential, then it cannot be given a truth value because, in Ḥat $\text{\=i}$ bz $\text{\=a}$ de's terms, "this sentence (qawl) is not a proposition, even if it is in a propositional form"  $(Solution, \S 5)$ . That is to say, Ḥat $\text{\=i}$ bz $\text{\=a}$ de does not regard self-referential sentences as propositions even if they are formulated in a propositional structure. He explains, depending on the premises above, why this kind of sentence is to be excluded from the category of propositions as follows:

[t4] [This is] because, according to the first premise, the judgment [attributing] truth or falsity to a thing is a judgment [that predicates] of it one of its states; according to the second premise, the judgment [predicating] of the thing one of them [i.e., truth and falsity] is a judgment [that predicates] of it one of its states that is essentially different from the thing; and the judgment [that attributes] one of its states to the thing that is essentially distinct from it [i.e., the thing] is essentially different from that thing, according to the third premise. No doubt, when we mean by that the proposition itself, the judgment of truth or falsity would not essentially differ from the thing, which is their subject, namely the judgment. [Solution, § 5]

Ḥaṭībzāde seems to mean here, again in a puzzling way, that self-referential sentences are not propositions according to the third premise, on the ground that in them the proposition and its subject are not essentially distinct from each other. The judgment of the proposition turns back to the proposition itself because its subject-term refers to it. He proves his argument through a *reductio*:

[t5] Otherwise, the predicate of the proposition that is taken as a mirror for the assessment of the other proposition, as a state of which the truth is taken, would be essentially different from the predicate of the proposition to be assessed. This is impossible because it entails that one and the same proposition, which includes a report of its truth or falsity, should be two essentially different propositions, one of which is a mirror for assessment of the other, as in our statement "Zaid is standing is false." [Solution, § 5]

Having stipulated in the third premise that the subject-term should not refer to the proposition itself, Haṭībzāde explains here the impossible situation that would otherwise arise. For him, if the proposition, for example, "This is false" is to be taken as self-referential, then there would be two predicates and therefore two propositions, one of which is judged while the other judges, as is easily distinguishable in the unproblematic example "Zayd is standing' is false." For there are clearly two propositions here: one of them ("Zayd is standing' is false") makes a judgment about the truth value of the other, which is in the position of the subject. Haṭībzāde calls this second-order proposition "the mirror"  $(mir^3at)$ , a common allegory in medieval Islam used in various contexts.

As is evident, the other one is the first-order proposition, "Zeyd is standing." However, when it comes to such paradoxical cases as "This is false," or "The sentence I am uttering now is not true," are we still allowed to make such a distinction between the first-order and second-order propositions? For Ḥaṭībzāde, it is impossible because it amounts to regarding one and the same proposition as two essentially distinct ones, which is inconceivable.

To sum up, Hatībzāde's solution to SLP arising from "The sentence I am uttering now is not true," relies on his argument that such selfreferential statements are not propositions on the ground that the judgment and its subject in them are not essentially distinct from each other. In his account, however, there is something unclear, namely whether he is extending this argument to cover all self-referential sentences or just ones whose predicates are "true" or "false." In the third premise in [t4], he seems to be talking about all self-referential sentences, denying categorically their propositionality. However, despite the wording of his premises, I think this reading would be too sweeping for him to support. This is because if we read his premises in the context, that is, if we remember that he is talking about the liar paradox and the liar sentences, whose predicates are "false" or "not true," then we can reach a more charitable conclusion that the object of his argument was not all self-referential sentences, but only those with truth and falsity as their predicates. Notice here that he begins the first premise by speaking of truth and falsity and his examples are always sentences of that kind. If I am correct, then his diagnosis of the fallacious element in the liar sentences can be more accurately read as the combination of selfreference and the predication of truth and falsity: A sentence, which is self-referential and whose predicate is "true" or "false" is not a proposition. Let us take this as the formulation of Hatībzāde's solution, at least, for the time being, because I will have to qualify it shortly.

An anonymous <code>husn-qubh</code> treatise, which is apparently contemporaneous with <code>Ḥat̄ībzāde</code>'s and dedicated to Mehmed II, provides a further corroboration for this reading of <code>Ḥat̄ībzāde</code>'s argument. In complete agreement with <code>Ḥat̄ībzāde</code> about the first and second premises, the anonymous author says about the third that the essential differentiation stipulated by the premise cannot be realized when the extension of the subject-term includes the proposition itself <code>and</code> the predicate is "true" or "false." I think this is exactly what <code>Ḥat̄ībzāde</code> means: The sentences, in which these two problems come together, are not propositions. However, the anonymous author differs from <code>Hat̄ībzāde</code>, arguing that in

such sentences, the essential difference is not required and one and the same proposition can be regarded as two different ones considered from different aspects.<sup>28</sup> He seems to claim that such a sentence, for example "The sentence I am uttering now is not true," can be interpreted as two propositions, one of which assesses the truth of the other and this interpretational difference between the two is sufficient to render it a proposition in the proper sense.<sup>29</sup> A fuller discussion of this solution would take us off the point, but let me only state here that, by stressing so strongly the *essential* difference between the proposition and its subject, Ḥaṭībzāde might have tried to distance himself from solutions of this kind.

