Ground Resonance of the Helicopter

By

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SUMMARY

Self excited vibrations of helicopters on the ground have been the cause of several accidents It is, therefore, essential that the means of ensuring freedom from such phenomena should receive serious attention

The theory of R P COLEMAN[2], is briefly reviewed and compared with results obtained from model rotor tests This comparison affords general confirmation of the theoretical results

The theory of a rotor on idealised supports is applied to a single rotor helicopter of known hub impedance Then experimental methods for determining this impedance are described

Finally, the influence of Ground Resonance on undercarriage design is reviewed The influence of shock absorber struts and tyres on the coupled rigid body modes of the helicopter is discussed and three undercarriage schemes representing alternative approaches to ground resonance elimination are examined It is found that, with respect to ground resonance, the most advantageous system for land based operations is one in which all the helicopter-undercarriage modes have very low frequencies

An Appendix indicates the application of the theory to a twin rotor helicopter of any configuration

"GROUND RESONANCE" OF THE HELICOPTER

Let us start by defining what we mean by "Ground Resonance" of a helicopter

We refer to a divergent oscillation of the helicopter on its undercarriage, in which the rotor hubs move cyclically in the plane of rotation, it may occur when a helicopter rests on the ground with its rotor running or, more probably, when taxying, landing or taking-off

We are to consider a problem in which energy may be interchanged between that of the helicopter oscillation and the kinetic energy of rotor rotation Self excited or divergent oscillations occur if the coupling between the rotor freedom and one or more degrees of freedom of the helicopter allow energy to be transferred from the stored kinetic energy of rotation to

The authors wish to acknowledge their debt to the Aircraft Division of the M O S for having given permission to show the film and use the experimental data presented They also wish to thank their colleagues at The Bristol Aeroplane Company for much helpful guidance

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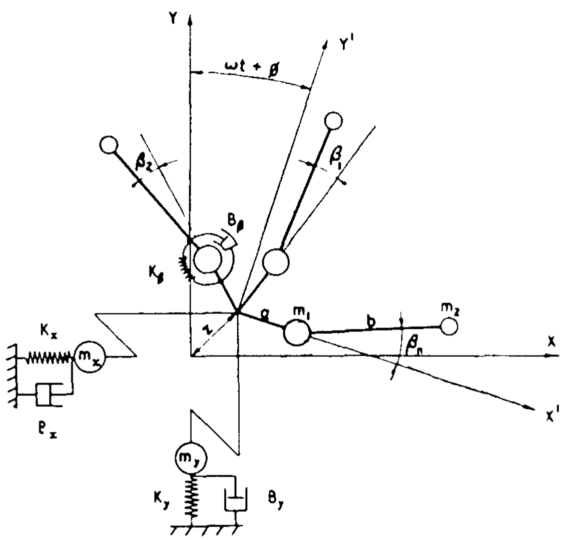
the helicopter oscillation at a greater rate than energy is dissipated at the undercarriage and at the drag hinge dampers   The problem is analogous in some respects to the flutter of a wing

It has been shown that self excitation can only occur at rotor speeds in the vicinity of a natural frequency of the helicopter, so we shall confine our attention to these lower frequencies

The equations of motion relating to the vibration of a rotor in its plane of rotation, and the dynamic system on which it is mounted, apply for any frequencies and are of use in the study of flight vibrations   However, it is general practice to ensure that the lowest natural frequencies of the helicopter, when airborne, should be well above 1 x rotor   This confines the phenomenon of self excitation to modes of vibration of the helicopter on its undercarriage—hence the title of this paper

Those of you, who like ourselves, have witnessed instances of this phenomenon will not need to be reminded of its possibly disastrous consequences   Fortunately, the instances at Bristol have not been of an explosive nature , other Firms have not been so lucky and some helicopters have been wrecked in a matter of a very few seconds after the onset of " ground resonance "

We would like, now, to show you a film of " ground resonance " of our Type 173 Mk 1 Helicopter   The instance you will see marked the beginning of our real interest in the problem   Parts of this film were shown at a



DYNAMIC SYSTEM FOR A SINGLE n BLADED ROTOR

FIG 1

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previous[1] meeting of this Association by our Chief Helicopter Test Pilot, " Sox " HOSEGOOD

Theory of Coupled Drag Hinge Oscillations

Instances of instability occurred with Rotaplanes before World War II Such instances were thought to be whirling problems, and a simple theory was formulated which gave the whirling speed of an articulated rotor on isotropic supports With the arrival of the helicopter, however, numerous incidents occurred which could not be attributed to whirling—the frequency of this oscillation was not the same as that of rotor rotation and, sometimes, the motion was violent in the extreme

In 1943, R P COLEMAN[2] produced his paper " The Theory of Self Excited Mechanical Oscillations of Hinged Rotor Blades " This paper has been for a long time the standard work on the subject

We propose now, to give a brief outline of the Theory for a single rotor on idealised supports This is a resume of Coleman's Theory

Theory of Drag Hinge Oscillation

Notation With reference to Fig 1, we define

X, Y = Fixed axes

X^1, Y^1 = Rotating axes

z = $x + iy$ relative to fixed axes

n = number of blades

a = drag hinge offset

b = distance from drag hinge to the centre of percussion of the blade

$\beta_n, \beta_1, \beta_2,$ = drag hinge deflections

m_x, m_y = masses in x and y directions

K_x, K_y, K_β = stiffness at hub and about drag hinge

B_x, B_y = damping at hub and about drag hinge

$M = n(m_1 + m_2) + \frac{1}{2}(m_x + m_y)$ $\Delta M = \frac{1}{2}(m_x - m_y)$

$K = \frac{1}{2}(K_x + K_y)$ $\Delta K = \frac{1}{2}(K_x - K_y)$

$B = \frac{1}{2}(B_x + B_y)$ $\Delta B = \frac{1}{2}(B_x - B_y)$

ω = rotor angular velocity

ω_f = angular frequency of hub oscillation

ω_β = angular frequency of drag hinge oscillation

ω_{rx}, ω_{ry} = natural frequencies in x and y directions, if masses m_1 are considered concentrated at the hub

