2

The pre-QCD era

2.1 The quark model

- We know that hadrons have mainly strong interactions. However, the number of observed hadrons increases drastically in comparison with that of leptons. The classification of hadrons into multiplets has been facilitated by the discovery of *internal symmetries*, which play an important rôle for obtaining relations among masses, magnetic moments and couplings of the hadrons. The classification under the $SU(3)_F$ group (named *flavour* at present) [7] has been successful, where hadrons are characterized under their isospin I, hypercharge Y, baryon number B and strangeness S. Therefore, the pions are placed in the same pseudoscalar octet as the $K,\ \bar{K}$ and η, while the vector mesons $\rho,\ \omega,\ \phi$ fill another octet, . . . The splitting of hadron masses was expected, due to $SU(3)_F$ breaking that originated from strong interaction forces, whereas the $SU(2)$ isospin subgroup was found to be almost symmetric. This led to the concept of *charge independence*, which has played an important rôle in nuclear physics, where the proton and neutron form an $SU(2)$ doublet.
- However, none of the fundamental representations $SU(3)_F$ were realized by the observed hadrons, which led Gell-Mann and Zweig [8,9] to postulate that the observed hadrons, like the atoms, are not elementary, but are built by more elementary *quark*[1] constituents q having three flavours *up, down and strange*. Their charge Q in units of the one of the electron are:

$$Q_u = 2/3\ , \qquad Q_d = Q_s = -1/3\ . \tag{2.1}$$

In this picture, the mesons are bound states of quark–anti-quark, while the baryons are made by three quarks. The quarks internal quantum numbers are given in Table 2.1.

The $SU(3)_F$ decomposition into products of $\underline{3}$ and $\underline{3}^*$ representations gives for mesons:

$$\bar{q}q\ :\ \underline{3}^* \otimes \underline{3} = \underline{1} \oplus \underline{8} \tag{2.2}$$

and for baryons:

$$qqq\ :\ \underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10}\ , \tag{2.3}$$

[1] The name *quark* did not exist in the English dictionary, and may have been inspired from the following poetry *Finnengan's wake* of J. Joyce:

"Three quarks for Muster mark!
Sure he has'not got much of bark
and sure any he has it's all beside the mark."

However, *quark* is a well-known German word as it means curdy milk, but more commonly it means a mess.

Table 2.1. *Additive quark-quantum numbers*

Quark	u	d	s
Charge Q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
Third component of isospin I_3	$\frac{1}{2}$	$-\frac{1}{2}$	0
Hypercharge Y	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
Baryon number	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Strangeness	0	0	-1

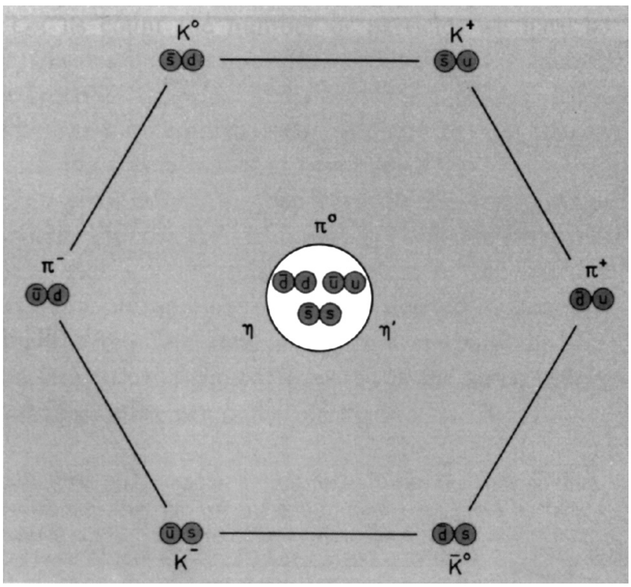

Fig. 2.1. The nine mesons built from the u, d, s quarks.

from which one can built a simple but complete Periodic Table of Hadrons. These classifications are given in Figs. 2.1 to 2.3. In this sense, the quark model was a modern version of the Sakata [10] model.

- Masses and mass-splittings of hadrons have been explained by using Gell-Mann–Okubo-like mass formulae [11], and by introducing the so-called *constituent* quark masses with the values [12]:

$$M_q \approx 300 \text{ MeV}, \tag{2.4}$$

and by assuming the quark-mass differences:

$$M_d - M_u \approx 4 \text{ MeV}, \qquad M_s - M_d \approx 150 \text{ MeV}. \tag{2.5}$$

- The compositeness hypothesis for the hadrons has been supported by the measurement of the proton magnetic moment which has a value of about 2.8 in units of $\mu_p = e\hbar/2M_p$, while it is expected to be unity from a point-like spin 1/2 object.

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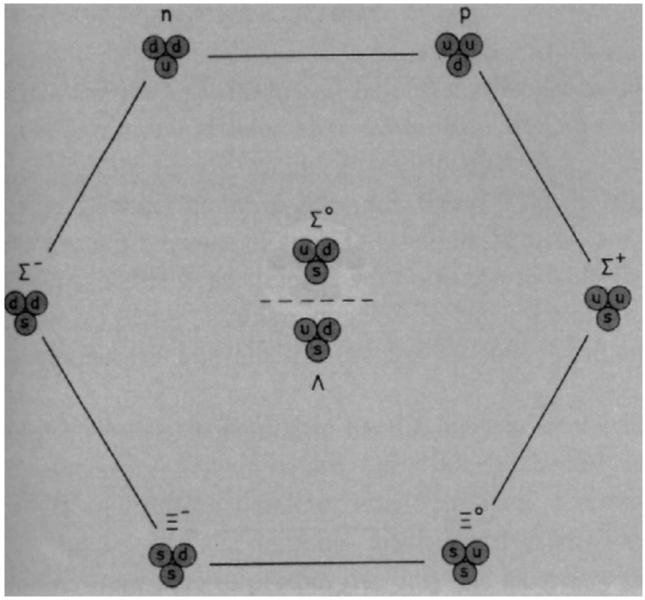

Fig. 2.2. The octet baryons built from the u, d, s quarks.

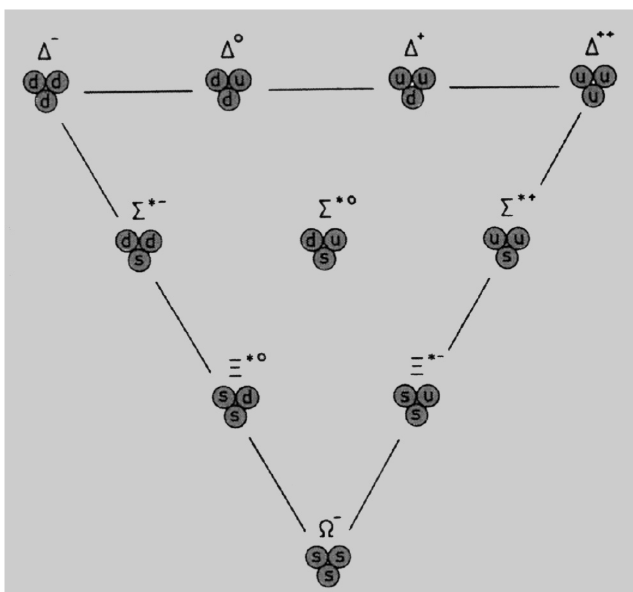

Fig. 2.3. The ten spin $3/2$ baryons built from the u, d, s quarks.

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2.2 Current algebras

Reviews on current algebras can be seen in [13]. In the following, we shall discuss some main features of the approach.

