CORRESPONDENCE.

ON CERTAIN FORMULÆ IN MR. DAVID JONES'S WORK ON ANNUITIES.

To the Editor.

Sir,—I am not aware whether what seems to me to be a serious error in one of the expressions given in Jones's work *On Annuities and Reversionary Payments*, for a deferred assurance, has ever been noticed or not.

On page 170, vol. i., of the work referred to, the general value of a deferred assurance on any number of lives is given as

$$A_{\frac{v}{(m,\,m_1,\,m_2,\,\&c.)}\rceil t}=r^{t+1}p_{\frac{v}{(m,\,m_1,\,m_2,\,\&c.),t}}-(1-r)a_{\frac{v}{(m,\,m_1,\,m_2,\,\&c.)}\rceil t}. \qquad (1)$$

An expression correct enough: but wishing, I presume, to give this result in a more convenient form, the author goes on to manipulate the latter term of the second member of this equation, thus—

$$(1-r)a_{\frac{v}{(m,\,m_1,\,m_2,\,\&c.)}\rceil t}=(1-r)r^t p_{\frac{v}{(m,\,m_1,\,m_2,\,\&c.),t}}\,.\,a_{\frac{v}{(m+t,\,m_1+t,\,m_2+t,\,\&c.)}};$$

and in this way he deduces

$$A_{\frac{v}{(m,\,m_1,\,m_2,\,\&c.)}\rceil t}=r^t p_{\frac{v}{(m,\,m_1,\,m_2,\,\&c.),t}}\,A_{\frac{v}{(m+t,\,m_1+t,\,m_2+t,\,\&c.)}}. \qquad (2)$$

This latter expression it seems to me is quite erroneous, in consequence of a fatal error in the process that leads to it. In fact, $(1-r)r^t p_{\frac{v}{(m,\,m_1,\,m_2,\,\&c.),t}}\,.\,a_{\frac{v}{(m+t,m_1+t,m_2+t,\&c.)}}$ is not $\left(\text{except in the single case when } v=m+m_1+m_2+\&c. \atop =\textit{number} \text{ of lives}\right)$ the equivalent of $(1-r)a_{\frac{v}{(m,\,m_1,\,m_2,\,\&c.)}\rceil t}$, as assumed in the work referred to.

This will at once be seen if we particularise the expression. Suppose two lives (m and m_1) concerned, and $v=1$; then

$$(1-r)a_{\frac{1}{(m,m_1)}\rceil t}=(1-r)(a_{m\rceil t}+a_{m_1\rceil t}-a_{m,m_1\rceil t})$$

$$=r^t(1-r)\left(\frac{l_{m+t}}{l_m}a_{m+t}+\frac{l_{m_1+t}}{l_{m_1}}a_{m_1+t}-\frac{l_{m+t}l_{m_1+t}}{l_m l_{m_1}}a_{m+t,m_1+t}\right),$$

while

$$(1-r)r^t p_{\frac{1}{(m,m_1)_t}}a_{\frac{1}{m+t,m_1+t}}=r^t(1-r)\left(\frac{l_{m+t}}{l_m}+\frac{l_{m_1+t}}{l_{m_1}}-\frac{l_{m+t}l_{m_1+t}}{l_m l_{m_1}}\right)(a_{m+t}+a_{m_1+t}-a_{m+t,m_1+t});$$

a very different expression indeed, and giving rise to a very serious error.

As an illustration, let us take an actual case:—Required the single premium for £1 payable on death of last of two parties now aged 30 and 40 respectively, provided that event happens after 10 years (Carlisle 3 per cent.)

The true value, *i.e.*, the result of equation (1) is ·3118
while the value given by equation (2) is ·2959

a result too little by nearly 2 per cent.

Hoping you will be good enough to inform me whether this point has ever been remarked on previously,

I am, Sir, your most obedient servant,

JAMES R. MACFADYEN.

City of Glasgow Life Assurance Company,
Glasgow, 18*th July*, 1866.

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