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$$\Lambda_1 = \frac{a}{b}, \quad \lambda = \frac{B}{M\omega r}$$

$$\Lambda_2 = \frac{K_\beta}{m_2 b^2 \omega r^2}, \quad \Delta\lambda = \frac{\Delta B}{M\omega r}$$

$$+\Lambda_4 = \frac{nm_2}{2[n(m_1 + m_2) + mx]}, \quad \lambda_\beta = \frac{B_\beta}{m_2 b^2 \omega r}$$

+ Provided $m_x = m_y$, or $K_y = \infty$

Other quantities will be defined in the text, as they arise

In general, the rotor has n degrees of freedom in the plane of rotation, if we treat each blade as a rigid body We wish to find which of these n freedoms can couple with motion of the hub

Torsional displacement of the " tip mass " nm_2 may be expressed

$$\theta_0 = \frac{ib}{n}(\beta_n + \beta_1 + \beta_2 + \quad) \tag{1}$$

Also, we can express the coordinates of the C G of the " tip masses " with respect to rotating axes

$$\theta_1 = \sum_{k=1}^{n} x_k^1 + iy^1_k$$

$$= \frac{1}{n} \sum_{k=1}^{n} \left[a\, e^{i\frac{2\pi}{n}k} + b\, e^{i\left(\beta_k + \frac{2\pi}{n}k\right)}\right] \tag{2}$$

Performing the summation and making the usual assumptions for small oscillations

$$\theta_1 = \frac{ib}{n}\left(\beta_n + \beta_1 e^{i\frac{2\pi}{n}k} + \beta_2 e^{i\frac{4\pi}{n}k} + \quad \right) \tag{3}$$

By analogy, the remaining n — 2 modes become

$$\theta_k = \frac{ib}{n}\left(\beta_n + \beta_1 e^{i\frac{2\pi}{n}k} + \beta_2 e^{i\frac{4\pi}{n}k} + \quad \right) \tag{4}$$

and, in particular,

$$\theta_{n-1} = -\bar{\theta}_1 \tag{5}$$

From an examination of these n modes it can be seen that the mode θ_1 and its complex conjugate *alone* are able to couple with lateral hub displace-

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ments All other modes than θ_0, θ_1, in fact, concern symmetrical oscillations within the disc The mode θ_0 represents the only mode which will couple with transmission torsional modes

Now, by symmetry, the phase displacement between the kth and (k + 1)th blade must be constant for a given mode, thus, for the n modes of vibration of the blades, the blade motion may be expressed

$$\beta_n = \beta_0 \sin \omega_\beta t = -\frac{i\beta_0}{2}\left(e^{i\omega_\beta t} - e^{-i\omega_\beta t}\right)$$

$$\beta_1 = \beta_0 \sin\left(\omega_\beta t + \frac{2\pi}{n} m\right) = -\frac{i\beta_0}{2}\left(e^{i\left(\omega_\beta t + am\right)} - e^{-i\left(\omega_\beta t + am\right)}\right)$$

$$\beta_k = \beta_0 \sin\left(\omega_\beta t + \frac{2\pi}{n} mk\right) = -\frac{i\beta_0}{2}\left(e^{i\left(\omega_\beta t + akm\right)} - e^{-i\left(\omega_\beta t + akm\right)}\right) \tag{6}$$

where m is any integer + ve or — ve and $a = \frac{2\pi}{n}$

Substituting into (3)

$$\theta_1 = \frac{\beta_0 b}{2n}\left[e^{i\omega_\beta t} \sum e^{iak(1+m)} - e^{-i\omega_\beta t} \sum e^{iak(1-m)}\right] \tag{7}$$

It can be shown that $\theta_1 = 0$ for any value of the integer m other than — 1 or + 1, and that

$$\theta_1 = \frac{\beta_0 b}{2} e^{i\omega_\beta t} \quad \text{if } m = -1 \tag{8}$$

$$\theta_1 = -\frac{\beta_0 b}{2} e^{-i\omega_\beta t} \quad \text{if } m = +1 \tag{9}$$

The symbol θ_1, used up to now, defines two modes corresponding to $m = -1$ and $m = +1$ We now limit the mode θ_1 to that obtained from $m = -1$

From equations (8) and (9) the mode obtained by putting $m = +1$ is minus the complex conjugate of that from $m = -1$ and hence is $-\bar{\theta}_1$, as now defined, *i e*, θ_{n-1} from equation (5)

β_n, β_1, are given by equations (6) substituting the appropriate value of m

It is seen, that the only mode which can couple with hub motion in the plane of rotation is one in which the blades move about their drag hinges

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with a phase displacement of $\frac{2\pi}{n}$ between each pair of adjacent blades, the C G of the tip mass rotates in a circle relative to the rotor shaft at an angular velocity ω_β the frequency of the drag hinge oscillations

Thus if ω_f is the frequency of hub lateral oscillations, we must have

$$\omega_f = \omega - \omega_\beta \tag{10}$$

The equations of Motion

The variable θ_1 may be expressed with respect to fixed axes by means of the relation

$$\zeta_1 = \theta_1 e^{-i(\omega t + \beta)} \tag{11}$$

Putting, $z = x + iy$, we can write expressions for the kinetic energy, T, of the blades and of the masses in terms of M, Δ M, and m_2 and variables ζ_1, z Similarly the potential energy, V, may be expressed in terms of K, ΔK, K_β, z and ζ_1 and a dissipation function, F, for damping may be expressed in terms of B, ΔB, B_β, z, ζ_1

The Lagrangean Equations of Motion may now be written, and lead to equations of motion

$$(m + nm_2)\ddot{z} + B\dot{z} + Kz + \Delta m\bar{\ddot{z}} + \Delta B\bar{\dot{z}} + \Delta K\bar{z} + nm_2\ddot{\zeta}_1 = 0 \tag{12}$$

$$\ddot{z} + \ddot{\zeta}_1 + 2i\omega\dot{\zeta}_1 - \omega^2\zeta_1 + \lambda_\beta\omega_r(\dot{\zeta}_1 - i\omega\zeta_1) + \omega_2\Lambda_1\zeta_1 + \Lambda_2\omega_r^2\zeta_1 = 0 \tag{13}$$