2.2.1 Currents conservation

- Although we have more forces in nature, *electromagnetism* plays a capital rôle. The theory of electron (muon) interacting with the photon field is the only one where the concepts of *quantum field theory* work in a satisfactory manner. Indeed, within Quantum ElectroDynamics (QED), one has been able to perform higher order approximate calculations which are confirmed by experimental measurements at an impressive, high level of accuracy (anomalous magnetic moment of the leptons, ...). Although more complicated, due to the presence of strong interactions, the study of the electromagnetic interaction of hadrons has been facilitated by the property of the *electromagnetic current conservation* leading to the *concept of universality*, which allows us to put, for example, at the same footing, an e^-, a π^- and a p^-, and to show, for instance, that the *physical charges* of these three particles remains the same after renormalizations. Moreover, current conservation allows the use of *soft photon theorems* in order to relate the cross-section to the static properties of the hadrons (charge, magnetic moments, ...). It is also one of the basis of the popular *Vector Meson Dominance Model* (VDM) [14]. As a consequence of the current conservation, the corresponding charge is a constant of motion, such that the only non-vanishing matrix elements of this charge are between equal-mass states.
- In the case of weak current, current conservation gives a well-defined meaning to the idea of universal weak coupling which has been successfully tested experimentally in the case of non-strange weak vector currents. However, difficulty arises when one tries to explain strangeness-violating transition such as the ratio of the $K^+ \to \pi^0 e^+ \nu_e$ over the $\pi^+ \to \pi^0 e^+ \nu_e$. It can only be explained by the introduction of the Cabibbo angle θ_c [15] allowing the mixing of the strange quark with the down quark, with the experimental value $\sin\theta_c = 0.220 \pm 0.003$ [16]. In this case, the idea of weak universality appears also to work in the process involving the strange quark.
- Inspired again by the quark model, Gell-Mann [7] suggested that the vector and axial charges satisfy a $SU(3) \otimes SU(3)$ algebra. This picture naturally leads to the existence of larger multiplets of particles having the same spins but with both parities, which has been confirmed by the data. The rôle of *partially conserved axial current* (PCAC) was found to be related to the existence of the light (compared with the ρ and p) pseudoscalar particle, the π, which has been understood, later on, from the spontaneous Nambu–Goldstone [17] nature of the symmetry breaking. More precisely, the exact current conservation of the axial current is realized when the pion is massless. Again inspired by the soft photon theorem which is a consequence of the conservation of the electromagnetic current, one can also derive *soft pion theorems* obtained from phenomenological Lagrangians satisfying the non-linear realizations of chiral symmetry.

2.2.2 Currents and charges

The next development is the construction of hadron currents built from quark fields in much the same way as one can write a current for lepton fields. The quark electromagnetic and charged weak currents can be written as:

$$J^\mu_{\rm em} = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{s}\gamma^\mu s + \cdots ,$$
$$J^\mu_{\rm weak} = \bar{u}\gamma^\mu (1-\gamma_5)\, d + \cdots , \tag{2.6}$$

where we ignore to a first approximation the mixing among quark fields due to the Cabibbo angle. In the massless quark limit ($m_j = 0$), the free quark Lagrangian density $\mathcal{L}_{\rm q}(x)$:

$$\mathcal{L}_{\rm q}(x) = i \sum_{j=1}^{n} \bar{\psi}_j \gamma_\mu \psi_j \ , \tag{2.7}$$

possesses a $SU(n)_L \times SU(n)_R$ global chiral symmetry and is invariant under the global chiral transformation:

$$\psi_i(x) \to \exp(-i\theta^A T_A)\psi_i(x) \ ,$$
$$\psi_i(x) \to \exp(-i\theta^A T_A \gamma_5)\psi_i(x) \ , \tag{2.8}$$

where $T^A (A \equiv 1, \ldots, n^2 - 1)$ are the infinitesimal generators of the $SU(n)$ group acting on the quark-flavour components. The associated Noether currents are the vector and axial-vector currents:

$$V^A_\mu(x) = \bar{\psi}_i \gamma_\mu \ T^A_{ij} \psi_i(x) \ ,$$
$$A^A_\mu(x) = \bar{\psi}_i \gamma_\mu \gamma_5 \ T^A_{ij} \psi_i(x) \ , \tag{2.9}$$

which are the ones of the algebra of currents of Gell-Mann [69,13] ($n = 3$ in the original paper). The corresponding charges which are the generators of $SU(n)_L \times SU(n)_R$ are:

$$Q^A = \int d^3x \ V^A_0(x) \ ,$$
$$Q^A_5 = \int d^3x \ A^A_0(x) \ . \tag{2.10}$$

The charges in Eq. (2.10) are conserved in the massless quark limit, and obey the commutation relations (simplified notations):

$$[Q^\alpha, Q^\beta] = i f_{\alpha\beta\gamma} Q^\gamma \ ,$$
$$\left[Q^\alpha_5, Q^\beta_5\right] = i f_{\alpha\beta\gamma} Q^\gamma \ ,$$
$$\left[Q^\alpha, Q^\beta_5\right] = i f_{\alpha\beta\gamma} Q^\gamma_5 \ , \tag{2.11}$$

i.e. Q_V and Q_A generate a closed algebra. They also imply:

$$[Q^\alpha, V^\beta] = i f_{\alpha\beta\gamma} V^\gamma \ ,$$
$$[Q^\alpha, A^\beta] = i f_{\alpha\beta\gamma} A^\gamma \ ,$$
$$\left[Q^\alpha_5, V^\beta\right] = i f_{\alpha\beta\gamma} A^\gamma \ ,$$
$$\left[Q^\alpha_5, A^\beta\right] = i f_{\alpha\beta\gamma} V^\gamma \ . \tag{2.12}$$

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2.2.3 Chiral symmetry and pion PCAC

In the Nambu–Goldstone [17] realization of chiral symmetry, the axial charge does not annihilate the vacuum, which is the basis of the successes of current algebra and pion PCAC [13]. In this scheme, the chiral flavour group $G \equiv SU(n)_L \times SU(n)_R$ is broken spontaneously by the light quark (u, d, s) vacuum condensates down to a subgroup $H \equiv SU(n)_{L+R}$, where the vacua are symmetrical:

$$\langle \bar{\psi}_u \psi_u \rangle = \langle \bar{\psi}_d \psi_d \rangle = \langle \bar{\psi}_s \psi_s \rangle \ . \tag{2.13}$$

The Goldstone theorem states that this spontaneous breaking mechanism is accompanied by $n^2 - 1$ massless Goldstone P (pions) bosons, which are associated with each unbroken generator of the coset space G/H. For $n = 3$, these Goldstone bosons can be identified with the eight lightest mesons of the Gell-Mann eightfoldway ($\pi^+,\ \pi^-,\ \pi^0,\ \eta,\ K^+, K^-,\ K^0,\ \bar{K}^0$). On the other hand, the vector charge is assumed to annihilate the vacuum and the corresponding symmetry is achieved à la Wigner–Weyl [18]. In the vector case, the particles are classified in irreducible representations of $SU(n)_{L+R}$ and form parity doublets. In addition to the electromagnetic mass which the Goldstone bosons can acquire [19], they get a mass mainly from an explicit breaking ($m_i \neq 0$) of the $SU(n)_L \times SU(n)_R$ global symmetry. In this case, the divergence of the axial-vector current does not vanish and reads (in the case of the $u,\ d$ quarks):

$$\partial_\mu A^\mu(x)^i_j = (m_i + m_j)\bar{\psi}_i (i\gamma_5)\psi_j \ , \tag{2.14}$$

to which are associated the quasi-Goldstone parameters defined as:

$$\langle 0|\partial_\mu A^\mu(x)^i_j|\pi\rangle = \sqrt{2} f_\pi m_\pi^2 \vec{\pi} \ , \tag{2.15}$$

where $\vec{\pi}$ is the pion field and $f_\pi = 92.4$ MeV is the pion decay constant which controls the $\pi \to \mu\nu$ decay width. In this case, the divergence of the vector current reads:

$$\partial_\mu V^\mu(x)^i_j = (m_i - m_j)\bar{\psi}_i (i)\psi_j \ , \tag{2.16}$$

to which is presumably associated the $a_0(980)$ scalar meson (the best experimental candidate).

