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FINITELY SORTING LIE ALGEBRAS

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Dedicated to G. E. (Tim) Wall, in recognition of his distinguished contribution to mathematics in Australia, on the occasion of his retirement

Abstract

Lie algebras whose finite-dimensional modules decompose into direct sums of modules involving only one type of irreducible are investigated. Some vanishing theorems for the cohomology of some infinite-dimensional Lie algebras are thereby obtained.

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1. Introduction

In [5] Mitra, Sitaram and Tripathy proved the following theorem.

THEOREM 1.1. *Let L be a Lie algebra and let V be an L-module. Suppose some element z in the centre L is represented by the identity transformation of V. Then $H^p(L, V) = 0$ for all p.*

In a recent paper [2, Corollary 6.3], Farnsteiner proved

THEOREM 1.2. *Let L be a Lie algebra and let N be a nilpotent ideal. Let V be a finite-dimensional L-module and let*

$$V_0(N) = \bigcap_{n\in N} V_0(n)$$

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where $V_0(n)$ is the Fitting null component for the action of n on V. Then $H^p(L, V) = H^p(L, V_0(N))$ for all p.

We show that these results can be obtained more easily and generalised using the method of [1].

2. Positive results

Throughout, L is a Lie algebra over the field F and V is an L-module. The element $x \in L$ is represented by the linear transformation $\rho(x)\colon V \to V$. We write $N \lhd L$ if N is an ideal of L. A subalgebra N of L is called subnormal, written $N \lhd\lhd L$, if there exists a finite chain of subalgebras N_i such that

$$N = N_0 \lhd N_1 \lhd \cdots \lhd N_r = L.$$

We shall make repeated use of the following well-known result.

LEMMA 2.1. *Suppose $N \lhd\lhd L$ and that V is an L-module. Suppose $H^p(N, V) = 0$ for all $p \leq k$. Then $H^p(L, V) = 0$ for all $p \leq k$.*

PROOF. We need only consider the case where $N \lhd L$, the result then following by induction over the length of the chain linking N to L. In the Hochschild-Serre spectral sequence, we have

$$E_2^{pq} = H^p(L/N, H^q(N, V)) = 0$$

for $q \leq k$. Hence $E_\infty^{pq} = 0$ for $q \leq k$. But

$$H^n(L, V) \simeq \bigoplus_{p+q=n} E_\infty^{pq} = 0$$

for $n \leq k$.

The following immediate consequence of Lemma 2.1 includes Theorem 1.1 as a special case.

THEOREM 2.2. *Suppose the element $x \in L$ acts invertibly on V and that the subspace $N = \langle x \rangle$ spanned by x is subnormal in L. Then $H^p(L, V) = 0$ for all p.*

PROOF. We have

$$H^0(N, V) = V^n = \ker\{\rho(x)\colon V \to V\} = 0, \quad H^1(N, V) = V/\operatorname{im}\rho(x) = 0$$

and $H^p(N, V) = 0$ for $p > 1$ for dimension reasons.

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COROLLARY 2.3. *Suppose $x \in L$ acts invertibly on V and that L is nilpotent. Then $H^p(L, V) = 0$ for all p.*

PROOF. Since L is nilpotent, $\langle x \rangle \lhd\lhd L$.

COROLLARY 2.4. *Suppose L is locally nilpotent and let V be a finite-dimensional L-module with $V^L = 0$. Then $H^p(L, V) = 0$ for all p.*

PROOF. Let K be the kernel of the representation ρ of L on V. Since L/K is a finite-dimensional nilpotent algebra, every composition factor p of V has $P^L = 0$. Thus we need only consider the case where V is irreducible. Take $x \in L$, $x \notin K$ such that $x + K$ is in the centre of L/K. Then $\rho(x)\colon V \to V$ is invertible. Now $C^q(K, V) = \mathrm{Hom}_F(\Lambda^q(K), V)$ where $\Lambda^q(K)$ is the component of degree q of the exterior algebra on K. Since $\mathrm{ad}(x)$ is locally nilpotent, so is the induced linear transformation of $\Lambda^q(K)$. By Farnsteiner [2, Lemma 4.3], x acts invertibly on $C^q(K, V)$. It follows that x acts invertibly on $H^q(K, V)$. By Corollary 2.3, $H^p(L/K, H^q(K, V)) = 0$ for all p, q and the result follows by the Hochschild-Serre spectral sequence.