Let the solution of equations (12) and (13) be expressed as an elliptic whirling motion

$$\left.\begin{aligned} z &= C_1 e^{i\omega_f t} + C_2 e^{-i\omega_f t} \\ \zeta_1 &= C_3 e^{i\omega_f t} + C_4 e^{-i\omega_f t} \end{aligned}\right\} \tag{14}$$

Substituting equations (14) in equations (12) and (13) and eliminating the arbitrary constants, a frequency equation is formed This is a complex equation, thus the solution requires both the real part and the imaginary part to be satisfied at once Both the real and the imaginary equations are cubic in ω^2 and quadratic in ω_f^2, though the imaginary equation is also satisfied by $\omega_f = 0$ It is usual to solve for ω for various values of ω_f

The Nature of Coupled Drag Hinge—Hub oscillations

Fig 2 taken from COLEMAN'S report shows the real and imaginary equations, plotted for a special case of Polar symmetry It will be seen that in general, there are three real roots, for each value of ω There is a range of ω in which only one real root exists, there will then be one real root and a complex root and its conjugate One of the latter pair has a negative imaginary part, indicating instability

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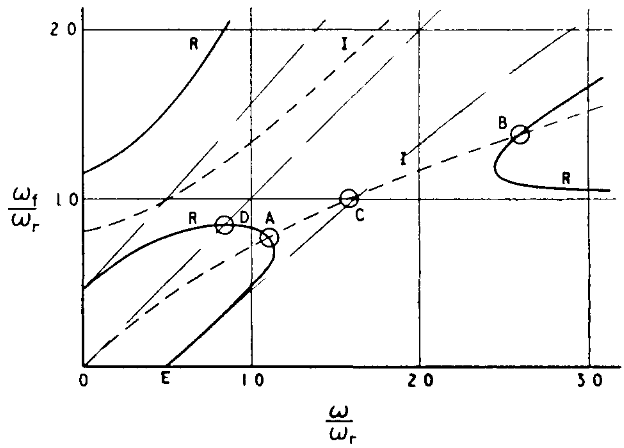
Now the assumed solution (equation (14)) was a solution appropriate to a boundary condition, *i e* , one of neutral stability where an oscillation would persist without increase or decay Thus, a condition of neutral stability exists only when both the real and the imaginary equations are satisfied, *i e* , at points A, B and E , no other complete solution of the form



PLOT OF REAL AND IMAGINARY EQUATIONS

$\omega_{r_x} = \omega_{r_y}$, $m_x = m_y$, $\Lambda_1 = 07$, $\Lambda_2 = 22$ $\Lambda_3 - 20$

FIG 2

of equation (14) exists if damping is present, either in the drag hinges or at the hub mounting, thus the motion is always damped except in the range from A to B The point E, may be of interest with respect to propellers or tail rotors, since it is the point at which an oscillation can be excited by a steady force, *e g* , gravity

If there is no damping either in the drag hinges or laterally at the hub, then there is no imaginary equation The real equation then gives the natural frequencies of the system

Two other points of interest , it will be seen that the centre of the unstable range, C, occurs at the intersection of lines representing the uncoupled natural frequency of the rotor mounting, (the frequency of the helicopter, if the " tip mass," nm_2 were concentrated at the hub), and of the natural frequency of a blade whose drag hinge rotates at constant angular velocity about a fixed axis

The point D, where $\omega_f = \omega$ is known as " shaft critical " or the critical whirling speed of the rotor , here, the frequency of drag hinge

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oscillation is zero, since $\omega_f = \omega - \omega_\beta$, and the blades rotate with uneven spacing

As far as purely lateral motion of the hub is concerned, this is not a condition of instability and will be positively damped if there is damping at the rotor mounting (B_x and B_y) However, large excitation at 1 x rotor frequency is almost invariably present due to rotor unbalance, errors in drag hinge spacing and blade track The resulting forced oscillation may reach a dangerous amplitude The oscillation once started will inevitably lead to progressive lack of track and possibly to instability on this account

The Influence of Damping

It is found from the frequency equation that if damping is included, then in general terms, the product of damping at the hub and at the drag hinge λ, λ_β exerts a powerful influence in drawing the two loops of the real equation together and so reducing the extent of the unstable range Damping at the drag hinge alone, λ_β has a small effect in modifying the real equation , whereas, hub damping, λ modifies the imaginary equation

Neither λ_β nor λ on their own appear to have a significant influence on the extent or severity of instability

Coleman has proposed a criterion for determining the product $\dfrac{\lambda\lambda_\beta}{\Lambda_3}$ necessary to eliminate instability He assumes that the motion is stable if the real equation is made to pass through the point of intersection of the uncoupled drag hinge frequency of the blades and uncoupled hub lateral frequency

It appears that this criterion must be slightly pessimistic and that instability may, in fact, be achieved at a lower value of $\lambda\lambda_\beta/\Lambda_3$ when the real equation passes some distance to the right of the common uncoupled frequency

The effect of lack of symmetry of the rotor mounting

Fig 3 illustrates the coupling effect between freedom in the x and y directions In general there must be as many unstable ranges as there are natural frequencies of the hub mounting As two natural frequencies converge, the two unstable ranges merge, until at the condition of polar symmetry, one unstable range remains, and is very similar to that for $K_x = \infty$

Fig 3 does serve to give warning of the wide separation of natural frequencies which is necessary, if one hopes to operate a helicopter with one natural frequency above and another below the operating range of rotor angular frequency

Experiments with a Model Rotor

At this point we would like to mention some experiments conducted at Bristol with a model rotor[3]

The model was built to represent the dynamic system of Fig 1 without the freedom in the y direction

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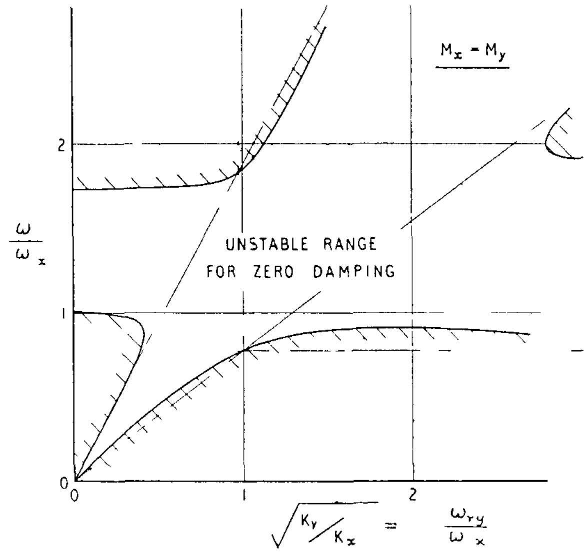