Current algebra also tells us that the two-point correlator associated with Eq. (1.15) is related to the axial-current one via a current algebra Ward identity [20,13], up to equal-time commutator terms (in the following we shall suppress flavour indices):

$$\begin{aligned} q_\mu q_\nu \Pi_5^{\mu\nu} &= \Psi_5(q^2) - q^\nu \int d^4x e^{iqx} \delta(x_0) \langle 0[A^0(x), (A^\nu(0))^\dagger]0\rangle \\ &+ i \int d^4x e^{iqx} \delta(x_0) \langle 0[\partial_\mu A^\mu(x)\ , (A^0(0))^\dagger]0\rangle \ , \end{aligned} \tag{2.17}$$

with:

$$\begin{aligned} \Psi_5(q^2) &= i \int d^4x e^{iqx} \langle 0|\mathbf{T}\partial_\mu A^\mu(x)(\partial_\mu A^\mu(0))^\dagger|0\rangle \ , \\ \Pi_5^{\mu\nu}(q^2) &= i \int d^4x e^{iqx} \langle 0|\mathbf{T}A^\mu(x)\,(A^\nu(0))^\dagger\,|0\rangle \ . \end{aligned} \tag{2.18}$$

At $q = 0$, the previous identity reduces to:

$$\Psi_5(0) = -i(m_u + m_d)\langle 0[\bar{\psi}_d(0)i\gamma_5\psi_u(0), Q_5^\dagger]0\rangle\ , \tag{2.19}$$

where Q_5 is the axial-charge generator. In the Nambu–Goldstone realization of chiral symmetry, one has:

$$Q_5|0\rangle \neq 0\ . \tag{2.20}$$

Therefore, we get:

$$\Psi_5(0) = -(m_u + m_d)\langle\bar{\psi}_d\psi_d + \bar{\psi}_u\psi_u\rangle\ . \tag{2.21}$$

Using Eq. (2.15) in the definition of $\Psi_5(q^2)$ and equating this with Eq. (2.19), we have the well-known pion PCAC (Gell-Mann *et al.* [21]) relation at $q = 0$ (recall that $f_\pi =$ 92.4 MeV):

$$-(m_u + m_d)\langle\bar{\psi}_d\psi_d + \bar{\psi}_u\psi_u\rangle = 2m_\pi^2 f_\pi^2\ . \tag{2.22}$$

2.2.4 Soft pion theorem and the Goldberger–Treiman relation

Let's consider the matrix element of the axial-vector current between two nucleon states shown in Fig. 2.4.

Using invariance properties, it can be parametrized as:

$$\langle N(p_2)|A_\mu|N(p_1)\rangle = \bar{u}(p_2)[\gamma_\mu g_A(q^2) + q_\mu g_P(q^2)]\gamma_5 u(p_1)\ , \tag{2.23}$$

where $q = p_2 - p_1$ is the momentum transfer between the nucleon states, and where experimentally $g_A(0) = 1.26$. The matrix element of the current divergence reads:

$$\mathcal{A} \equiv \langle N(p_2)|\partial^\mu A_\mu|N(p_1)\rangle = \bar{u}(p_2)[2M_N g_A(q^2) + q^2 g_P(q^2)](i\gamma_5)u(p_1)\ , \tag{2.24}$$

where the relation for the Dirac spinors:

$$q_\mu\bar{u}(p_2)(i\gamma^\mu\gamma_5)u(p_1) = 2M_N\bar{u}(p_2)\gamma_5 u(p_1)\ , \tag{2.25}$$

has been used. The PCAC hypothesis in Eq. (2.15) yields in the massless pion (chiral) limit:

$$2M_N g_A(q^2) + q^2 g_P(q^2) = 0\ . \tag{2.26}$$

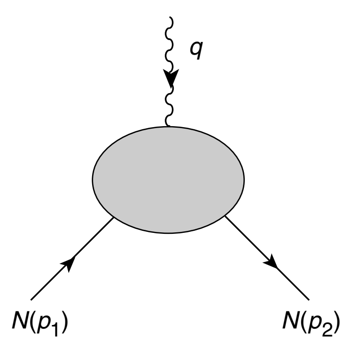

Fig. 2.4. Axial-vector scattering with nucleon.

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where the divergence of the axial-vector current is zero. If $g(q^2)$ has no singularity at $q^2 = 0$, then Eq. (2.26), would imply either $M_N = 0$ or $g_A = 0$. However, none of these requirements are true. Therefore, g_P should have a pole at $q^2 = 0$:

$$\lim_{q^2 \to 0} g_P(q^2) = -\frac{2M_N g_A}{q^2} \,. \tag{2.27}$$

The matrix element in Eq. (2.24) between a one pion state and the vacuum is the same as if there were a term in $A_\mu(x)$ of the form $\sqrt{2} f_\pi \partial_\mu \vec{\pi}(x)$. Therefore, in the chiral limit, the matrix element has a pole, and reads:

$$\langle N(p_2)|A_\mu|N(p_1)\rangle = \sqrt{2} f_\pi q_\mu \langle N(p_2)|\vec{\pi}|N(p_1)\rangle = \frac{2 f_\pi q_\mu}{-q^2} g_{\pi NN}(q^2)\bar{u}(k_2)(i\gamma_5)u(k_1) \,, \tag{2.28}$$

where $g_{\pi NN}(q^2)$ is the πNN vertex function. Its *physical coupling* is defined at $q^2 = m_\pi^2$ at has the experimental value of 13.50 ± 0.15 [16]. Solving these last two equations, one can derive the *Golberger–Treiman relation* (GT) [22] in the chiral limit:

$$f_\pi g_{\pi NN}(0) = M_N g_A(0) \,. \tag{2.29}$$

In the case of massive quarks, one can write the matrix element in Eq. (2.24) as:

$$\mathcal{A} = \sqrt{2} f_\pi m_\pi^2 \langle N(p_2)|\vec{\pi}|N(p_1)\rangle = \frac{2 f_\pi m_\pi^2}{-q^2 + m_\pi^2} g_{\pi NN}(q^2)\bar{u}(k_2)(i\gamma_5)u(k_1) \,. \tag{2.30}$$

By identifying Eqs. (2.24) and (2.30), and setting $q^2 = 0$, one would obtain the previous GT relation in Eq. (2.29), which one can identify with the physical coupling assuming that the coupling is a smooth function of q^2 from 0 to m_π^2, which is valid as there is no one-pion pole in this function. One should remark that only $g_P(q^2)$ has a pion pole term, and it is of the form:

$$g_P(q^2) = \frac{\sqrt{2} f_\pi}{m_\pi^2 - q^2} \sqrt{2} g_{\pi NN} \,, \tag{2.31}$$

such that at $q^2 = m_\pi^2$, Eqs. (2.30) and (2.24) leads to a trivial equality.

2.2.5 The Adler–Weisberger sum rule and soft pion theorems

In the case of the Golberger–Treiman relation, we have used a one-pion soft theorem for estimating the pion-nucleon-nucleon matrix element. Here, we shall be concerned by low-energy theorems for pion-nucleon scattering amplitudes involving two soft pions. The process is depicted in Fig. 2.5.