Hatībzāde's solution reminds us of that of Nasīr al-dīn al-Tūsī (d. 1274), who also claimed that self-referential propositions cannot be qualified as true or false. In his  $Ta^{c}d\bar{\imath}l \ al\text{-}mi^{c}y\bar{a}r$ , a critical commentary on Atīr al-Dīn al-Ahbarī's (d. 1265) Tanzīl al-afkār, Tūsī contends that truth and falsity cannot be applied to a sentence when the declarative sentence (habar) and its subject (muhbar canh) are not distinct from each other. This is because, for him, the truth value of a proposition is determined by its correspondence to reality, and this correspondence can only be realized between two things. When the declarative sentence and its subject are the same, it is not possible to talk about the existence or non-existence of such a correspondence. Therefore, such propositions are neither true nor false. 30 As evident, Hatībzāde concurs with Tūsī that self-referential sentences are not apt for the assignment of a truth value. But would Tūsī also have excluded self-referential sentences from the category of propositions? In other words, is there a category of propositions that is neither true nor false in his logic? Answering these questions is not so easy because, unfortunately, the relevant passage of his text is so flawed that it is difficult to derive a coherent theory from it. However, Tūsī's statement here that the paradox arises from "mistaken consideration of predication" ( $s\bar{u}^{\circ}i^{c}tib\bar{a}r$  al-haml) can be read

<sup>&</sup>lt;sup>28</sup> Risāla fī al-ḥusn wa al-qubḥ al-caqliyayn (Istanbul, Süleymaniye Yazma Eser Kütüphanesi, MS Ayasofya 973), fols. 36v-37r.

<sup>&</sup>lt;sup>29</sup> Actually, this is a solution to the paradox that has been proposed in similar forms by scholars such as Šams al-Dīn al-Samarqandī (Kistâsu'l-efkâr Düşüncenin Kistası, ed. Necmettin Pehlivan [Istanbul, 2014], p. 553) and Mollā Ḥayālī (Šarh al-ʿAllāma al-Ḥayālī alā al-Nūniyya, p. 235–236).

<sup>&</sup>lt;sup>30</sup> Tūsī, "Tacdīl al-micyār fi naqd *Tanzīl al-afkār*," in M. Mohaghegh & T. Izutsu (eds.), *Collected Texts and Papers on Logic and Language* (Tehran, 1974), p. 237. For an analysis of Tūsī's solution, see Alwishah & Sanson, "The early Arabic liar," esp. p. 113–123.

to suggest that the liar sentences have no predication and therefore are not propositions. This makes sense also given the principle generally accepted by Arabic logicians that there should be a difference between the subject and predicate of a proposition because, otherwise, it becomes a tautology. However, his constant use of the term "declarative sentence" (habar) to refer to such sentences brings to mind the possibility that he might have accepted them as propositions.

In addition, to trivialize Dawānī's position, Daštakī claims that no one before him claimed that the liar sentences were not propositions. Answering this claim, Dawānī could have well resorted to the authority of Tūsī if he had known Tūsī was of that view. Instead, he speaks about different uses of the term *habar*. This seems to imply that the prevalent view in the tradition has been that Tūsī considered self-referential and therefore paradoxical sentences as propositions. In this case, Ḥaṭībzāde can be credited with raising for the first time the argument that the liar sentences are not propositions, at least until one discovers an earlier figure who came up with the same idea.

As we have seen, Ḥaṭībzāde agrees with Tūsī that the liar sentences cannot be given any truth value, and it is perfectly possible that the former was influenced by the latter in this regard. This is the easier part of the question of influence because the relationship between Ḥaṭībzāde's solution and Dawānī's proves rather problematic. However, I will set it aside awhile and rather move on to Ḥaṭībzāde's treatment of the second, circular form of the paradox.

## 3. ḤAṬĪBZĀDE ON THE CIRCULAR LIAR PARADOX (CLP)

As stated above, the two sentences of the same person, namely "The sentence I will utter tomorrow is not true" (yesterday-sentence, YS)<sup>34</sup> and "The sentence I uttered yesterday is true" (tomorrow-sentence, TS) lead to the circular form of the paradox because the truth of each sentence implies its falsity but, this time, takes a circular way through the

<sup>&</sup>lt;sup>31</sup> Taşköprülüzâde, "Qawā<sup>c</sup>id al-hamliyyāt," p. 156–161.

<sup>&</sup>lt;sup>32</sup> Daštakī, "Risāla fī ḥall šubhat ğadhr al-aṣamm," p. 52.

<sup>33</sup> Dawānī, "Al-Muntaḥab min ḥāšiyatih calā al-šarḥ al-ğadīd li-t-Tağrīd," in Qarama-likī, Davāzdah risāla, p. 87.

<sup>&</sup>lt;sup>34</sup> Both Taftāzānī and Ḥaṭībzāde have also universal forms of the yesterday-sentence, such as "Nothing I will say tomorrow is true" and "Everything I will say tomorrow is false," but they seem to use the particular and universal forms interchangeably, without discerning any significant difference between them.

other. To my best knowledge, Ḥaṭībzāde is the first to deal with this form of the paradox independently and offer a comprehensive solution to it.

Before proceeding to discuss Hatībzāde's solution, let me linger on a point, made by both Taftāzānī and Hatībzāde, that the YS should be taken as an external proposition. According to the distinction between the essentialist  $(haq\bar{\imath}q\bar{\imath})$  and externalist  $(h\bar{a}ri\check{g}\bar{\imath})$  readings of propositions, if the referents of the subject-term that exist in the external world are taken into account, this gives us the external reading of the proposition. However, only assuming the existence of those referents provides its essential reading. Accordingly, "The phoenix is a bird" is false if read externally because "phoenix" does not exist in the external world, but it is true if read essentially, that is, on the assumption that "If the phoenix existed, it would be a bird."35 Then, the YS here should be read externally because otherwise the hypothetical, externally non-existent referents of the subject-term could be taken into consideration, which would break the circle between the sentences and thus render them unparadoxical. We can hypothetically suppose, for example, that the speaker utters the next day "The world is empty," which does not imply the YS being false and then there would occur no paradox. This is why Taftāzānī and Hatībzāde emphatically stipulate that the YS should read as an external proposition and the only sentence the speaker utters the next day should be "The sentence I uttered yesterday is true." 36