THE EFFECT OF COUPLING BETWEEN x & y FREEDOMS

FIG 3

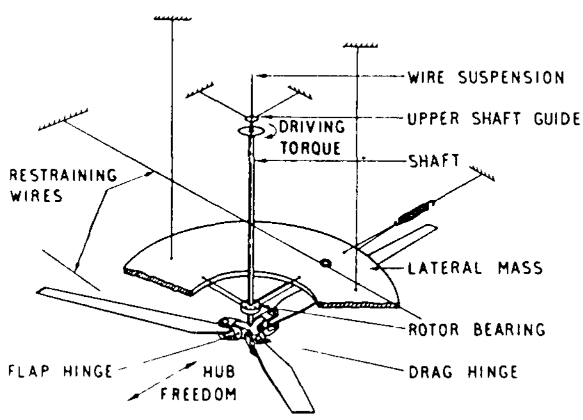

DIAGRAM OF MODEL ROTOR $K_y = \infty$

FIG 4

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Fig 4 shows the mechanism of the model in diagrammatic form The rotor diameter was 6 feet, and the scales employed, from Model Helicopter were

Frequency	1	1
Mass	014	1
Length	125	1

The mass at the rotor bearing was given freedom in one direction, but restrained by tensioned piano wires in the other, great pains were taken to avoid unwanted friction, and in fact, the model behaved particularly well at zero damping

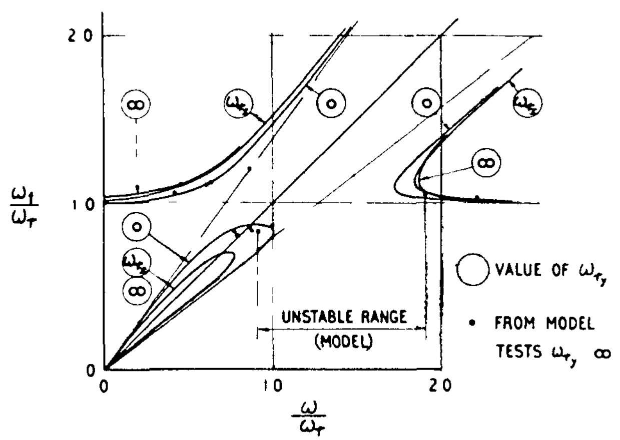
A damper for lateral motion was lashed-up in which a flat plate connected to the lateral mass moved in a narrow gap between the two static plates immersed in oil Provision of a viscous damper for the drag hinges proved a very much more difficult problem After some abortive attempts to make hydraulic viscous dampers, it was decided to make multi-plate stepped friction dampers These dampers had 12 moving plates each and gave a characteristic work diagram whose area was proportional to (amplitude)^2

The damper was considered to represent a viscuos damper, the equivalent viscous damping coefficient, was proportional to

$$\frac{\text{clamping load across the plates}}{\text{frequency of drag hinge oscillation}}$$

Test Results for Zero Damping

Experimental results are compared in Fig 5 with theoretical frequencies



NATURAL FREQUENCIES OF A TYPICAL ROTOR

ZERO DAMPING

Λ_1 052 Λ_2 0 Λ_3 058 $m_x - m_y$ n - 3

FIG 5

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for a particular case The model indicates too small an unstable range for the case $K_y = \infty$, though the pattern of the measured frequencies is in agreement with the case $K_y = 0$ We think the explanation is that we did not achieve a high positive impedance in the y direction This was due partly to the flexibility of the overhung rotor shaft, partly due to too low a stiffness at the end attachments for the restraining wires

This discrepancy emphasises the importance of finding the actual impedance of a rotor mounting in both directions rather than yield to the temptation of assuming that the equivalent stiffness in the fore and aft direction is either zero or infinite

Test results with damping

It was established with the model that neither drag hinge damping nor hub damping alone had any significant effect on the unstable range However, the range was closed at a value of the parameter $\lambda\lambda_\beta/\Lambda_3$ only 14% of the value obtained from the criterion at $K = \infty$ or 28% of the crtierion at $K_y = 0$

This discrepancy has been a puzzle for some time we think perhaps the explanation is, two-fold The effective drag hinge damping was probably greater than we thought , this was due partly to friction effects at the hinge and partly to aerodynamic damping due to flapping of the blades in sympathy with precession of the shaft

We also believe that the Coleman criterion is a little pessimistic and that the unstable range closes less and less rapidly with increase in damping, so that the range is extremely small and the divergence very gentle some time before one achieves the damping required theoretically

The model tests in retrospect

Looking back on these tests, we feel glad to have this independent check on the theory It was the acid test ! Furthermore, we were very impressed with the desirability of preserving low natural coupled frequencies of the rotor mounting The reason for this was that the self excited range occurred at lower rotor r p m , when the kinetic energy of rotation was less For instance, when the natural frequency of the mounting, ω_r, was halved, the rate of divergence in the centre of the self excited range was relatively gentle , at the higher frequency it had been explosive

This effect may be illustrated from Coleman's stability criterion Assuming viscous damping, the value of the product of the actual damping constants B_x B_β is proportional to ω_r^2 , whereas, if multi-plate friction dampers are used, B_x B_β is approximately proportional to ω_r^3

Thus, with Type 173 Mk 1, when the frequency of the mainly rolling mode was halved, the product B_x, B_β required was only $\frac{1}{8}$ the original requirement We found that this lower quantity could be achieved without overstressing the blades

Application to a Single Rotor Helicopter

An actual helicopter has 6 degrees of freedom as a rigid body on a flexible undercarriage Of these, the freedoms of lateral displacement, rolling and yawing must always be considered , furthermore, we have