The amplitude can be written as:

$$\langle \pi_i(q_2)N(p_2)|\pi_j(q_1)N(p_1)\rangle = i(2\pi)^4\delta^4(p_1 + q_1 - p_2 - q_2)T_{ij} \,, \tag{2.32}$$

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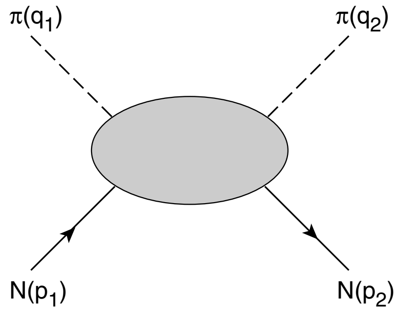

Fig. 2.5. Forward pion-nucleon scattering process.

which can be decomposed in terms of two invariants (isospin-even and -odd):

$$T_{ij} = \delta_{ij} T^{(+)} + \frac{1}{2}[\tau_i, \tau_j] T^{(-)} , \tag{2.33}$$

where i, j are isospin indices. Using standard reduction formula discussed in the next section, one can apply the soft pion theorem, which gives:

$$\begin{aligned} T_{ij} &= i\left(-q_2^2 + m_\pi^2\right)\langle N(p_2)|\vec{\pi}^{\,i}(0)\vec{\pi}^{\,j}(q_1)N(p_1)\rangle \\ &= \frac{q_2^\mu\left(-q_2^2 + m_\pi^2\right)}{\sqrt{2} f_\pi m_\pi^2}\langle N(p_2)|A_\mu^i(0)|\pi^j(q_1)N(p_1)\rangle . \end{aligned} \tag{2.34}$$

For $q_2 \to 0$, we can take $T^{(-)} = 0$ since it is odd under crossing. Also, the non-singular part of the amplitude vanishes (*Adler's consistency condition*) [23]:

$$T^{(+)}\left(\nu = 0,\ \nu_B = 0,\ q_1^2 = m_\pi^2,\ q_2^2 = 0\right) = 0 , \tag{2.35}$$

where:

$$\nu \equiv q_1(p_1 + p_2)/2 , \qquad \nu_B = -q_1 \cdot q_2/2 , \tag{2.36}$$

are kinematic variables. Similarly, when $q_1^2 \to 0$, one obtains:

$$T^{(+)}\left(\nu = 0,\ \nu_B = 0,\ q_1^2 = 0,\ q_2^2 = m_\pi^2\right) = 0 . \tag{2.37}$$

Applying two times the soft pion theorems, one can reduce the amplitude as:

$$T_{ij} = i\left(q_1^2 - m_\pi^2\right)\left(q_2^2 - m_\pi^2\right)\frac{1}{2m_\pi^4 f_\pi^2}\int d^4x\, e^{iq_1x}\langle N(p_2)|\mathcal{T}\partial^\mu A_\mu(x)\partial^\mu A_\mu(0)|N(p_1)\rangle . \tag{2.38}$$

Using the current algebra Ward identity:

$$\begin{aligned} q_1^\mu q_2^\nu \int d^4x\, e^{iq_1x}\mathcal{T} A_\mu^i(x) A_\nu^j(0) = \int d^4x\, e^{iq_1x}\big[&\mathcal{T}\partial^\mu A_\mu^i(x)\partial^\mu A_\mu^j(0) \\ &- iq_1^\mu\delta(x_0)\left[A_0^j(0), A_\mu^i(x)\right] + \delta(x_0)\left[A_0^i(0), \partial^\mu A_\mu^j(x)\right]\big] , \end{aligned} \tag{2.39}$$

one can see after sandwiching between two nucleon states that the first term is the nucleon matrix element of a time-ordered product of two-pion operators; the second term can be evaluated from the current algebra commutation relation:

$$\delta(x_0)\left[A_0^i(0), A_\mu^j(x)\right] = -i\delta(x)\epsilon^{ijk} V_{k,\mu}(x) , \tag{2.40}$$

while the last term gives the *pion-sigma term*, which is symmetric in i, j, and then this t-channel state must have isospin 0 or 2 since the pion has isospin 1. However, since the nucleon has isospin 1/2, only $I = 0$ state can contribute, and therefore:

$$\sigma^{ij} = \delta^{ij}\sigma_N . \tag{2.41}$$

In the low-energy limit, the following soft-pion theorems are obtained:

$$\lim_{\nu\to 0} \nu^{-1} T^{(-)}(\nu, 0, 0, 0) = \left(1 - g_A^2\right)/f_\pi^2 , \tag{2.42}$$

and

$$\lim_{\nu\to 0} \nu^{-1} T^{(+)}(0, 0, 0, 0) = -\sigma_N/f_\pi^2 . \tag{2.43}$$

It is also expected and assumed that $T^{(-)}$, which is odd under the change $\nu \to -\nu$, obeys an unsubtracted dispersion relation in the variable ν:

$$\frac{T^{(-)}(\nu, q^2 = 0)}{\nu} = \frac{2}{\pi}\int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2}\,\mathrm{Im}T^{(-)}(\nu', 0) . \tag{2.44}$$

Its imaginary part can be related to the πN cross-section if one assumes a smoothness assumption:

$$\mathrm{Im}T^{(-)}(\nu, 0) \simeq \mathrm{Im}T^{(-)}\left(\nu, m_\pi^2\right) = \nu\left[\sigma_{\rm tot}^{\pi^+ p}(\nu) - \sigma_{\rm tot}^{\pi^- p}(\nu)\right] . \tag{2.45}$$

Using the previous GT relation in Eq. (2.29) for eliminating f_π in Eq. (2.42), the dispersion relation gives the *Adler–Weisberger relation* [24]:

$$1 - \frac{1}{g_A^2} = \frac{2M_N^2}{\pi g_{\pi NN}^2}\int_{\nu_0}^{\infty} \frac{d\nu}{\nu}\left[\sigma_{\rm tot}^{\pi^+ p}(\nu) - \sigma_{\rm tot}^{\pi^- p}(\nu)\right] , \tag{2.46}$$

which is an interesting low-energy sum rule.

2.2.6 Soft pion theorem for $\rho \to \pi^+\pi^-$ and the KSFR relation

We discuss here a further use of soft pion theorems. We consider the process in the chiral limit where the pions are massless:

$$\rho^0 \to \pi^+\pi^- . \tag{2.47}$$

It is described by the amplitude:

$$T_{\nu\mu}^{ij} = i\int d^4x\ \exp(iqx)\langle 0|\mathcal{T} A_\nu^i(x) A_\mu^j(0)|\rho(p)\rangle , \tag{2.48}$$

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where $i,\ j$ are isospin indices. Taking its divergence, one obtains:

$$q^\nu T^{ij}_{\nu\mu} = \left\{ U^{ij}_\mu \equiv -\int d^4x\ \exp(iqx)\langle 0|\mathcal{T}\partial^\nu A^i_\nu(x)A^j_\mu(0)|\rho(p)\rangle \right\}$$
$$-\int d^4x\ \exp(iqx)\delta(x_0)\langle 0|\left[A^i_0(x),\, A^j_\mu(0)\right]|\rho(p)\rangle\ . \tag{2.49}$$

Using the commutation relation given previously, one can deduce the Ward identity:

$$q^\nu T^{ij}_{\nu\mu} = U^{ij}_\mu - if^{ijk}\langle 0|V_{\mu,k}|\rho(p)\rangle\ , \tag{2.50}$$

where V_μ is the vector isovector current. In the massless pion limit, the axial current is conserved such that U^{ij}_μ vanishes. The coupling of the *neural* ρ-meson to the isovector current is introduced as (from now, we shall suppress the isospin indices):

$$\langle 0|V_\mu|\rho(p)\rangle = \frac{M^2_\rho}{2\gamma_\rho}\epsilon_\mu\ . \tag{2.51}$$

where, experimentally, $\gamma_\rho = 2.55$, with the normalization:

$$\Gamma_{\rho\to e^+e^-} \simeq \frac{2}{3}\pi\alpha^2\frac{M_\rho}{2\gamma^2_\rho}\ . \tag{2.52}$$