Ḥaṭībzāde has two premises for the solution of the CLP and states them as follows:

[t6] As for [the solution of] the second [form], it builds on two premises: [1] First, the ability to accept ( $ihtim\bar{a}l$ ) truth and falsity is a concomitant feature of the declarative sentence (habar); [2] second, the cause of this ability is the possibility of the judgment's realization ( $tahaqquq\ al-hukm$ ), according to the thing-itself ( $nafs\ al-amr$ ), with either one of the two [truth values], one replacing the other. The pre-eminent scholar al-Šarīf [al-Ğurğānī] dwells on this in his commentary on the  $Mift\bar{a}h$ . [Solution, § 6]<sup>37</sup>

First of all, note that Ḥaṭībzāde abandons here the logical terminology by using the term "declarative sentence" (*habar*) instead of "proposition" (*qaḍiyya*). Ḥabar can be said to be the equivalent of *qaḍiyya*, generally used in the Islamic and linguistic sciences. However, we have no

<sup>&</sup>lt;sup>35</sup> On this distinction made first by Fahr al-Dīn al-Rāzī, see Rāzī, Manţiq al-Mulahhas, ed. Āḥad F. Qaramalikī & Adīna 'Asgarīnazhād (Tehran, 1381), p. 140–141.

<sup>&</sup>lt;sup>36</sup> Taftāzānī, *Šarh al-Magāsid*, p. 287.

<sup>&</sup>lt;sup>37</sup> For Ḥaṭībzāde's reference, see Ǧurǧānī, *Šarḥ al-Miftāh* (Istanbul, Süleymaniye Yazma Eser Kütüphanesi, MS Fatih 4655), fol. 8v ff. This copy of the commentary is important because it once belonged to Hatībzāde, as noted on its title page.

textual evidence that Ḥaṭībzāde could have seen any significant difference between the two terms, but he seems to follow the terminology of his source, as he makes clear in the final sentence of the passage, namely al-Sayyid al-Šarīf al-Ğurǧānī's (d. 1413) commentary on the third part of Abū Yacqūb al-Sakkākī's (d. 1229) classical handbook of rhetoric, *Miftāḥ al-culūm*.

Another term in the text that is in need of clarification is "the thingitself" or "nafs al-amr." In the treatise, Hatībzāde provides no explanation as to what he means exactly by the term, but in his super-glosses on Ğurğānī's glosses on Šams al-Dīn al-Isfahānī's (d. 1349) commentary on Nasīr al-Dīn al-Tūsī's (d. 1274) philosophical kalam handbook, the Tağrīd al-i<sup>c</sup>tiqād, we find significant clues. In this work, Ḥaṭībzāde seems to agree with Ğurğānī's account of the nafs al-amr given that he raises no objection against it although he is usually critical of Ğurğānī throughout the glosses. For Čurǧānī, "nafs al-amr" signifies the states of affairs of the thing in itself with no reference to any particular mind or its supposition. If it is said, for example, "x is existent in the nafs alamr," this means that it remains existent when no consideration ( $i^ctib\bar{a}r$ ) or supposition (fard) is taken into account. This does not mean, however, that it is synonymous with extramental reality (hāriğ) because the former is ampler than the latter: somethings existent in the thing-itself may not existent extramentally. As for its relationship with the mental existence, there is a partial overlap between them because the mind can suppose counter-factual creations that would not obtain in the nafs alamr. Besides, both Čuršanī and Hatībzāde go against the view of the nafs al-amr held by the author, Tūsī, namely that it is identical to the Active Intellect (al- $^{c}aql$  al-fa $^{c}\bar{a}l$ ), the tenth of the celestial intellects. $^{38}$ 

Ḥaṭībzāde's first premise in the text declares that the ability to be true or false is an inseparable concomitant of declarative sentences, while the second tells us the cause of this ability: the possibility that the judgment either corresponds to the thing-itself (*nafs al-amr*) and,

<sup>&</sup>lt;sup>38</sup> For Ğurğānī's glosses, see the page bottom of Šams al-Dīn al-Isfahānī, *Tasdīd al-qawā'cid fī šarh Tağrīd al-caqā'cid*, ed. Eşref Altaş et al., 3 vol. (Istanbul, 2020), II, p. 202; for their translation, see Moiz Hasan, "Foundations of science in post-classical era: The philosophical, historical, and historiographical significance of Sayyid al-Sharīf al-Jurjānī's (d. 1413) project" (unpublished Ph.D. dissertation, University of Notre Dame, 2017), p. 425–426. For Ḥaṭībzāde's comments, see *Ḥawāšī Ḥaṭībzāde calā Ḥawāšī šarh al-Tağrīd* (Istanbul, Beyazıt Yazma Eser Kütüphanesi, MS Veliyüddin 2006), fols. 105r-106r. For a comprehensive study of the subject, see Hasan Spiker, *Things as they are: Nafs al-amr and the metaphysical foundations of objective truth* (Abu Dhabi, 2021).

therefore, becomes true, or it does not and, therefore, becomes false. But "ability"  $(i\hbar tim\bar{a}l)$  and "possibility"  $(imk\bar{a}n)$  are almost synonymous terms. Then what kind of distinction might Ḥaṭībzāde have drawn between them so that the latter would be the cause of the former? We can find an answer to this question in Ḥaṭībzāde's source in [t6], Ğurǧānī's commentary. "Ability" here, Ğurǧānī explains, refers to mental possibility, that is, the hesitation of the mind about the truth and falsity of the sentence whereas "possibility" is related to the state of affairs in the nafs al-amr. Therefore, "possibility" is extramental, but "ability" is mental, and the mental is supposed to correspond to the extramental in order for a proposition to be true, which is why the possibility is the ontological ground of that ability of declarative sentences.