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already seen that it is unwise to assume that infinite impedance exists in the fore and aft direction, thus the rigid body fore and aft and pitching freedom must be allowed

The conception of mass, stiffness and damping properties in each direction at the hub is now inconvenient Instead, it is necessary to know the impedance at the rotor hub in both x and y directions Our real interest is to determine the boundaries between stable and unstable oscillations, so we do not need to know the total energy the complete dynamic system may be represented by equivalent stiffness and damping in the x and y directions and by the articulated tip masses, m_2, with their drag hinge stiffness and damping

In the general case, equations (12) and (13) are rewritten putting $nm_1 + m_x = nm_1 + m_y = 0$
and

$$\lambda_3 = \frac{1}{2}$$

$$\omega_{rx} = \left(\frac{Kx}{nm_2}\right)^{\frac{1}{2}} , \qquad \omega_{ry} = \left(\frac{Ky}{nm_2}\right)^{\frac{1}{2}}$$

The procedure is to calculate or measure the helicopter impedance at the rotor, *i e*, the constants K_x, K_y, B_x, B_y

The real and imaginary equations are both quadratic in ω^2 and may be solved using the appropriate constants for each frequency ω_f As before, boundary conditions are determined by intersections of the real and imaginary equations

The analysis of a tandem helicopter is indicated in an Appendix to this paper

Impedance Tests

It is unfortunate that the oscillatory properties of undercarriages are very difficult to assess with accuracy, thus even if a helicopter may be shown to be stable on a basis of calculated impedance at the rotor hub, an impedance test should still be made

The purpose of the test is to find the equivalent dynamic stiffness and equivalent damping constant at the rotor in the fore and aft and lateral directions for a periodic force applied at the rotor axle in the plane of rotation of the rotor

DEUTSCH[4] has suggested that it is sufficient to run up the helicopter rotor with cables attached to the rotor mounting, and fixed to posts driven into the ground, so that if instability sets in, the motion may be arrested by tightening the cables We believe this method to be both dangerous and inconclusive , dangerous, because the cables themselves are flexible and may well make an unstable condition even less stable , inconclusive, because we know of several instances where ground running has been successfully completed, but ground resonance has occurred on landing In fact we had one instance with Type 173 where one landing was successfully made, but the second was the subject of the film we saw earlier

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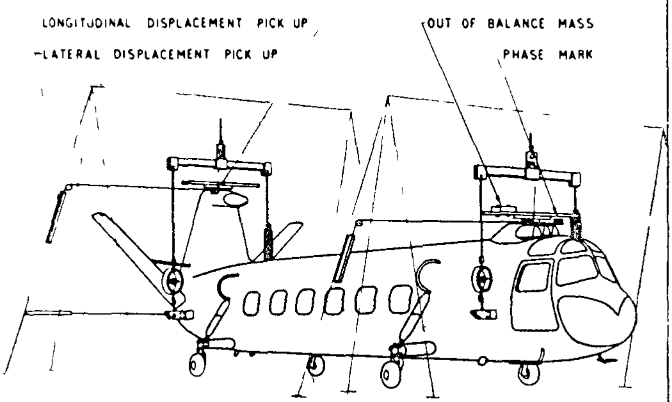
An impedance test must take account of the following variables

1 The frequency of unstable oscillations may be anywhere between, say 4 and 1 2 x rotor frequency
2 Oleo stiffness and damping vary with amplitude frequency and total mean load
3 Tyre stiffness varies with normal load on the tyre
4 Undercarriage geometry may change considerably with total wheel reaction
5 Variation of all-up weight

Items 2, 3, 4 demand that the impedance test be made for various magnitudes of periodic applied force and also over a range of attitudes and wheel reactions corresponding to a complete take-off or landing cycle

To reduce the magnitude of the test, it is usual to test for one all-up weight condition only—that which is expected to be least stable



RIG FOR IMPEDANCE TEST

FIG 6

Fig 6 shows a tandem helicopter mounted for impedance test The machine is supported from jacking points by a suspension which has very low pendular stiffness, low vertical stiffness and zero rolling stiffness The load in the suspension may be varied as required and is measured by proving rings, thus a complete cycle of landing or take-off may be represented

Each rotor is now replaced by a mass equal to the total rotor mass less the tip mass

The mass is bolted to the rotor mounting and carries out of balance masses Up to the present time, we have used one of the helicopter engines

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to drive the out of balance blade mass This has the advantage of simplicity and of adding no extra restraint, however it limits the possible range of excitation from the frequency of clutch engagement to that corresponding to maximum permissible engine r p m

Impedance is measured by displacement pick-ups at the rotor in the x and y directions and by a phasing mark relating the position of the out of balance weight to the displacement

The frequency of oscillation lies within the useful range of a pen recorder, thus the pick up response is at once evident and frequencies at which the phase lag between force and displacement passes through 90° may be seen Such regions may then be explored more thoroughly

For a tandem helicopter the displacements at both hubs must be recorded for excitation separately at each hub

Undercarriage Design

It will be assumed in this discussion that the modes of the helicopter which have frequencies of the 1 x rotor order can be treated as rigid body modes of the helicopter on its undercarriage

The modes which are of most trouble with a single rotor machine are mainly lateral and mainly rolling modes , to these, the mainly torsion mode must be added for a tandem helicopter It will often be impossible to arrange for the torsional mode to be uncoupled from the lateral Thus, in the design stage, it is necessary to calculate the coupled modes in these three degrees of freedom

Tyre and Shock Absorber Leg Characteristics

As far as ground vibrations of a helicopter are concerned, it is an unfortunate circumstance that, if we represent a shock absorber leg by a spring K_2, and damper, B_2 in parallel, the values of K_2 and B_2 are functions of mean axial load, frequency and amplitude

Let us consider the characteristics of the familiar oleo-pneumatic strut, for example

The axial load in the strut must reach a certain value before any movement takes place This load is the frictional resistance of the glands plus the product of piston area and air pressure in the fully extended position

When the strut is partly compressed under a constant mean load, no periodic change of length will result unless the periodic applied force exceeds the static friction force The strut has " infinite " stiffness

Further increase in the periodic applied force will cause periodic telescoping The strut will have a stiffness, K_2, which is virtually that of its air column and is a function of mean axial load