ϵ^μ is the polarization of the ρ meson which ensures the conservation of the vector current. Contracting again with the pion momentum q', one obtains:

$$q^\nu q'^\mu T_{\nu\mu} = (\epsilon\cdot q')\frac{M^2_\rho}{2\gamma_\rho}\ . \tag{2.53}$$

Introducing the $\rho\pi\pi$ coupling as:

$$\langle\pi(q'),\pi(q)|\rho(p)\rangle = \epsilon^\nu(q'-q)_\nu g_{\rho\pi\pi}\ , \tag{2.54}$$

and taking the limit $q'\to q\to 0$, one obtains the soft pion relation:

$$\frac{M^2_\rho}{2\gamma_\rho} = 4f^2_\pi g_{\rho\pi\pi}\ . \tag{2.55}$$

If one assumes ρ-universality from the vector meson dominance model [14], one has:

$$\frac{M^2_\rho}{2\gamma_\rho} = \frac{M^2_\rho}{g_{\rho\pi\pi}}\ . \tag{2.56}$$

The two equations give the Kawarabayashi–Suzuki–Ryazuddin–Fayazuddin (KSFR) relations [25]:

$$g^2_{\rho\pi\pi} = \frac{M^2_\rho}{4f^2_\pi}\ , \tag{2.57}$$

or:

$$f^2_\pi = \frac{M^2_\rho}{16\gamma^2_\rho}\ , \tag{2.58}$$

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which are useful in different phenomenological applications. One can check from the data that the predictions given by these two relations are unexpectedly good despite the crude approximation used for deriving them.

2.2.7 Weinberg current algebra sum rules

Another important consequence of the commutation relation of currents are the different current algebra dispersion sum rules, based on the assumption that the $SU(3)\otimes SU(3)$ symmetry is realized asymptotically. Though conceptually difficult to digest, this *asymptotically free* hypothesis has been very successful in different applications [13] (Weinberg and Das–Mathur–Okubo (DMO) sum rules [26,27], Adler–Weisberger sum rule [24] discussed previously, ...). Here, we shall discuss briefly the Weinberg and DMO sum rules. They are based on the assumed asymptotic behaviour of the absorptive amplitudes, with the assumption that the $SU(2)_L\times SU(2)_R$ chiral symmetry is asymptotically realized in nature. Weinberg has derived two superconvergent sum rules, well-known as Weinberg sum rules (WSR) [26]. In order to show this result, it is appropriate to study the two-point correlator:

$$\begin{aligned} W^{\mu\nu}_{LR} &\equiv i\int d^4x e^{iqx}\langle 0|\mathcal{T}J^\mu_L(x)\left(J^\nu_R(0)\right)^\dagger|0\rangle \\ &= -(g^{\mu\nu}q^2-q^\mu q^\nu)\Pi^{(1)}_{LR}+q^\mu q^\nu\Pi^{(0)}_{LR}\ , \end{aligned} \tag{2.59}$$

where J^μ_L and J^μ_R are left- and right-handed charged currents, which read in terms of the quark fields:

$$J^\mu_L\equiv\bar u\gamma^\mu(1-\gamma_5)d\ ,\qquad J^\mu_R\equiv\bar u\gamma^\mu(1+\gamma_5)d\ . \tag{2.60}$$

$\Pi^{(1)}_{LR}$ and $\Pi^{(0)}_{LR}$ are respectively the transverse and longitudinal parts of the correlator. In the asymptotic limit ($q^2\to\infty$) or in the chiral limit ($m_{u,d}\to 0$), where the $SU(2)_L\times SU(2)_R$ chiral symmetry is realized, $W^{\mu\nu}_{LR}$ tends to zero. Using the Källen–Lehmann representation of the two-point correlator:

$$\left(\Pi^{(J)}_{LR}\equiv\left(\Pi ij^{(J)}\right)_{LR}\right)\left(q^2,m_i^2,m_j^2\right)=\int_0^\infty\frac{dt}{t-q^2-i\epsilon}\,\frac{1}{\pi}\,\mathrm{Im}\Pi^{(J)}_{LR}(t)+\cdots, \tag{2.61}$$

where $\cdots$ represent subtraction points, which are polynomial in the q^2-variable, one can transform the previous property of $W^{\mu\nu}_{LR}$ into superconvergent sum rules for its absorptive parts [26]:

$$\begin{aligned} &\int_0^\infty dt\ \mathrm{Im}\left(\Pi^{(1)}_{LR}+\Pi^{(0)}_{LR}\right)\approx 0\ , \\ &\int_0^\infty dt\ t\ \mathrm{Im}\Pi^{(1)}_{LR}\approx 0\ , \end{aligned} \tag{2.62}$$

where the first WSR comes from the $q^\mu q^\nu$ component of $W^{\mu\nu}_{LR}$ and the second WSR comes from its $g^{\mu\nu}$ part. These WSR express in a clear way, the *global duality* between the long-range (spectral function measurable at low-energy) and the high-energy (asymptotic theory)

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parts of the hadronic correlators. This quark-hadron duality is one of the basic idea behind QCD spectral sum rules, which we shall discuss in detail in the next part of the book.

In order to parametrize the spectral functions, we use a narrow-width approximation and assume that the π, A_1 and ρ dominate the spectral functions. In this way, one can derive the constraints:

$$\frac{M_\rho^2}{2\gamma_\rho^2} - \frac{M_{A_1}^2}{2\gamma_{A_1}^2} - 2f_\pi^2 \approx 0 \,,$$
$$\frac{M_\rho^4}{2\gamma_\rho^2} - \frac{M_{A_1}^4}{2\gamma_{A_1}^2} \approx 0 \,, \tag{2.63}$$

where $f_\pi = 92.4$ MeV is the pion decay constant governing the $\pi \to \mu\nu$ decay; γ_V is the V-meson coupling to the corresponding *charged* current:

$$\langle 0|V^\mu|\rho\rangle = \sqrt{2}\frac{M_\rho^2}{2\gamma_\rho}\epsilon^\mu \,, \tag{2.64}$$

where experimentally $\gamma_\rho \simeq 2.55$. Notice the extra $\sqrt{2}$ factor coming from the different normalizations of the charged and neutral current discussed in the analysis of the $\rho^0 \to \pi^+\pi^-$ decay. From the above crude assumptions, one can predict by solving the two WSR equations and by using the experimental values of the ρ and π parameters:

$$M_{A_1} \simeq 1.1 \text{ GeV} \,, \tag{2.65}$$

which is in good agreement with the present data [16]. If, in addition, one uses the relation between f_π, γ_ρ and M_ρ (approximate KSFR relation [25]) discussed previously:

$$f_\pi^2 \simeq \frac{M_\rho^2}{16\gamma_\rho^2} \,, \tag{2.66}$$

deduced, from $\rho \to \pi\pi$ decays, using soft pion techniques, one arrives at the successful Weinberg mass formula:

$$M_{A_1} \simeq \sqrt{2}M_\rho \,, \tag{2.67}$$

although one should notice that the data from hadronic experiments give a slightly higher value [16].