Another crucial point to notice here is that according to Ḥaṭībzāde's premises, a sentence is declarative if and only if it is apt to take both values, namely truth and falsity, in the *nafs al-amr*, in the sense that none of them is necessary or impossible for it in itself. In other words, it should be neither necessarily true nor necessarily false, but possible in both directions. Accordingly, if a sentence is always true or always false in itself, not due to an extrinsic cause, it would not be declarative. <sup>40</sup> This point greatly matters to us because it lies at the very heart of Ḥaṭībzāde's solution, as will be seen shortly.

If the subject-term of the YS ("The sentence I will utter tomorrow"), Ḥaṭībzāde goes on, refers to something which by no means implies this sentence's falsity, then the YS is a proposition in the proper sense, not leading to any paradox whatsoever. However, if it refers to a sentence that necessitates YS to be false, then the YS is neither a proposition nor a declarative sentence, even if it seems so formally. Ḥaṭībzāde's argument runs as follows:

[t7] [I] The ability to accept truth and falsity is a concomitant feature of the judgment and declarative sentence, [II] but it is absent here because of the non-existence of its cause, which is the possibility of the judgment's realization (tahaqquq) with truth and falsity, one replacing the other. [III] [This is] because if the judgment that the tomorrow-sentence is false to become realized with its truth, this would amount to its realization with the falsity of the tomorrow-sentence. If the tomorrow-sentence happens to be such that its falsity necessitates the yesterday-sentence to be false, then the yesterday-judgment's realization with its truth would be equivalent to its realization with its falsity. [IV] In this case, on the assumption of its re-

<sup>&</sup>lt;sup>39</sup> Ğurğānī, Šarh al-miftāh, fol. 10r.

<sup>&</sup>lt;sup>40</sup> See Fadl Ḥasan ʿAbbās, Al-balāga funūnuhā wa afnānuhā (ʾilm al-maʾānī) (Yarmūk, 1417/1997), p. 101.

alization, the judgment could not be characterized by the possibility of two realizations, one replacing the other, but characterized all the time by the possibility of one of them alone. [Solution, § 7]

In this dense argument, Hatībzāde first repeats (in [I]) the first premise above, but notice here that having ability to be true and false is a necessary concomitant  $(l\bar{a}zim)$  of declarative sentences so that its non-existence is a strong indication that the sentence is not declarative. 41 In [II], Hatībzāde predictably puts as a second premise that these sentences, that is, the sentences of the CLP do not have this necessary concomitant. Now, if he succeeds in proving that they lack this necessary concomitant, his argument will serve the purpose proving that the CLP sentences are not declarative. To do so, he tries in [III] to explain why he thinks that these sentences are devoid of that necessary concomitant: because its cause does not exist therein. According to the principle of sufficient reason, nothing comes to existence without its sufficient reason being already present. The cause in question is the possibility of the sentence's being true and false in the nafs al-amr. However, there is no such possibility in this case because the subjectterm of the YS refers to the TS, and the latter implies the former's falsity. In other words, supposing the YS to be true leads necessarily to its falsity, which means that this sentence must be false when true. But is this enough to justify Hatībzāde's argument that these sentences lack the possibility in question?

At this point, however, Ḥaṭībzāde seems to have made an unwarranted leap. For if the truth of the liar sentences requires their falsity, the only conclusion that follows from this is that they are both true and false at the same time. Yet Ḥaṭībzāde deduces that they are either always true or always false, not undeterminedly open to truth and falsity at the same time. He does not specify which one of truth and falsity the YS and TS necessarily assume, but he may have held that they are always false, tacitly assuming the premises that they always lead to a contradictory and impossible conclusion and that whatever implies the impossible is itself impossible. In addition, there are Arabic logicians, like Aṭīr al-Dīn al-Abharī, who argued that liar sentences are always

<sup>&</sup>lt;sup>41</sup> This understanding of the declarative sentence compelled Muslim scholars to discuss whether the Quranic verses and the sayings of the Prophet that are always true are declarative at all. Consequently, they had to emphasize that the extrinsic properties of the sentences, such as their utterers or sources, should not be taken into account. Only on this condition are the Quranic verses declarative. See Ğurğānī, Šarḥ al-miftāḥ, fol. 7v; Abū al-Baqā° al-Kafawī, Al-kulliyyāt, ed. °Adnān Darwīš & Muhammad al-Misrī (Beirut, 1998), p. 414–416.

false on the ground that their falsity implies no contradiction.  $^{42}$  In any case, he concludes that the liar sentences do not have in the *nafs al-amr* the possibility of being true *and* false, and therefore they are neither declarative sentences nor propositions. To reiterate, this is because they lack the ontological basis of being true and false and thus are deprived of the relevant ability, which is a necessary concomitant of statements. A sentence therefore devoid of this possibility cannot be a proposition. According to Ḥaṭībzāde, what needs to be done here is to deny the existence of a judgment, as well as truth and falsity, in YS and TS. "He who grants [the existence of] one of them and tries to solve the paradox," he claims, "is not on the wise path ( $^cal\bar{a}\ bas\bar{\imath}ra$ ) (Solution, § 7).