Let $N \lhd\lhd L$ and let A be an irreducible N-module. As defined in [1], an A-component of the finite-dimensional L-module V is an L-submodule $A(V)$ such that every N-composition factor of $A(V)$ is isomorphic to A while $V/A(V)$ has no N-composition factor isomorphic to A. If an A-component exists for every A, then V is their direct sum and is called N-sortable. We denote by F_N the ground field, regarded as N-module with trivial N-action. Clearly, if $F_N(V)$ exists, then $F_N(V) = V_0(N)$. In [1], only finite-dimensional Lie algebras were considered. We modify terminology slightly to allow for infinite-dimensional Lie algebras.

DEFINITION 2.5. We say that (L, N) is a *finitely sorting pair*, abbreviated to FS pair, if $N \lhd\lhd L$ and every finite-dimensional L-module is N-sortable. We say that L is *absolutely finitely sorting*, abbreviated to AFS, if every finite-dimensional L-module is L-sortable, that is, if (L, L) is an FS pair.

In [1], a number of conditions were shown to be equivalent to the assumption that (L, N) is an FS pair. By separating out the points at which use was made of the assumption that L is finite-dimensional, we can rephrase that result as follows.

THEOREM 2.6. *Let L be a Lie algebra and let $N \lhd\lhd L$. Then the following conditions are equivalent.*

(a) *(L, N) is an FS pair.*

(b) *$F_N(V)$ exists for every finite-dimensional irreducible L-module V.*

(c) *If* V *is a finite-dimensional irreducible* L*-module which does not contain* F_N *as* N*-composition factor, then* $H^1(L, V) = 0$.

(d) *If* V *is a finite-dimensional* L*-module and* $V^N = 0$, *then* $H^1(L, V) = 0$.

If L *is finite-dimensional, then also equivalent to these are the following conditions.*

(e) *In the case where* char $F \neq 0$, N *is nilpotent. In the case where* char$F = 0$, $N = S \oplus R$ *where* S *is semi-simple and* R *is nilpotent.*

(f) *If* V *is a finite-dimensional* L*-module and* $V^N = 0$, *then* $H^p(L, V) = 0$ *for all* p.

Every finite-dimensional L-module is an L/K-module for some ideal K with L/K finite-dimensional. Clearly, (L, N) is an FS pair if and only if for every ideal K of L with L/K finite-dimensional, $(L/K, (N+K)/K)$ is an FS pair. Thus (L, N) is an FS pair if and only if for every ideal K of L with L/K finite-dimensional, $(N+K)/K$ has the structure 2.6(e).

COROLLARY 2.7. *Suppose* $N \lhd\lhd L$. *If either* L *or* N *is AFS, then* (L, N) *is an FS pair.*

PROOF. Let $K \lhd L$ with L/K finite-dimensional. We have to show that $(N+K)/K$ has the structure 2.6(e). If L is AFS, then L/K has that structure and $(N+K)/K$, being subnormal in L/K, also has that structure. If N is AFS, then $(N+K)/K$, being a finite-dimensional homomorphic image of N, has that structure. In either case, it follows that (L, N) is an FS pair.

We can now generalise Theorem 1.2.

THEOREM 2.8. *Let* (L, N) *be an FS pair. Suppose that, for every finite-dimensional irreducible* L*-module* P *with* $P^N = 0$, *we have* $H^p(L, P) = 0$ *for all* p. *Then* $H^p(L, V) = H^p(L, F_N(V))$ *for all* p *and every finite-dimensional* L*-module* V. *In particular, if* $N \lhd\lhd L$ *and* N *is locally nilpotent, then* $H^p(L, V) = H^p(L, F_N(V))$ *for all* p *and every finite-dimensional* L*-module* V.