2.2.8 The DMO sum rules in the $SU(3)_F$ symmetry limit

Electromagnetic current

Weinberg-inspired sum rules have been also derived from the asymptotic realization of the flavour symmetry. The Das–Mathur–Okubo (DMO) sum rules [27] can be studied from the two-point correlator:

$$\Pi_i^{\mu\nu}(q^2) \equiv \int d^4x e^{iqx} \langle 0|\mathcal{T} V_i^\mu(x) \left(V_i^\nu(0)\right)^\dagger |0\rangle$$
$$= -(g^{\mu\nu}q^2 - q^\mu q^\nu)\Pi_i^{(1)}(q^2) \,, \tag{2.68}$$

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where $V_i^\mu(x) \equiv \bar{\psi}_i\gamma^\mu\psi_i$ $(i \equiv u, d, s, \ldots)$ are the flavour components of the electromagnetic current:

$$J^\mu_{EM}(x) = \frac{2}{3}V_u^\mu - \frac{1}{3}V_d^\mu + \frac{2}{3}V_c^\mu - \frac{1}{3}V_s^\mu + \cdots \tag{2.69}$$

In the asymptotic limit ($q^2 \to \infty$) or in the chiral limit ($m_i \to 0$), one can derive the DMO sum rule [27]:

$$\int_0^\infty dt[\mathrm{Im}\Pi_3(t) - \mathrm{Im}\Pi_8(t)] \equiv \int_0^\infty dt\ \mathrm{Im}(\Pi_u + \Pi_d - 2\Pi_s)(t) = 0\ , \tag{2.70}$$

which corresponds to the difference between the isovector and isoscalar spectral functions associated with the $SU(3)_F$ symmetry. Saturating the spectral functions by the lowest mass resonances, one can derive the well-known successful phenomenological relation among vector mesons:

$$M_\rho\Gamma_{\rho\to e^+e^-} - 3(M_\omega\Gamma_{\omega\to e^+e^-} + M_\varphi\Gamma_{\varphi\to e^+e^-}) \simeq 0\ . \tag{2.71}$$

One can also re-write the DMO sum rules in terms of the total cross-section for $e^+e^- \to$ hadrons by using the optical theorem:

$$\sigma(e^+e^- \to \text{ hadrons}) = \frac{4\pi^2\alpha}{t}e^2\frac{1}{\pi}\mathrm{Im}\Pi(t)\ . \tag{2.72}$$

This relation is useful for testing the breaking of $SU(3)_F$, as we shall see later on, because we have complete data for the total cross-section.

Charged current

In the case of the charged vector or axial current:

$$V^\mu(x)^i_j = \bar{\psi}_i\gamma^\mu\psi_j\ , \qquad A^\mu(x)^i_j = \bar{\psi}_i\gamma^\mu\gamma^5\psi_j\ , \tag{2.73}$$

the DMO sum rules read in the chiral limit:

$$\int_0^\infty dt\ \mathrm{Im}\Pi^{(1)}(t)^d_u = \int_0^\infty dt\ \mathrm{Im}\Pi^{(1)}(t)^s_u\ , \tag{2.74}$$

where the spectral function can be measured in the $\tau \to \nu_\tau +$ hadrons decays. By saturating the spectral function with the lowest resonances, one can deduce the constraint:

$$\frac{M_\rho^2}{\gamma_\rho^2} \approx \frac{M_{K^*}^2}{\gamma_{K^*}^2}\ . \tag{2.75}$$

Using $\gamma_\rho = 2.55,\ M_\rho = 0.776$ GeV and $M_{K^*} = 0.892$ GeV, it gives:

$$\gamma_{K^*} = 2.93\ , \tag{2.76}$$

which is already an interesting constraint as compared with the data from τ decay [16]. On can notice that, as in the case of the WSR, the DMO sum rules give constraints between the low-energy behaviour of the spectral functions and their asymptotic one.

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2.2.9 π^+-π^0 mass difference

Hadronic contributions to the electromagnetic π^+-π^0 mass difference have been derived by Das *et al.* [19] by assuming a good realization of the $SU(2)_L \times SU(2)_R$ chiral symmetry at short distance. In this way, by integrating the virtual photon with momentum q^2, they derive the result, in the chiral limit:

$$\begin{aligned} m^2_{\pi^+} - m^2_{\pi^0} &\simeq -i\frac{6\pi\alpha}{f^2_\pi}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\int_0^\infty \frac{dt}{q^2+t-i\epsilon}\frac{t}{\pi}\mathrm{Im}\Pi^{(1)}_{LR} \\ &\simeq -\frac{3\alpha}{4\pi f^2_\pi}\int_0^\infty dt\left(t\,\ln\frac{t}{\nu^2}\right)\frac{1}{2\pi}\mathrm{Im}\Pi^{(1)}_{LR} \end{aligned} \tag{2.77}$$

where the spectral functions enter the second WSR and ν is an arbitrary UV cut-off. Using a lowest resonance saturation of the spectral functions in the narrow width appproximation (NWA), and the constraints provided by the first and second sum rules, which guarantee the convergence of the integral, one can derive the relation:

$$m^2_{\pi^+} - m^2_{\pi^0} \simeq \frac{3\alpha}{4\pi}\frac{M^2_{A_1}M^2_\rho}{M^2_{A_1}-M^2_\rho}\ln\frac{M^2_{A_1}}{M^2_\rho}\ . \tag{2.78}$$

Using the WSR relation $M^2_{A_1} = 2M^2_\rho$, one can deduce the result of [19]:

$$m_{\pi^+} - m_{\pi^0} \simeq \frac{3\alpha}{4\pi}\frac{M^2_\rho \ln 2}{m_\pi}\ , \tag{2.79}$$

which is in good agreement with the data $m_{\pi^+} - m_{\pi^0} = 4.5936(5)$ MeV [16]. The improvements of these prototype current algebra sum rules in the QCD context have been done in [28–34] and will be discussed in details in the following sections.

2.3 Parton model and Bjorken scaling

Different deep-inelastic scattering experiments such as the unpolarized electroproduction process $ep \to eX$ (X being the sum of inclusive produced hadrons) at high-energy virtual photon with momentum Q, have been used to explore the quark structure of the proton. This unpolarized process can be characterized by two measurable *structure functions* $W_{1,2}$, which parametrize the hadronic tensor and contains all strong interaction information about the response of the target nucleon to electromagnetic probes:

$$\frac{d\sigma}{dQ^2d\nu} = \frac{\pi\alpha^2}{4M_pE^2\sin^4\theta EE'}\left\{2\sin^2\frac{\theta}{2}W_1(Q^2,\nu)+\cos^2\frac{\theta}{2}W_2(Q^2,\nu)\right\}\ . \tag{2.80}$$

As shown in Fig. 2.6, they depend on the usual kinematic variables $-q^2 \equiv Q^2$ and ν:

$$\nu \equiv p\cdot q = M_p(E-E')\ , \tag{2.81}$$

where ν/M_p is the energy transfer in the proton rest frame; p and M_p are the proton momentum and mass; E and E' are the energies of the incident and scattered electrons in the proton rest frame, and θ is the scattering angle ($Q^2 = 4EE'\sin^2\frac{\theta}{2}$). For a point-like

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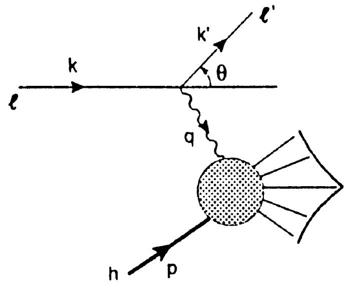

Fig. 2.6. $ep \to e$+hadrons process.

proton, the structure functions are δ-functions:

$$W_1(Q^2,\nu) = \frac{Q^2}{4M_p^2}\delta\left(\nu - \frac{Q^2}{2}\right), \qquad W_2(Q^2,\nu) = \delta\left(\nu - \frac{Q^2}{2}\right). \tag{2.82}$$