Once Hatībzāde's two premises are accepted, it is not difficult to achieve the conclusion he meant to reach. However, by putting these premises at the very outset and removing the paradoxical sentences from the category of declarative sentences, could be solve the problem or just move the discussion to the premises, given that his premises are not self-evident at all? On the contrary, there have always been views inconsistent with these premises in both Islamic and Western thought. To cite a few examples, 'Abd al-Qāhir al-Baġdādī (d. 1037) argues that the liar sentences are both true and false and, as such, constitute an exception to the principle of bivalence and even to the principle of noncontradiction.<sup>43</sup> Ibn Kammūna (d. 1284), on the other hand, extends the category of declarative sentences beyond the sentences that are true and false, talking about declarative sentences neither true nor false. 44 As discussed above, Tūsī avoids attributing a truth value to the paradoxical sentences even though he seems to accept them as declarative. Today, solutions to the paradox generally reflect one of these two tendencies, characterizing such sentences as either "true and false" or "neither true nor false." 45 Also, contemporary with Ḥatībzāde, the author of the aforementioned anonymous *husn-qubh* treatise raises objections to both of his premises. 46 Therefore, his premises seem to

<sup>&</sup>lt;sup>42</sup> Zarepour, "Abharī's Solution," p. 5.

<sup>&</sup>lt;sup>43</sup> See Alwishah & Sanson, "The early Arabic liar," p. 100–101.

<sup>&</sup>lt;sup>44</sup> See El-Rouayheb, "The liar paradox," p. 256.

<sup>&</sup>lt;sup>45</sup> For a general account of the contemporary solutions falling into one of these two categories, see Jc Beall, Michael Glanzberg and David Ripley, "Liar paradox," in Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, https://plato.stanford.edu/entries/liar-paradox/#ParaParaLogi, esp. § 4.1. Paracomplete and paraconsistent logics (accessed on 12 August 2021).

<sup>&</sup>lt;sup>46</sup> However, the expressions of the treatise are too cryptic to make sense of the objections. See *Risāla fī al-husn wa al-aubh al-caplivayn*. fol. 37v.

have served to change the focus of the discussion, rather than provide a solution.

Probably aware of this, Ḥaṭībzāde addresses a possible objection (Solution, § 8): What if this paradox is put forward as an objection to the idea that declarative sentences are necessarily capable of being true and false, or if this possibility or capability is interpreted in a way that is compatible with this form of the paradox? This question forces him to qualify his solution and present his identification of the problem in the paradoxical sentences in a much narrower, but, at the same time, more accurate way. He still insists that such sentences are not declarative because they do not have any judgments:

[t8] The way to this solution is through denying [the existence of] any judgment in this form of the paradox on the ground that hypothetically  $(fard\bar{a})$  any sentence  $(kal\bar{a}m)$  that declares itself to be false includes no judgment in the thing-itself  $(nafs\ al-amr)$  and, therefore, is neither a proposition nor a declarative sentence, even if it seems so in form. [Solution, § 8]

In this passage, Ḥaṭībzāde has arrived at the narrower description of problematic sentences. He first found the problem in self-referential sentences, and then, by the examples, he narrowed down his accusation to apply only to the self-referential sentences with the predicates "false" and "true." But now he limits the set of problematic sentences to "any sentence that declares itself to be false." Although he reaches this proposition in the context of the CLP, his solution applies to the other form of the paradox, the SLP, as well: "This solution covers, and works for, every form of the paradox" (*Solution*, § 9). Ḥaṭībzāde's position becomes clearer here: If a sentence requires its falsity, directly or indirectly, it is not a proposition. He puts this idea into words thus:

[t9] The signification  $(dal\bar{a}la)$  of a declarative sentence is nothing but what it signifies, namely what it involves of the nexus [between its subject and predicate] and affirmative or negative judgment. Two significations, I mean that of the affirmative judgment and that of the negative one, cannot co-exist in the same declarative sentence. [...] If we are to assume any declarative sentence that declares itself to be false, there would follow the combination of two significations in that sentence. [This is] because the report that any proposition is false, for example, includes the report that the affirmative nexus between the subject and predicate of the proposition does not obtain  $(w\bar{a}qi^ca)$  and this report and signification are in contradiction to the report and signification that that nexus obtains. It is inconceivable therefore to combine them in the same declarative sentence. [Solution, § 8]

According to the passage, every proposition contains an affirmative or negative nexus and tacitly implies that this nexus obtains in the extramental reality. However, if a sentence implies its falsity, despite the implied assertion of its truth, this amounts to saying that its nexus both does and does not accord with the factual reality, which is impossible. To repeat, since the implicit assertion of the proposition's truth is inevitably assumed, any new assertion that contradicts it will damage the nature of the proposition and render it a non-proposition. The most obvious example of this is the liar sentence with different variants that directly or indirectly state that it is false.

At this point, as a possible source or, at least, inspiration of Hatībzāde's solution that any sentence implying its own falsity is not a proposition, I would like to refer to a discussion of self-referent and self-destructive sentences in Qutb al-Dīn al-Rāzī al-Tahtānī's (d. 1365) commentary on Nağm al-Dīn al-Kātibī's (d. 1276) logic primer, the *Al-šamsiyya*, and Ğurğānī's glosses thereupon. In his commentary, Qutb al-Dīn counters an objection against the possibility of predication by maintaining that it amounts to saying "Predication is impossible" (al-hamlu muhalun),<sup>47</sup> which itself includes a predication. This would, he goes on, a case of the falsification of the thing through itself (ibtāl al-šay<sup>o</sup> bi-nafsih), which is impossible. Glossing this, Čurǧānī states that the objection that predication is impossible is inefficient because it includes a predication; it predicates the term "impossible" of "predication." The argument therefore falsifies itself and "whatever falsifies itself is false because if it were true, then it would be true and false at the same time, which is impossible."48 Moreover, the same idea occurs later in 'Alī Kuşcu's commentary on the Tağrīd al-i'tiqād, but this time Kuşcu puts it slightly different way: "What implies its own falsity on the assumption of its being true is absolutely false" (mā valzamu butlānuhu calā tagdīri sihhatihi fa-huwa bātilun gatcan). 49 In his superglosses upon Ğurğānī's glosses upon Tağrīd commentary by Šams al-Dīn al-Isfahānī, Hatībzāde uses this formulation, with the addition "and [it is] nothing at all" (wa lavsa bi-šavoin). 50 Taking into account

<sup>&</sup>lt;sup>47</sup> Note here that this sentence is not paradoxical because it makes no trouble when it is taken to be false. However some Muslim logicians, like Abharī, argue that the liar sentences are not different in this respect, leading to no paradox if they are false. On this, see Zarepour, "Abharī's Solution."