PROOF. Since $V = \bigoplus_A A(V)$, we have $H^p(L, V) = \bigoplus_A H^p(L, A(V))$. If $A \neq F_N$, then $H^p(L, P) = 0$ for every L-composition factor P of $A(V)$, whence it follows that $H^p(L, A(V)) = 0$. Thus $H^p(L, V) = H^p(L, F_N(V))$.

Suppose N is locally nilpotent and that P is an irreducible L-module with $P^N = 0$. Since P is sortable as an N-module, it follows that every N-composition factor Q of P satisfies $Q^N = 0$. By Corollary 2.4,

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$H^p(N, Q) = 0$ for all p. It follows that $H^p(N, P) = 0$ for all p. By Lemma 2.1, $H^p(L, P) = 0$ for all p and the result follows.

Theorem 2.9. *Suppose the Lie algebras N_i for $i \in I$ are AFS. Then $N = \bigoplus_{i \in I} N_i$ is AFS.*

Proof. Let K be an ideal of N and let $f: N \to N/K$ be the natural homomorphism. Suppose N/K is finite-dimensional. Then $f(N_i)$ has the structure of 2.6(e) and $f(N)$ is the sum (not necessarily direct) of the ideals $f(N_i)$. But a finite-dimensional sum of nilpotent ideals is nilpotent. A finite-dimensional sum of semi-simple ideals is semi-simple. Thus $f(N)$ also has the structure 2.6(e).

By Theorem 2.6, locally nilpotent algebras and, if $\mathrm{char} F = 0$, semi-simple algebras are AFS. So too, trivially, are infinite-dimensional simple algebras. If N is AFS and V is an N-module having no non-zero finite-dimensional quotients, then any extension of V by N is AFS. Some more interesting examples are provided by a well-known construction (see Jacobson [3, Chapter VII, §1], Serre [5, VI · 19–VI · 26]).

Example 2.10. Let F be a field of characteristic 0 and let $A = (A_{ij})$ be an $l \times l$ Cartan matrix. Let L be the Lie algebra over F generated by elements H_i, X_i, Y_i for $i = 1, \dots, l$, with defining relations

$$H_iH_j = 0, \quad X_iY_j = \delta_{ij}H_j, \quad H_iX_j = A_{ij}X_j, \quad H_iY_j = -A_{ij}Y_j,$$

where δ_{ij} is the Kronecker delta. Then $L = H \oplus X \oplus Y$ as vector space, where H is an abelian subalgebra spanned by the H_i and X, Y are free Lie algebras freely generated by the X_i and the Y_i respectively. Put

$$P_{ij} = \mathrm{ad}(X_i)^{1-A_{ij}} X_j, \quad Q_{ij} = \mathrm{ad}(Y_i)^{1-A_{ij}} Y_j$$

for $i \neq j$. Let P be the ideal of X generated by the P_{ij} and let Q be the ideal of Y generated by the Q_{ij}. Then P, Q are ideals of L and $L/(P+Q)$ is the split semi-simple Lie algebra with Cartan matrix A. For an ideal K of L, L/K is finite-dimensional if and only if $K \supseteq P + Q$. Since every finite-dimensional quotient L/K is semi-simple, L is AFS.

3. Negative results

From the finite-dimensional case, one might conjecture that if (L, N) is an FS pair, then N is AFS. This is false.

Example 3.1. Let L be the Lie algebra over a field of characteristic 0 constructed in Example 2.10 from a Cartan matrix A of rank $l \geq 2$. Then

(L, P) is trivially an FS pair since P is contained in the kernel of every finite-dimensional representation of L. But P, being a subalgebra of the free Lie algebra X, is a free Lie algebra by the Širšov-Witt Theorem [4, page 331]; P is infinite-dimensional, so it is free on more than one generator and therefore has finite-dimensional quotients not of the type 2.6(e).

