PHOTO-ELASTIC METHODS OF MEASURING STRESS.

Lecture given by Professor E. G. Coker, D.Sc., F.R.S., M.Inst.C.E., Univerity College, London, before the Institution at University College, Gower Street, W.C.1, on 20th February, 1924. Mr. W. O. Manning in the Chair.

INTRODUCTION.

PROFESSOR COKER said:

The study of the stresses in materials due to load, although one of the oldest branches of science, is continually presenting new aspects requiring examination, and this has been especially marked since the invention of the steam engine and the free use of iron and its alloys, which brought in their train a host of problems for solution.

Every later advance in experimental and theoretical knowledge has but widened the field of investigation, and in particular the comparatively recent advent of the internal combustion engine has opened up innumerable avenues for research.

In the field of aeronautics alone almost every branch of physical science has been pressed into service, and although the small field of applied science which it is proposed to consider now has not played much part in aeronautics, yet there are indications that it may, as photo-elastic laboratories for the general study of stress problems are now established in many parts of the world, and some of these are particularly interested in aeronautical problems.

It is therefore not unfitting to give a brief account of this form of experimental study before this Institution, since it may be regarded as one branch of the varied activities with which it may be properly concerned. It is not proposed here to spend much time on the elementary theory of the behaviour of polarized light when transmitted through stressed bodies, since this is well known, but rather to outline some standard methods of treating a variety of problems which have been developed for the use of engineers and others engaged in the design and construction of all kinds of engineering work, for whatever their nature there are almost invariably some difficult questions in stress distribution to consider, and usually it is necessary to solve them with great celerity.

Now photo-elastic experiment is particularly well qualified to help a busy engineer, because it gives within its limits a complete picture of stress distribution in a model and, with some reservations in particular cases, this picture represents to scale exactly what the stress distribution is in the metal or other body of the same form when the laws of similarity are taken into account.

If, for example, a stiff-jointed frame is loaded, consisting of a simple rectangle (Fig. 1) of uniform breadth and thickness on three sides and merging into a solid plate on the fourth, a definite colour pattern appears upon it when the frame is viewed in polarised light. This colour picture now shown tells us what the stress distribution is at every part, and no matter how and where the model is loaded in its own plane we get a corresponding picture which can be interpreted with comparatively little trouble. It is not easy even in such a simple case to calculate the stress distribution, while in the frames of airships and aeroplanes, which are very much more complicated than this, the calculations are naturally more difficult. If, for example, a designer wishes to compare the merits of a number of unlike stiff frames built up, let us say, of quadrilateral cells, the problem may be quite impossible to solve by calculation in a reasonable amount of time, and may prove very costly if actual frameworks are tested to destruction.

In order to understand the stress picture obtained with a loaded transparent model, and the simple laws which it obeys, it is convenient to commence by taking the simplest case of a tension member under load, which shows, as may now be observed, a specific colour for a definite stress. Having established in this manner a colour scale in relation to stress, it is now possible to read off the stress at any point of the contour of this model, and the accompanying Fig. 2 shows the results of these observations. Almost invariably the maximum stresses occur at the edges or contours of a model, while if it becomes overstressed at any point a black patch develops there and gives a very ready means of showing the weak place in a structure without any measurements whatever.

If, however, we desire to go more fully into the question of the stress distribution in the interior of a body such as this frame, it is not quite so simple, for, as is well known, the stress at a point in a plate loaded in its own plane has in general two principal components, p and q, at right-angles, and they tend to destroy one another as the colour effect is proportional to $(p-q)$. It is, however, not difficult to separate p from q by observations of the colour bands and their directions, which latter are shown in plane polarised light by dark bands as marked on the drawing, Fig. 3.

Given these, we can find both p and q separately* at every point and so complete the analysis. There are, however, other ways of carrying this out, and perhaps the best of these is to determine the sum (p + q) of the stresses by the aid of an extensometer, which measures the change of thickness of the model at any place, and by comparing this change with the effect produced in a simple tension member the value of (p + q) is obtained.

It is not necessary here to describe the lateral extensometer used for measuring (p + q) and now shown in the accompanying photograph, as its construction is well known, and it may be briefly dismissed as a measuring machine of great precision for determining very small changes of thickness at any point of a plate and mounted in a rather elaborate slide rest, which allows it to be set in any position.

The principles which have been briefly illustrated in this simple case to determine stress distribution apply to any other plane frame, and as nearly every practical case is much more complicated in design, it may be worth while to indicate how such problems might be solved by photo-elastic means.

In this connection it is most interesting to study some of the published drawings of various airship frameworks which have successfully stood the test of use, and although these are solved problems they give an opportunity of speculating whether photo-elastic experiment could have helped their solution or is likely to be of use in the further problems which are bound to arise as aeronautical science and practice advance. From a scientific point of view, it is also of interest to examine any problem with a view to its solution in different ways, even if alternative solutions offer no practical advantages.