Let B_2 be the equivalent viscous damping constant giving the same rate of energy dissipation as the strut for a given frequency and amplitude

The part of B_2 due to friction is inversely proportional to the product of amplitude and frequency , that is to the maximum sliding velocity of the cycle Whereas, that part of B_2, due to the orifices is directly proportional to the maximum sliding velocity

Thus, as the maximum sliding velocity per cycle increases from zero, B_2 falls from infinity to a minimum and then rises again We should at this stage also briefly review the influence of the tyre-oleo combination

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We have attempted in Fig 7 to indicate the range over which the equivalent stiffness K_v and damping B_v of a combined tyre and " oleo " system may operate The curves are intended to represent the following cases, for a representative tyre-oleo combination

Case 1 Helicopter almost airborne, *i e*, low air stiffness, oleo still partly compressed, and small maximum velocity of sliding, representing small amplitude of oscillation

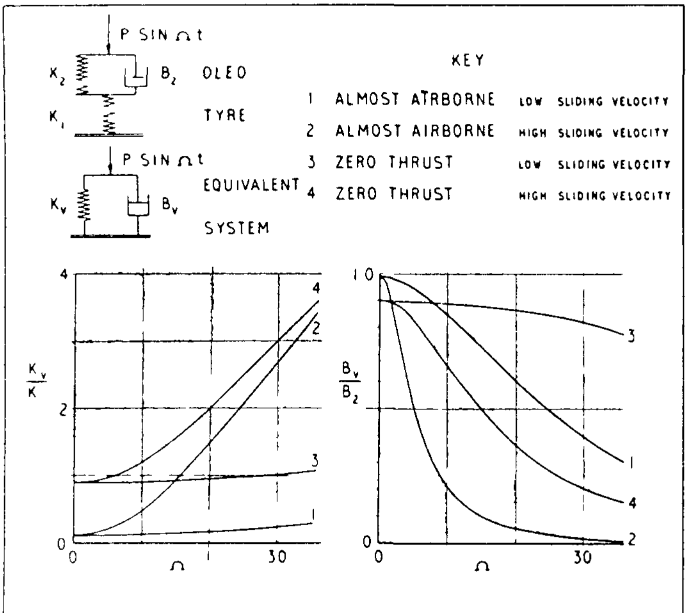
Case 2 Helicopter almost airborne, oleo still partly compressed and high maximum velocity of sliding, representing high amplitude of oscillation

Case 3 Small rotor thrust, *i e*, high air stiffness, otherwise as Case 1

Case 4 Small rotor thrust, *i e*, higher air stiffness, otherwise as Case 3

The vertical stiffness will become equal to K_1 before the helicopter becomes airborne, in fact a helicopter may often remain at this condition for some time

Fig 7 illustrates the large variation in K_v and B_v during a take-off cycle



OLEO - TYRE CHARACTERISTICS

FIG 7

For small amplitudes of oscillation (low sliding velocity) $\frac{K_v}{K_1}$ drops from curve (3) to (1), then rises to 1 0 as the oleos reach their full extension If the amplitude is large, however, the stiffness K_v increases almost linearly with frequency

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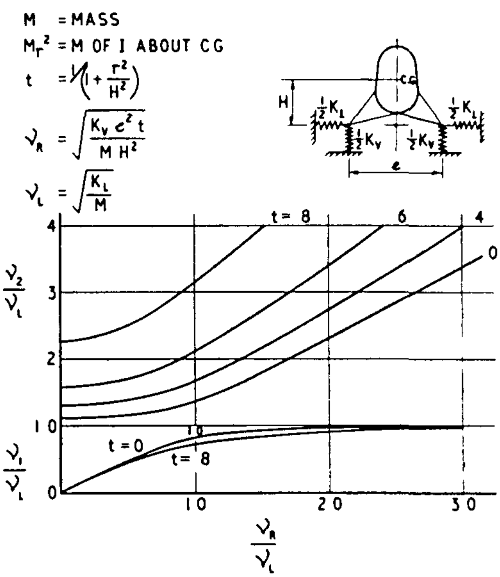
Damping B_V falls with increasing frequency and falls further still with increasing amplitude, both undesirable effects

Thus, we see that the shock absorber strut in series with a tyre provides a very troublesome system for analytical treatment The designer must, however, endeavour to ensure that his helicopter will not suffer self-excited vibration whatever the oleo characteristics may be

The influence of coupling between pure lateral and pure roll modes

For simplicity, we have ignored coupling with yawing and have plotted in Fig 8 the frequencies of the coupled lateral and rolling modes for a range of values of the ratio of pure rolling frequency to pure lateral frequency, ν_R/ν_L for various values of coupling coefficient $t = \frac{1}{\left(1 + \frac{r^2}{H^2}\right)}$ for conventional designs t is about 0 7



FREQUENCIES OF COUPLED LATERAL CUM ROLLING MODES

FIG 8

The curves of Fig 8 show that the coupled frequencies diverge rapidly as the ratio ν_R/ν_L increases It may also be noticed that if the pure rolling frequency ν_R is reduced to zero, the frequency of the mainly rolling mode is conventionally well above the uncoupled lateral frequency ν_L

Designing for avoidance of " Ground Resonance "

We have already noticed that it is most unlikely that sufficient damping

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could be provided to eliminate self-excitation, if this occurs at rotor r p m within the operational range , that is the r p m range within which the operations of taxying, landing or take-off may be performed In this range, we must depend for stability on a suitable selection of the frequencies of coupled modes

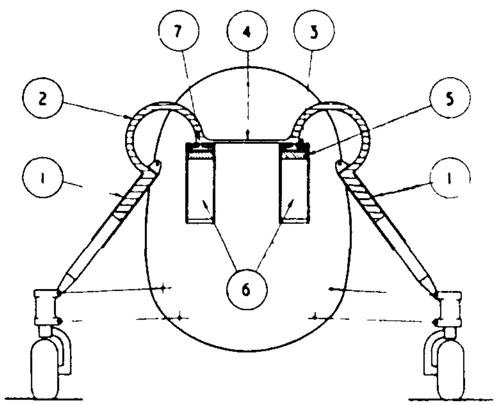
Three schemes will be considered representing some of the alternatives open to the designer In the first two schemes, freedom from ground resonance is achieved irrespective of the characteristics of the device for absorbing energy of the vertical drop landing design cases We shall also consider the behaviour of these schematic units in relation to