<sup>&</sup>lt;sup>48</sup> For both Qutb al-Dīn's commentary and Ğurğānī's glosses, see Qutb al-Dīn al-Rāzī, Taḥrīr al-qawā'id al-manṭiqiyya šarḥ al-risāla al-Šamsiyya (Cairo, 1367/1948), p. 92.

<sup>&</sup>lt;sup>49</sup> <sup>c</sup>Alā<sup>o</sup> al-Dīn <sup>c</sup>Alī Qūšǧī, Šarḥ Taǧrīd al-<sup>c</sup>aqā<sup>o</sup>id, ed. Muḥammad Ḥusayn al-Zirā<sup>o</sup>ī al-Raḍāyī (Qum, 1393), p. 332.

<sup>&</sup>lt;sup>50</sup> Hatībzāde, *Hawāšī Hatībzāde*, fol. 106r.

the similarity between this principle articulated by Ğurğānī and Kuşcu and what Ḥaṭībzāde said in [t8] and [t9], I speculate that he may have applied this principle to the liar paradox, which can be taken as his original contribution to the liar discussions.

As stated above, there are also remarkable parallels between the solutions proposed by Ḥaṭībzāde and by Dawānī. Given that they were contemporary and acquainted with each other, I think these paralells matter and invite the possibility of influence between them, in either direction. Therefore, the final section of the paper, after a brief account of Dawānī's solution, will make a comparison between their respective solutions and try to shed light on the problem of the relationship between them, following some historical clues.

#### 4. HATĪBZĀDE AND DAWĀNĪ

With the discussions he had with Daštakī and his son Ġiyāt al-Dīn al-Mansūr (d. 1542) in fifteenth-century Širaz, <sup>51</sup> Dawānī made notable contributions to the liar paradox literature of Islamic thought. His solution relies first on his view of the declarative sentence (habar). For him, the definitive nature of the *habar* is to report a factual nexus between its subject and predicate. If the report corresponds to the reality of the nexus, then it is true; otherwise, it is false. Here, this feature of declarative sentences distinguishes them from performative sentences  $(in\check{s}\bar{a}^{\circ})$  because in the latter the nexus does not exist independently of the speaker. Thus, if a sentence reports a factual nexus independent of both itself and the mind of the speaker, then it is declarative and thus apt to take a truth value.<sup>52</sup> As he was evidently committed to this theory of habar, I think Dawānī should have dismissed self-referential sentences as not propositions, for the sake of consistency. But curiously enough, he instead talks about true self-referential sentences, like "This sentence ( $kal\bar{a}m$ ) is compound" and false ones, like "This sentence is not compound."53 What is, then, his criterion to distinguish between selfreferential sentences that are propositions and that are not? In other words, how could he draw the distinction between the liar sentences

<sup>&</sup>lt;sup>51</sup> For a comprehensive analysis of the discussions, see El-Rouayheb, "The liar paradox," p. 251–275. On Dawānī's position and influences, see also Rezakhany, "Jalāl al-Dīn ad-Dawānī's solution," p. 183–220.

 $<sup>^{52}</sup>$  For a concise account of the theory of habar in Islamic thought, see El-Rouayheb, "The liar paradox," p. 252–253.

<sup>&</sup>lt;sup>53</sup> Dawānī, "Nihāyat al-kalām," p. 147.

and other unproblematic cases of self-reference, without falling into arbitrariness or, as Daštakī puts it,  $^{54}$  obstinacy ( $muk\bar{a}bara$ )?

For Dawānī, self-reference is part of the problem, but not the whole. The problem consists of using truth and falsity as predicates in self-referential sentences because such a sentence, for example, "This statement is true," when used to refer to itself, becomes unable to meet the condition of reporting a factual relationship, which is the definitive feature of declarative sentences. Like the performatives, this sentence has a nexus created by the speaker and thus is dependent on her: it ceases to exist as soon as the speaker and her mind are not taken into consideration. Therefore, self-referential sentences whose predicate is "true" or "false" should fall into the category of performative sentences, not declarative ones. Yet this distinction is not arbitrary since it depends on the fact that the formal features of the predicate affect the status of the proposition. Dawānī, therefore, seems to have taken the peculiarities of the predicates "true" and "false" to be a formal feature of the proposition, through which he tries to ward off claims of arbitrariness.

Setting aside the question of how tenable this defence of Dawānī's is, the parallels between his solution and Hatībzāde's are abundantly clear. Above all, they agree that self-reference plus predication of truth and falsity suffice to disqualify a sentence from the set of propositions, and therefore the liar sentences are not propositions. But when it comes to the reason why they are not so, the two philosophers differ from each other discernably. Dawānī argues that this is because these sentences do not report a factual nexus (nisba wāqi<sup>c</sup>ivya), which is supposed to be independent of the speaker and her mind. On the contrary, they themselves create the nexus, just as in the performative sentences.<sup>56</sup> Hatībzāde, on the other hand, bases his position on the premise that the liar sentences do not have the possibility in the nafs al-amr to be true and false, and that they therefore lack the necessary concomitant of propositions, which is the ability to take each truth value alternately. In addition, Dawani is of the view that the predicate "true" in selfreferential sentences is as problematic as "false," although he avoids explaining the reason, just saying that "Conscience  $(vi\check{g}d\bar{a}n)$  gives the right judgment"<sup>57</sup> on this matter. He justifies this evasive attitude with

<sup>&</sup>lt;sup>54</sup> Daštakī, "Min fawā<sup>3</sup>id al-Amīr Ṣadr al-Dīn al-Daštakī fī šubhat ğadhr al-aṣamm," in Qaramalikī, *Davāzdah risāla*, p. 19.