A mere casual acquaintance with the structure of a rigid airship is sufficient to convince anyone that one of the most interesting stress problems is that of a stiff joint. Any drawing of such a structure shows that these occur many hundreds of times. It is proposed here to approach this and other questions quite apart from practical considerations and purely from the scientific interest of the problem itself.

In order to present a complete picture of the stress distribution in such joints for illustration here, a practical limitation is imposed by the size of the photographic plate which can be employed in a projection lantern and the amount of detail which can be rendered visible, but no such limitation exists in an actual investigation, as will be described later, but for demonstration purposes it is convenient to take a few simple cases, such as the square cell now shown, stressed by a load across the diagonals and affording a picture of stress distribution which has been approximately analysed very

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* "Experimental Determination of the Distribution of Stress and Strain in Solids." Profs. Filon and Coker.

"Report of the Committee on Stress Distributions in Engineering Materials" B.A.Report. 1914.

rapidly to show how the lines of principal stress are directed, and also how the stress varies at the contours where the maximum values occur. Bearing in mind the colour scale, it will be seen at once that these latter diagrams can be visualised quite easily by inspection of the colour photograph just shown. A slightly more complicated case is presented by a hexagon-shaped cell, stressed in a similar way across a diagonal, and the next figure, a polygon with ten equal sides, although even more complicated, has also been rapidly analysed by one of my research students, Mr. Jenvey, in order to determine the directions of the stresses in the interior of the frame and their magnitudes at the inner and outer contours. This information is shown on Fig. 4, and illustrates very well the ease with which a fairly correct map of stress distribution can be obtained in a few hours which would probably be difficult to obtain in any other way. These examples serve to illustrate the general nature of the complex problem which is presented in the next case, Fig. 5, which is a diagram of one of the frames used in the airship R.80. This consists of a polygonal frame of eleven sides, ten of which are each provided with a central strut on each side, tied back to the ends of the member, while the eleventh forms one side of a triangular cell connected by transverse booms to adjacent members, and forming with succeeding cells the spinal column of the airship. In addition, there is an elaborate system of wire bracing, as indicated in the figure, imposing a variety of loads upon the framework. This is, in fact, a typical case of the complex nature of an aeronautical problem of stress distribution, even when limited to one plane, and in general the problem is three dimensional. Now the reason why it is not possible here to project a stress picture of the whole frame is not the size of a lantern plate, but the size of the polariscope available for photographic purposes. The apparatus available for this lecture will not accommodate any object having any linear dimensions of more than 3 inches. Purely mechanical considerations, therefore, limit us here, owing to the necessity of constructing a model within this size. This is a task not beyond the power of a skilled workman, but a much better way would be to construct a much larger model and take photographs section by section and then cut and assemble the plates so that the whole of the frame is shown. Although, for demonstration purposes, such photographs are useful, they are not employed in actual investigation, as all the work is done by projecting an enlarged image on a screen, so that whatever the size of the object may be it is always possible to examine each part successively, and in this case some simple device for bringing any part of a large model into the field of view as required would suffice. Should the necessity ever arise, however, of viewing a very large model as a whole, there would be no great difficulty, as models up to 3 feet in length have been shown under stress in a special form of polariscope.

Assuming the necessity of such a procedure for airship work, it might be possible to effect an improvement on existing forms of polariscope by using as the source of light an arc lamp of sufficient power set at the focus of a parabolic reflector, such as is used in a searchlight, instead of a bank of incandescent lights as now employed.

A suggested arrangement is shown in diagrammatic form, Fig. 6, in which the light from an arc lamp A is reflected as a parallel beam by a parabolic reflector B, and is polarised by reflection from a black glass plate C, and then traverses a quarter-wave plate D before passing through the model E. The circularly polarised beam then passes through a second quarter-wave plate F and plate analyser G in the usual manner.

Given a sufficiently powerful source of light, it is quite possible that an image of the object could be projected on a screen, as is usual in the more efficient arrangements using Nicol's prisms or their equivalents, which latter are necessarily of very limited size. If each element of such an arrangement is considered it is clear that each part can be constructed on a large scale without much difficulty, with perhaps the possible exception of the quarter-wave plates, which, so far as is known, have not been constructed larger than about 20 inches in diameter.

If, however, this difficulty is overcome, a very large frame could be built and viewed at one time in a circularly polarised field, and the effects of all kinds of combinations of loads due to the wires studied in a convenient manner.

Nothing has been said as yet concerning the manner in which the members of the frame are to be constructed in detail, and considerable simplification is obtained by applying the principles of dynamical similarity, which show,* for example that if the elastic constants E are in the ratio q, the applied stresses must also be in this ratio, that cross-sections under bending need not be geometrically similar provided the radii of gyration of the cross-sections are in the ratio of geometrical similarity. The frame can therefore be constructed of members of rectangular cross-section, and the direct stresses, shears and bending moments found therefrom for application to a built-up structure.