(*a*) Heavy one-wheeled landings
(*b*) Taxying
(*c*) Punctures
(*d*) Burst Tyres

SCHEME 1

We will take the undercarriage of Type 173 Mk 1 as an example of this scheme The undercarriage structure of this machine is relatively flexible in the lateral direction , this feature, together with the use of high pressure tyres with a high overall diameter/hub diameter ratio gives a low K_L, hence the yawing frequency and the pure lateral frequency are low, about 80 and 60 c p m respectively

Fig 9 shows the mechanism of the unit Each telescopic leg (1) is partly filled with oil and connected by large bore pipe (2) to the chamber (3)



UNDERCARRIAGE SCHEME I

FIG 9

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The two chambers (3) are then connected by a small bore pipe (4) Each chamber (3) is virtually an oleo-pneumatic cylinder in which a piston (5) is preloaded by air pressure in the space (6) When oil pressure in the pipe (2) exceeds air pressure in space (6) the piston (5) moves, and oil passes into the chamber (3) via an orifice (7) The orifice (7) is the usual double orifice in which the area is reduced for flow out of the chamber

Thus, for rolling oscillations of the helicopter, under constant mean wheel reaction, oil passes from one leg to the other, giving zero stiffness, but substantially viscous damping

In the event of simultaneous impulsive loading at a pair of wheels, *viz* symmetrical landing, the system acts very much as though normal ' oleos ' were fitted

Rolling stiffness is achieved in two complementary ways

(*a*) by low rate, steel coil springs external to the legs ,

(*b*) a small stiffness exists due to gravity, since, by virtue of the under-carriage geometry, the C G rises if the helicopter is displaced in roll

This stiffness is proportional to wheel reaction and therefore drops to zero when the helicopter is on the point of leaving the ground

By this means the coupled mainly rolling frequency remains at about 120 c p m throughout the take-off or landing cycle Moreover, the product of undercarriage damping and drag hinge damping is more than enough to eliminate instability when the rotor is run up

We should mention here that a stepped friction damper similar in principle to that used in model rotor tests, is fitted to each drag hinge on Type 173

One must note that on the run up, the blades will usually remain on the trailing drag hinge stop, so that the λ_2 is greatly increased , the frequencies of the unstable range are increased and, in fact, the lower fringe of instability is well above the value of ω at which the blades leave their drag stops A similar state of affairs exists on the run down , if the rotor brake is applied, the blades then swing forward to the leading drag hinge stop

If the oscillation is of sufficiently small amplitude friction will prevent any telescoping of the legs (1) Thus, the mainly rolling frequency is very high, due to the high K_1 of the high pressure tyre and the infinite K_2 of the leg However, the transition from high rolling frequency to low rolling frequency is completed at very small vibration amplitude and gives no trouble

We also observe that, in a one wheel landing the wheel making contact must give a lower reaction, due to flow through the cross pipe, than would occur without cross coupling, *i e* , in a normal tyre-oleo system

Furthermore, the second wheel moves downwards towards the ground, thereby reducing the time which must elapse before the two wheels share load

The great advantage of this system is that the helicopter will receive relatively small angular acceleration and the tendency to bounce from one wheel to the other will be reduced If bouncing does occur, the fundamental frequency of the resulting oscillating must be lower than that of the mainly rolling mode , hence, the condition is stable and becomes more stable still, should bouncing occur

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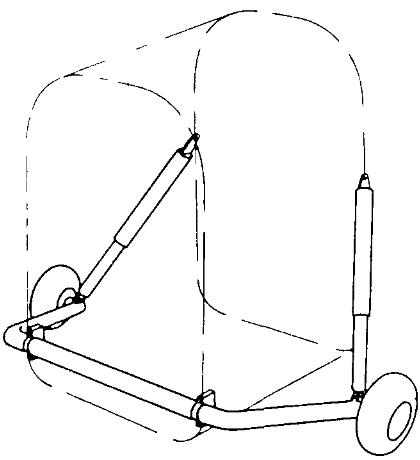
When the helicopter is taxied, the effective lateral stiffness of the tyres will be reduced, thereby making the helicopter more stable dynamically Shimmy of castoring wheels is not a low frequency phenomenon, so we may assume that this, too, will not affect rotor stability

From similar arguments, one concludes that a slow puncture will not be dangerous However, a tyre burst may lead to high lateral stiffness and cause " ground resonance " to set in

SCHEME 2

In this case, a high rolling stiffness is employed , the frequencies of coupled modes are separated so that the mainly rolling frequency is well above the 1 x rotor range, whilst the mainly lateral frequency must be well below the ½ x rotor range, say



UNDERCARRIAGE SCHEME 2

FIG 10

Fig 10 shows an example of this scheme in which a torsion bar is used to give high mechanical stiffness in roll The frequency separation possible in such a scheme is limited by the ratio of vertical to lateral tyre stiffness

Such an undercarriage will become less stable with increase of taxying speed, because the frequency of the mainly roll mode must depend, in part, on tyre lateral stiffness

More serious are the results of a puncture giving a soft tyre , the mainly rolling frequency is then reduced due to reduction in both lateral and vertical stiffness at the wheel However, the system is safe in the event of a burst tyre, provided lateral structure stiffness is not too great

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One observes that, in a one wheel landing, two ' oleos ' virtually operate on one wheel, thus the rolling accelerations are high There appears to be a possibility of " bouncing resonance " in the event of a heavy enough rolled landing

This scheme does not appear to have the docility of Scheme 1, though it may be that for certain operational roles, *e g* , ship board use, a high rolling stiffness will be imperative

SCHEME 3

This scheme is in use in most of the successful smaller helicopters The Bristol Type 171, Sycamore is a typical example In this case, reliance is placed in the lateral tyre stiffness and the vertical stiffness of an oleo, tyre combination to achieve high enough mainly rolling and low enough mainly lateral frequencies