<sup>&</sup>lt;sup>55</sup> Dawānī, "Nihāyat al-kalām," p. 155.

<sup>&</sup>lt;sup>56</sup> El-Rouayheb, "The liar paradox," p. 266ff.

<sup>&</sup>lt;sup>57</sup> Dawānī, "Nihāyat al-kalām," p. 136.

reference to the fact that the predicate "true" is not the focus of the liar paradox. Ḥaṭībzāde, however, does not seem to have found any problem with the predicate "true," at least in his final analysis, as I discussed above.

As another point of comparison, Ḥaṭībzāde takes the two forms of paradox and treats each relatively separately, although he offers a solution at the end that is intended to apply to both. His language in his treatment of these two forms varies slightly. While dealing with the first form (SLP), he heavily uses the terminology of logic, but in his solution to the CLP, his terminology is mainly borrowed from the works on semantics ( ${}^c$ ilm al- $ma^c$ ānī). His frequent use of "declarative sentence" instead of "proposition" is an example of this variation. Dawānī's terminology is also from semantics. In addition, he states that self-reference in the paradox can be indirect as well, even through multiple intermediary propositions, but such propositions suffer from the same problem as the direct self-referential ones, such as "What I am saying now is false." He therefore does not feel the need to devote a separate treatment to them.  $^{58}$ 

As for the question of whether this close similarity indicates a relationship of influence between the two philosophers, let us try to discuss it in the light of the historical data available. First of all, let me note that neither Hatībzāde nor Dawānī gives any hint concerning the source of his solution. In addition, Dawānī discusses the solutions put forward before him one by one but makes no mention of Hatībzāde and his solution. However, as we mentioned above, the two philosophers knew of each other, though not in person. In the entry of Hatībzāde of his  $\check{S}aq\bar{a}^{3}iq$ , the Ottoman bio-bibliographer Taşköprīzāde (d. 1561) narrates that Dawānī sent his greetings to Hatībzāde and Hocazāde in a letter he wrote to a friend of his, living in Ottoman Anatolia. In this greeting note, Hatībzāde's name comes first, which he interprets as a sign of his superiority to Hocazāde in the eyes of Dawānī. 59 Whether this interpretation is correct or not, we learn from this anecdote not only that Dawānī knew of him but also that Dawani was held in high esteem among contemporary Ottoman scholars. Even the order of names in his greetings could be a subject of different interpretations and debates.

In the absence of any historical testimony on the meeting of the two philosophers, I reason that their acquaintance must have been through

<sup>&</sup>lt;sup>58</sup> Dawānī, "Al-muntaḥab min ḥāšiyatih calā al-šarḥ al-ğadīd li-al-Tağrīd," 83; "Ni-hāyat al-kalām," 104, 146.

 $<sup>^{59}</sup>$  Taşköprülüzâde, Al-š $aq\bar{a}$ °iq, p. 255.

their works. In the life story of Müeyyedzāde 'Abdurrahmān Efendī (d. 1516), one of the leading scholars in the court of Bayezid II and disciples of Dawānī, we find a significant testimony about the book exchange between Iran and Ottoman countries of the period. 60 Taşköprīzāde relates that with Mehmed II's issuing a death warrant for Müeyyedzāde, he had to escape first to Aleppo in 1476 and then to Širaz in the same year, where he studied with Dawānī for about a decade. 61 When he arrived there, Müeyyedzāde gifted Dawānī Hocazāde's well-known work, *Tahāfut*, which, according to Taşköprīzāde, he liked very much. <sup>62</sup> Judging from this, I would conjecture that Müeyvedzāde may have also taken together some of Hatībzāde's works to Dawānī, and through these works, he may have been aware of Hatībzāde's solution. However, for one thing, on that assumption, Dawani would be expected to have mentioned Hatībzāde among the solutions preceding him, which is not the case. Second, I doubt that Hatībzāde's treatise of the liar paradox was perceived then as a work important enough to be taken to Iran as a gift. Therefore, it seems hardly possible for Dawānī to have been aware of Hatībzāde's treatise.

The possibility of Ḥaṭībzāde having been influenced by Dawānī does not seem very likely, either. As I discussed in detail in another paper, <sup>63</sup> Ḥaṭībzāde's treatise seems to have been a part of, or an appendix to, a set of glosses he wrote on the Four Principles (al-muqaddimāt al-arbac') propounded by Ṣadr al-Šarīca al-Maḥbūbī (d. 1346) in his Tawḍāḥ. Given that the glosses were dedicated to Meḥmed II, as noted in the preface, <sup>64</sup> they must have been written before 886/1481 when the sultan died. Indeed, the oldest copy of his treatise on the paradox is dated to 884/1480. However, the earliest written record of the debate between Daštakī and Dawānī on the paradox is found in the former's second set of glosses on Alī Ḥuṣcu's (d. 1474) commentary on Naṣīr al-Dīn al-Ṭūsī's Taǧrīd al-ictiqād. These glosses, dedicated to Bāyezīd II, were written in, or just before 887/1482, as we learn from Müeyyedzāde's bi-

<sup>&</sup>lt;sup>60</sup> See Judith Pfeiffer, "Teaching the learned; Jalāl al-Dīn al-Dawānī's *ijāza* to Mu³ayyadzāda 'Abd al-Raḥmān Efendi and the circulation of knowledge between Fārs and the Ottoman Empire at the turn of the sixteenth century," in Maurice A. Pomerantz & Aram A. Shahin (eds.), *The heritage of Arabo-Islamic learning* (Leiden, 2016), p. 284–332.