This makes it possible to construct the main frame apart from bracing wires in one piece if required, and for this condition the limit of size is imposed by the dimensions of the sheets of transparent material, commercially obtainable free from initial stress. Rectangular sheets, about 36 inches by 20 inches, can be obtained free from stress to within a short distance of the edges, and these allow of a circular model of about 18 inches diameter, or if made in two halves jointed together, a model 36 inches diameter. A built-up model could be constructed of much larger size, and is perfectly feasible if the necessity arises.

Photo-elastic Design.—Another aspect of photo-elastic work arises in design, and a good many years ago I ventured on a remark, which I repeat here, that the science and art of engineering design was put into its most concentrated form by the poet, Oliver Wendell Holmes, in his story of the

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* "On the principal of dynamical similarity applied to deformable elastic structures" by Major Filon. The Aeronautical Journal, November 1918.

Deacon's wonderful " One Hoss Shay," which " ran a hundred years to a day " and then collapsed into a heap, every part giving way exactly at the same moment. The principles on which it was constructed are a counsel of perfection.

> " Fur," said the Deacon, " 't's mighty plain
> Thut the weakes' place mus' stan' the strain;
> 'n' the way t' fix it uz I maintain
> Is only jest
> To make that place uz strong uz the rest."

The Deacon used the very best materials and brought great skill and wide experience to his task, and although we have no detailed guidance of his methods of design, there is little doubt of the soundness of his views. If it were now possible to ensure so successful a result in the air as he obtained on the road, there would be little further to say. The task of the aeronautical engineer is, however, infinitely more difficult if only for the reason that every part of a structure which is to fly must be as light as possible, but the principle enunciated still holds. Now there are various ways of approach to this end, and calculation, experience and experiment can all play a part, and in fact do so. On the experimental side it is not uncommon to test a number of different designs to destruction in order to discover which is the most perfect for the purpose. Although this is apparently a very safe procedure, it is not altogether free from criticism, for it is not always easy to say what are the exact reasons which caused failure of a structure, and further, there is generally the tacit assumption that one of the designs must be the best possible, which may not be the case. It can only be the best of those submitted to test and no more; there may still be room for further improvement due to the possibility that one or more different defects in design occur in each, or are common to all and may be possible to eliminate. Although this is obvious, it is not always taken into account, nor do the ordinary tests to destruction bring out such facts very well.

Photo-elastic experiments to destruction have this advantage over other means, that stress distribution is visible as a changing picture during the whole process, and that up to the breakdown of the material this can be accurately interpreted in terms of stress intensity. Beyond this point, although stress effects continue to be visible, the optical laws are not yet known with any precision, but there is no difficulty in determining where failure commences.

From time to time an endeavour has been made to put into practice the idea of making some mechanical detail equally strong in every part to resist the loads which come upon it. Although it is in general an impossible task, it is an ideal worth striving for, and photo-elastic experiment is of considerable help in attaining to some approximation thereto. As an instance of this, let us take a simple eye-bolt of some standard design and stress a transparent model in a beam of polarised light. It is apparent from the photograph that this form causes great concentration of stress to occur at the innermost points of a transverse cross-section through the centre of the pin, and now that the load is released you will observe that the model shows signs of overstress in this region. If a detailed survey of the stress distribution is made along this section, it is found that the stress is distributed in the manner shown by Fig. 7. It is, as we have seen, very great at the pin and diminishes very rapidly until at a point J near the outer contour the material is not stressed at all, while beyond this the material is under a slight compression stress. These stress indications are confirmed by measurements of the lines of principal stress, Fig. 8, which show that at a point K these lines are arranged in such a manner that there are equal like principal stresses in every direction, and from the former figure we have already seen that in this neighbourhood the stresses are very small.

The design is therefore such that the material near the outer contour along this section is of very little use, and would be more effective if placed around the pin in the form of a boss. This does, in fact, effect an improvement in the stress distribution, and although it does not produce uniformity along this section, the eye-bolt is stronger than before. It can be made still stronger by alterations in the outer contour, which allow the lines of principal stress to arrange themselves so that the focal point K of Fig. 8 disappears. The lines springing normally from the inner contour then intersect the outer boundary normally and the orthogonal set lose their points of inflection, with the result that a better distribution of normal stress is developed at the cross-section A—B, Fig. 7, owing to more load being thrown on to the outer part. It is probable that very little further photo-elastic experiment on these lines would evolve a still better form of eye-bolt, giving still better approximation to maximum strength for a given weight of material.

Another example in which possibly some useful work might be done relates to the struts used to connect the wings of aeroplanes. A model of one of these is now shown on the screen under load, which shows how great and uneven is the stress near the pin-jointed ends, while even far away from these points the stress distribution is far from uniform. This brief review is, I hope, sufficient to indicate the use of photo-elastic experiment and the nature of the help which is afforded by its means for solving problems of stress distribution, and also the aid which it can give in the production of designs where it is essential to distribute material in the most efficient manner.