Not only is the system likely to give trouble in the event of a tyre puncture or burst, and particularly, following very heavy rolled landings, but careful matching of tyre and oleo characteristics are necessary if a decrease in stability with amplitude is not to occur

One should observe that in these smaller helicopters, the pilot is closer to the wheels and can, generally, make better landings than he could with, say, a tandem

It is very interesting to note that many helicopters with undercarriages of this type are known to have critical taxying speeds above which ground resonance occurs This fact may serve to show how narrow a margin of stability is often considered acceptable

CONCLUDING REMARKS

It is believed that the assumptions on which the theory rests are reasonable and that, if the impedance of a rotor mounting is known, then the extent of the unstable range may be determined with confidence

Non-linear characteristics of the undercarriage are responsible for the principal difficulty, that of determining rotor mounting impedance Because of this, the undercarriage design must be governed by its ground resonance characteristics as much as by the vertical drop landing cases

The available data on the dynamic properties of shock absorber struts and tyres is very slender More information on the dynamic stiffness of tyres in all three directions, vertical, lateral and tangential, would be particularly helpful

It is clear that as bigger and better helicopters come along—some with wings, the problem of ground resonance will assume ever greater importance

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APPENDIX I

Application to a Tandem Helicopter

Let x_1, y_1 be co-ordinates of rotor hub (1) and let q_1, r_1 be displacements of hub (1) due to an exciting force at hub (1)

When hub (1) is excited by a periodic force, displacements will also occur in hub (2) The helicopter will be symmetrical about its fore and aft datum, so we may assume that, in the absence of the rotor, there is no coupling between the x and y directions

For a boundary condition between stable and unstable motion, the periodic motion is of constant amplitude and we may write for the displacement of hub (2) due to excitation at hub (1)

$$x_{21} = a_{21}\, q_1 + b_{21}\, q_1 \qquad \text{(A1)}$$

$$y_{21} = c_{21}\, r_1 + d_{21}\, r_1$$

then the displacements at rotor (1) become

$$x_1 = q_1 + a_{12}\, q_2 + b_{12}\, q_2 \qquad \text{(A2)}$$

$$y_1 = r_1 + c_{12}\, r_2 + d_{12}\, r_2$$

and at rotor (2)

$$x_2 = q_2 + a_{21}\, q_1 + b_{21}\, q_1 \qquad \text{(A3)}$$

$$y_2 = r_2 + c_{21}\, r_1 + d_{21}\, r_1$$

Let $z_1 = x_1 + iy_1 \quad , \quad z_2 = x_2 + iy_2 \qquad \text{(A4)}$

$$S_1 = q_1 + ir_1 \quad , \quad S_2 = q_2 + ir_2$$

S_1 and S_2 may now be regarded as variables for the fuselage motion

Let $U_1 = \frac{1}{2}(a_{12} + c_{12}) \quad , \quad \Delta U_1 = \frac{1}{2}(a_{12} - c_{12})$

$$V_1 = \frac{1}{2}(b_{12} + d_{12}) \quad , \quad \Delta V_1 = \frac{1}{2}(b_{12} - d_{12}), \text{ etc} \qquad \text{(A5)}$$

Then displacements at the two hubs may be written

$$Z_1 = S_1 + U_1\, S_2 + \Delta U_1\, \bar{S}_2 + V_1\, S_2 + \Delta V_1\, \bar{S}_2$$

$$Z_2 = S_2 + U_2\, S_1 + \Delta U_2\, \bar{S}_1 + V_2\, S_1 + \Delta V_2\, \bar{S}_2 \qquad \text{(A6)}$$

Now let the impedance at rotor (1) due to excitation at rotor (1) be represented by the stiffnesses Kx_1, Ky_1, and the dampers, Bx_1, By_1

The reactive forces exerted at rotor (1) are

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$$Px_1 + Qx_1 + q_1 Kx_1 + q_1 Bx_1, \text{ etc}$$

and, if $P_1 + Q_1 = Px_1 + i Py_1 + Qx_1 + iQy_1$ we obtain

$$P_1 + Q_1 = K_1 S_1 + \Delta K_1 \bar{S}_1 + BS_1 + \Delta B \bar{S}_1 \qquad \text{(A7)}$$

The energy expressions for the helicopter complete with rotors may now be written in terms of the variables

S_1, S_2, ζ_1, ζ_2 and their complex conjugates where ζ_1, ζ_2 are variables, one for each rotor, representing the generalised co-ordinate ζ_1 of single rotor theory

One arrives at equations of motion relative to a final co-ordinate system of the form

$$\frac{nm_2}{2}(\zeta_1 + U_2 \zeta_2 + \Delta V_2 \bar{\zeta}_2) + K_1 \bar{S}_1 + \Delta K_1 S_1 + B_1 \bar{S}_1 + \Delta B_1 \bar{S}_1$$

$$+ A_{11} S_2 + A_{12} \bar{S}_2 + A_{13} S_2 + A_{14} \bar{S}_2 + A_{15} S_2 + A_{15} \bar{S}_2 = 0$$

for the variable $\bar{S}_1$ and a similar equation for the variable $\bar{S}_2$, also, for the variable ζ_1, one obtains

$$S_1 + U_1 S_2 + \Delta U_1 \bar{S}_2 + V_1 S_2 + \Delta V_1 \bar{S}_2 - \zeta_1 + A_{31} \zeta_1 + A_{32} \zeta_1 = 0 \qquad \text{(A13)}$$

and a similar equation for the variable ζ_2

Where A_{11}, A_{12}, etc, are constants depending on the constants U_1, U_2, etc, and A_{31}, A_{32}, etc, are constants depending on the rotor parameters λ_1, λ_2, λ_β and K_ρ

The solution may be assumed to be an elliptic whirling motion for each variable, leading to an 8th order frequency determinant It would be necessary to solve this determinant separately for each value of ω_r and plot the real and imaginary parts of the resulting frequency equation in order to find points of intersection, *i e*, boundaries between stability and instability

It will be appreciated that the amount of labour involved in this treatment is very considerable, one has to consider the whole range of wheel loadings and helicopter attitudes from zero rotor thrust to the airborne condition

Equations A12 and A13 may be represented by an electronic analogue, however, and the case for constructing such an analogue is obviously a strong one

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