<sup>&</sup>lt;sup>61</sup> Taşköprülüzâde, *Al-šaqā iq*, p. 469.

<sup>&</sup>lt;sup>62</sup> Taşköprülüzâde, *Al-šaqā³iq*, p. 235.

<sup>&</sup>lt;sup>63</sup> Daşdemir, "A fifteenth-century Ottoman treatise on the liar paradox."

<sup>&</sup>lt;sup>64</sup> Hatībzāde, *Hāšiya li-hall al-muqaddimāt al-arba*<sup>c</sup> *min al-Talwīh*, fol. 1v.

ography,  $^{65}$  and arrived in Istanbul not earlier than 888/1483.  $^{66}$  Therefore, even if we assume that with these glosses Ḥaṭībzāde learned about both Daštakī's and Dawānī's solutions, this could have happened only after he wrote his treatise on the paradox. We do not know, on the other hand, the exact period in which Dawānī authored his commentary on  $Tahd\bar{a}b$  al-mantiq, which includes a concise form of his solution,  $^{67}$  let alone whether, or when, it was at Ḥaṭībzāde's disposal. Thus, the possibility that Ḥaṭībzāde was influenced by Dawānī in his treatment of the liar paradox seems rather weak. Therefore, these two philosophers seem to have almost simultaneously proposed solutions to the paradox that were quite similar, but independent from each other, in Istanbul and Širaz, two important intellectual centers of the Islamic world of the time.

#### 5. CONCLUSION

The present paper has dealt with a treatise on the problem of al- $\check{g}adr$ al-asamm or the liar paradox, which was written by Hatībzāde as a part of theological debates around the issue of good and evil during the reign of the Ottoman Sultan Mehmed II. As a study aiming to introduce the treatise to the contemporary discussions on the conception of the paradox in Islamic thought, the paper has reached the conclusion that the author of the treatise seems to have taken as a point of departure the relevant passage of Taftāzānī's theological work Šarh al-Magāsid and discussed the two different forms of the paradox accordingly. However, unlike Taftāzānī, who thinks that it is best not to try to solve it, Ḥatībzāde offers slightly different solutions to these two forms. To solve the first form (SLP), which originates from the sentence "The sentence I am uttering now is not true," Ḥatībzāde starts from three basic premises. The first premise states that truth and falsity can be inseparable features of the proposition as well as its direct predicate while the second declares that the proposition is essentially different from its concomitant that is truth and falsity. These two premises are not very problematic, but

<sup>&</sup>lt;sup>65</sup> Reza Pourjavady, "Jalāl al-Dīn al-Dawānī (d. 908/1502), Glosses on 'Alā' al-Dīn al-Qūshjī's commentary on Naṣīr al-Dīn al-Ṭūsī's *Tajrīd al-i*ctiqād," in Khaled el-Rouayheb & Sabine Schmidtke (eds.), *The Oxford handbook of Islamic philosophy* (Oxford, 2017), p. 424.

<sup>&</sup>lt;sup>66</sup> Taşköprülüzâde, *Al-šaqā ³iq*, p. 469.

<sup>&</sup>lt;sup>67</sup> See Dawānī, "Šarḥ al-Muḥaqqiq al-Dawānī 'alā Tahdhīb al-manṭiq," in Šarḥā al-Muḥaqqiq al-Dawānī wa-al-Mullā 'Abdallāh al-Yazdī 'alā Tahdhīb al-manṭiq, ed. 'Abd al-Nasīr al-Malībārī (Kuwait, 2014), p. 173–174.

the third, which gives the impression that it leaves all self-referential sentences out of the category of propositions, seems open to objections. Despite the literal interpretation of the premise, I think we can deduce from the context of the subject, that is, the fact that Ḥaṭībzāde was dealing with the liar sentences in the treatise, and from the examples he used to explain the subject, that he only meant the self-referential sentences whose predicates consist of the terms "true" or "false." Therefore, Ḥaṭībzāde's solution to SLP can be expressed as denying that the relevant liar sentence is a proposition.

The second form of the paradox, CLP, arises from the two liar sentences, namely "The sentence I will utter tomorrow is not true" and "The sentence I uttered yesterday is true," verbalized by the same person one day after another. Ḥaṭībzāde has two premises for the solution of this form. The first states that having the ability to be true and false is a necessary concomitant of declarative sentences and the other makes clear the cause of this ability as the possibility that the statement in itself be open to both truth and falsity in the *nafs al-amr*. The two liar sentences have neither the possibility nor the ability, Ḥaṭībzāde reasons, because to suppose their truth leads inevitably to their falsity and contradictions cannot obtain in the *nafs al-amr*. Ḥaṭībzāde finally reaches the proposition that is valid for both forms of the paradox: Any sentence that implies its own falsity is not a proposition, which limits the problem to the self-referential sentences with the predicate "false."

His solution bears close parallels with that of his Persian contemporary Dawānī, but we do not have any historical or textual evidence to show that one of them was influenced by the other or was aware of the solution of the other. Historically, Ḥaṭībzāde's treatise seems to have preceded Dawānī's writings on the subject, but our knowledge of this is rather insufficient. What is obvious, however, is that both benefited from the same sources, especially the theory of declarative sentences of Muslim scholars, such as Taftāzānī and Ğurǧānī. From these sources, they reached similar conclusions independently of each other. This conclusion, of course, is quite tentative and open to be qualified or challenged by further research on the subject.

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