It has been observed that, at large Q^2, contributions from pointlike spin $1/2$ objects inside the proton still remain, while prominent contributions of resonances at low Q^2 die out quickly when Q^2 increases. A rough estimate of the *proton structure functions* can be done by assuming that the proton consists with pointlike spin $1/2$ quark constituents (called *wee partons* by Feynman [35]), each one carrying a given fraction ξ_i of the proton momentum. Defining by $f_i(\xi_i)$ the probability that a parton i has momentum fraction ξ_i, and by $W_j^{(i)}$ the parton contribution to the structure function, then the proton structure function becomes an incoherent sum of the one of the partons, and reads:

$$\begin{aligned} W_1(Q^2,\nu) &= \sum_i \int_0^1 d\xi_i\, f_i(\xi_i) W_1^{(i)}(Q^2,\nu) = \frac{1}{2}\sum_i e_i^2 f_i(x) \equiv F_1(x) \\ W_2(Q^2,\nu) &= \sum_i \int_0^1 d\xi_i\, f_i(\xi_i) W_2^{(i)}(Q^2,\nu) = \frac{M_p^2}{\nu} x \sum_i e_i^2 f_i(x) \equiv \frac{M_p^2}{\nu} F_2(x), \end{aligned} \tag{2.83}$$

where e_i is the electric charge and:

$$x \equiv \frac{Q^2}{2\nu}. \tag{2.84}$$

This simple parton description of the proton, where the structure function depends only on the kinematic variable x, is known as *Bjorken scaling* [36]. As a consequence of the spin-$1/2$ assumption of the constituent quarks, one also obtains the *Callan–Gross relation* [37]:

$$F_2(x) = 2xF_1(x). \tag{2.85}$$

These two *QCD sum rules* are well-satisfied by the data as shown in the Figs. 2.7 and 2.8,[2] which then surprisingly suggest the existence of free point-like partons inside the proton, in apparent contradiction with the confinement postulate.

[2] Small logarithmic deviations from the parton model prediction are also seen, and are well explained in QCD (as we shall see later on) after leading logs-resummation using the Altarelli–Parisi equation [38].

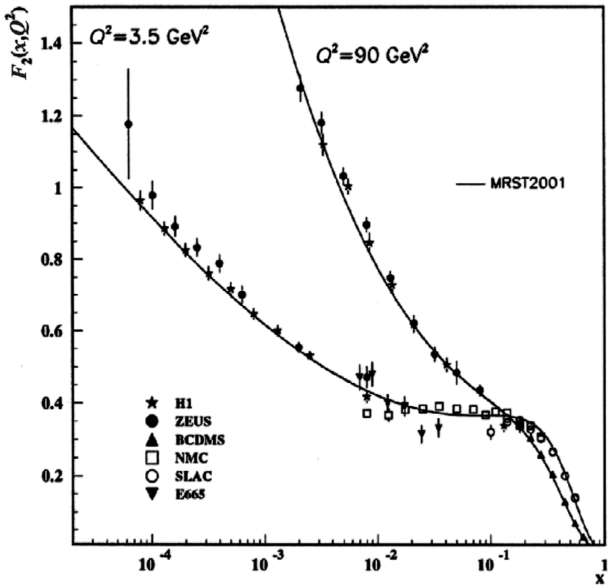

Fig. 2.7. The proton structure function F_2 versus x at two values of Q^2, exhibiting scaling at the pivot point $x \approx 0.14$.

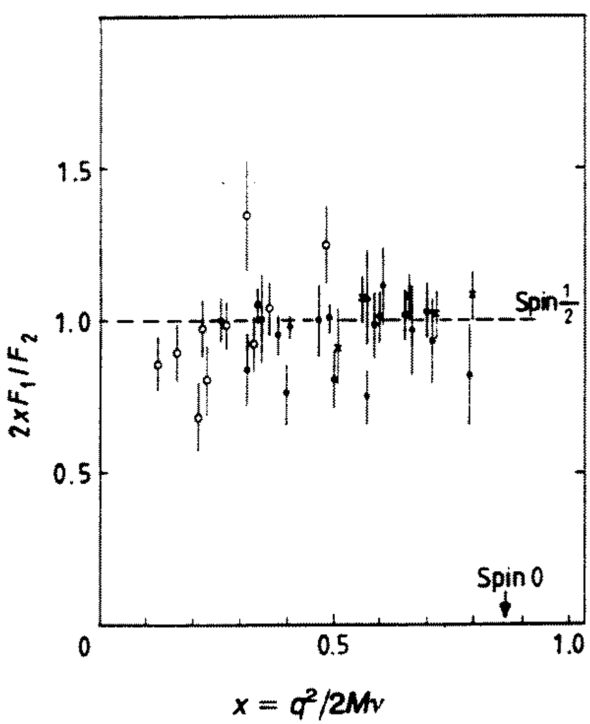

Fig. 2.8. The ratio $2xF_1/F_2$ versus x for Q^2 values between 1.5 and 16 GeV^2.

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2.4 The S-matrix approach and the Veneziano model

2.4.1 The S-matrix approach

An alternative to the quark model was the so-called S-matrix *(bootstrap)* approach which was very popular in the 1960s–1970s. It is based from a general Lagrangian, which should be constrained from general principles (relativistic covariance, substitution rule, unitarity and analyticity), and which limits the choice of the S-matrix. One of the main consequences of this approach is the *Regge poles* theory [39], which gives a general classification of hadrons *(Regge trajectories)* and predictions for high-energy data in terms of low-energy parameters from the study of resonances. This approach can be illustrated by the scattering process:

$$A + B \to C + D \tag{2.86}$$

and the crossed processes:

$$A + \bar{C} \to \bar{B} + D\,, \qquad A + \bar{D} \to \bar{B} + C\,, \tag{2.87}$$

characterized by the two kinematic variables s and t. The amplitude can be written in a dispersive form:

$$A(s,t) = \frac{1}{\pi}\int \frac{\mathrm{Im}A(s',t')}{s'-s}ds', \tag{2.88}$$

where one assumes that it converges for sufficiently large t, while it can be written as a sum of poles:

$$A(s,t) = \beta(t)\sum_{n=0}^{\infty}\frac{s^n}{\alpha(t)-n} \tag{2.89}$$

in the variable t at the solutions of the equations $\alpha(t_0) = 0$, $\alpha(t_n) = n$. Regge asymptotic law gives rise for fixed t to:

$$\lim_{s\to\infty} \mathrm{Im}A(s,t) \sim \beta(t)s^{\alpha(t)}\,, \tag{2.90}$$

where one can see a direct relation between the s and t-channels description of the scattering. This relation can also be seen more conveniently from the finite energy sum rule:

$$\int_0^L ds\, s^n\, \mathrm{Im}A(s,t) = \frac{L^{\alpha(t)+n+1}}{\alpha(t)+n+1}\,. \tag{2.91}$$

2.4.2 The Veneziano model and duality

The duality relation (crossing) between the s-channel resonance and t-channel Regge poles suggests the *duality bootstrap*. This has been achieved by the Veneziano approach [40],

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where a complete (though approximate) description of scattering can be obtained in terms of the s-channel resonances only. Inserting the resonance contributions from the particles contained in the trajectory $\alpha(s)$ and in its daughters, one obtains:

$$A(s,t) = \sum_n \frac{c_n(t)}{\alpha(s)-n} = \sum_n \frac{c_n(s)}{\alpha(t)-n}\ , \tag{2.92}$$

where the last equality is due to the duality constraints. $c_n(t)$ is a polynomial of order n in t. The contribution of highest spin $j=n$ comes from the $\alpha(s)=n$ intercept in the leading trajectory, while the ones of lower spin come from the presence of lower 'daughter' trajectories. The solution to this equation is given by the well-known *Veneziano beta-function amplitude*:

$$A(s,t) = \frac{\Gamma[-\alpha(s)]\Gamma[-\alpha(t)]}{\Gamma[-\alpha(s)-\alpha(t)]}\ . \tag{2.93}$$

The Veneziano dual-resonance model for the scattering amplitude can be summarized by the following conditions:

- Only infinitely *narrow* resonances appear, and the only singularities are poles on the real axis.
- There is an exact crossing symmetry.
- There is an asymptotic Regge behaviour with linear trajectories with universal slope.

However, one should notice that straight line trajectories are very far from the expectation from a field theoretical argument which suggests a Yukawa-like potential. Instead, they follow from a harmonic oscillator potential, and seem to be supported by the data.

2.4.3 Duality diagrams

The previous discussion can be visualized using duality diagrams introduced in [41]. It consists to represent the quark content of non-exotic (ordinary) hadrons as:

- Ordinary baryons composed of three quarks will be represented by three quark lines oriented in the same directions.
- Mesons composed by quark–anti-quark will be represented by quark lines going in opposite directions.

The process is represented by the topological structure of the graph:

- Planar diagrams can be drawn without crossing quark lines, which coïncide with the ones suggested by duality and ordinary hadrons, and which give a non-vanishing contribution to the imaginary part of the amplitude.
- Non-planar diagrams are the other possibility, but do not contribute to the imaginary part as they are real.

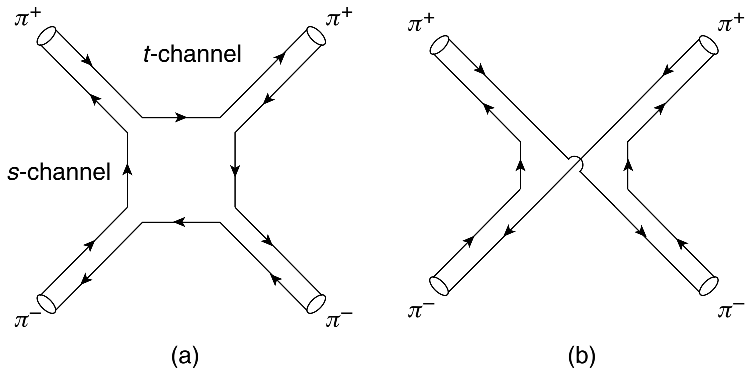

Fig. 2.9. Duality diagrams for $\pi^+\pi^-$ scattering: (a) planar (s, t) graph; (b) non-planar $(s, \bar{s})$ or $(t, \bar{s})$ graph.

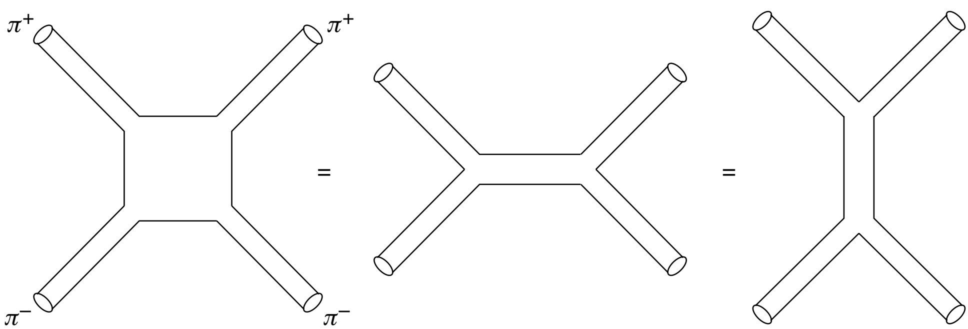

Fig. 2.10. Dual-resonance diagram for π-π scattering.

In order to illustrate these rules, we can consider the scattering process:

$$\begin{aligned} &\pi^+\pi^- \to \pi^+\pi^- \qquad (s - \text{channel}), \\ &\pi^+\pi^- \to \pi^+\pi^- \qquad (t - \text{channel}), \\ &\pi^+\pi^+ \to \pi^+\pi^+ \qquad (\bar{s} - \text{channel}), \end{aligned} \tag{2.94}$$

shown in Fig. 2.9.

From the previous discussion, only the planar diagram contributes to the imaginary part of the amplitude. Duality invokes that a sum of resonances (or Regge poles) exchanged in the s channel is equivalent to the sum of Regge poles (or resonances) exchanged in the t channel, which is shown in Fig. 2.10. Similar planar diagrams can be drawn for π-N scattering as shown in Fig. 2.11. In the case of N-N (or in general baryon–antibaryon) scattering, one has the dual-resonance diagram (Fig. 2.12).

It shows that the planar graph represents exchange of non-exotic objects in the s channel, but exchange of exotics in the t channel. This feature signals that without exotics, the approach cannot consistently explain the hadronic world.

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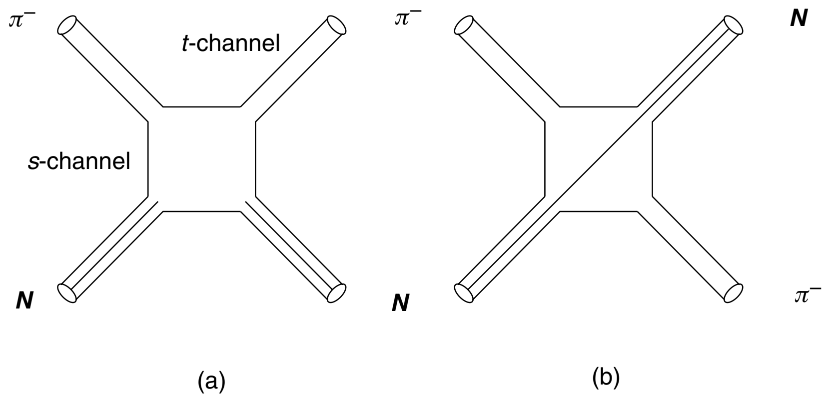

Fig. 2.11. Planar diagrams for π-N scattering: (a) ((s, t) or ($\bar{s}$, t); (b) (s, $\bar{s}$).

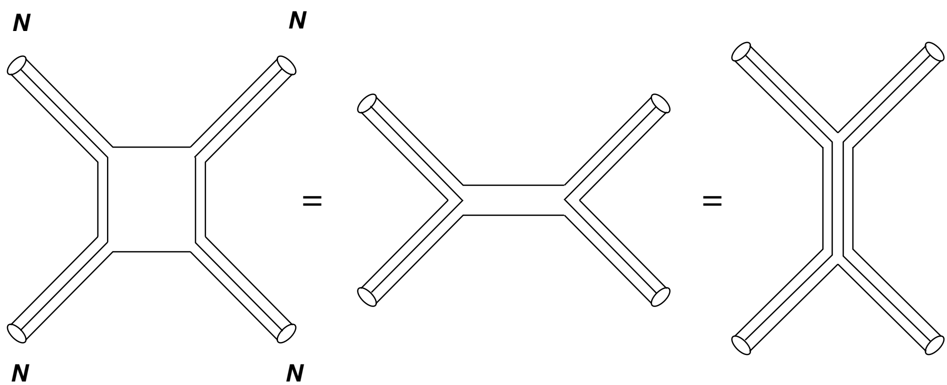

Fig. 2.12. Dual-resonance diagram for N-N scattering.

One has expected that the previous approach based on superconvergence and duality, and implemented by the dual-resonance model suggested by Veneziano [40] will bring new insights in the developments of the theory of strong interactions. Alas, after the discovery of QCD, such theories became unsuccessful, although we know, at present, that the Veneziano model (actually it can be viewed as a string model) revives as the basics of superstring theories with which one wishes to unify the three electromagnetic, weak and strong forces with gravitation.

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