EXAMPLE 3.2. Let X be the Lie algebra over the field F of characteristic $p \neq 0$ defined by $\langle x, y, z | xy = z, xz = yz = 0 \rangle$, and let $A = \langle a_0, a_1, \dots \rangle$. We make A into an X-module by defining

$$xa_i = a_{i-1}, \quad ya_i = (i+1)a_{i+1}, \quad za_i = a_i.$$

Then A has submodules $A_n = \langle a_0, a_1, \dots, a_{np-1} \rangle$, and it is easily seen that these are the only proper submodules of A. Let E be the split extension of A by X. If K is an ideal of E with E/K finite-dimensional, then $K \supseteq A$ and E/K is nilpotent. Thus E is AFS. Let $N = \langle z, A \rangle$. Then $N \lhd E$, (E, N) is an FS pair, but N has the non-abelian 2-dimensional algebra as homomorphic image and so is not AFS.

From Theorem 2.6(f) and Corollary 2.4, one might conjecture that if L is AFS and V is a finite-dimensional L-module with $V^L = 0$, then $H^p(L, V) = 0$ for all p. This is false.

EXAMPLE 3.3. We again use the construction and notations of Example 2.10, this time with Cartan matrix $\left(\begin{smallmatrix} 2 & 0 \\ 0 & 2 \end{smallmatrix}\right)$. Let $\alpha_i \colon H \to F$ be the root given by $\alpha_i(H_j) = 2\delta_{ij}$ for $i, j = 1, 2$, and put $\alpha_{mn} = m\alpha_1 + n\alpha_2$ for $m, n \in \mathbb{Z}$. Let V be the irreducible L-module with highest weight α_{11}.

We shall show that $H^2(L, V) \neq 0$ by constructing a 2-cocycle $z \colon L \times L \to V$ such that

$$(*) \qquad z(H, L) = z(P^2, L) = z(Q^2, L) = z(X, X) = z(Y, Y) = 0$$

which we shall show is not a coboundary. For this, it is convenient to use a different set of generators for L. We put $h_i = \frac{1}{2}H_i$ so $\alpha_i(h_j) = \delta_{ij}$, and put $x_i = \frac{1}{2}X_i$, $y_i = Y_i$. We put

$$x_{rs} = \mathrm{ad}(x_1)^{r-1}\,\mathrm{ad}(x_2)^{s-1}(x_1x_2) \quad \text{and} \quad y_{rs} = \mathrm{ad}(y_1)^{r-1}\,\mathrm{ad}(y_2)^{s-1}(y_1y_2)$$

for $r, s \geq 1$, and observe that the $h_i, x_i, y_i, x_{rs}, y_{rs}$ form a basis of L modulo $P^2 + Q^2$. We have

$$(**) \qquad \begin{aligned} x_1y_{rs} &\equiv -\binom{r}{2}y_{r-1,s} \bmod Q^2, & y_1x_{rs} &\equiv \binom{r}{2}x_{r-1,s} \bmod P^2, \\ x_2y_{rs} &\equiv -\binom{s}{2}y_{r,s-1} \bmod Q^2, & y_2x_{rs} &\equiv \binom{s}{2}x_{r,s-1} \bmod P^2. \end{aligned}$$

Now V is 9-dimensional and has a basis consisting of eigenvectors for the weights α_{mn} for $m, n = -1, 0, 1$. We shall denote the chosen eigenvector for the weight α_{mn} by $v_{\alpha_{mn}}$ or v_{mn} interpreting this to be 0 if either m or n is outside $\{-1, 0, 1\}$. As $\rho(y_1)$ and $\rho(y_2)$ commute, we may choose v_{11} arbitrarily and take $v_{1-r,1-s} = \rho(y_1)^r\rho(y_2)^s v_{11}$. We then have

$$\begin{aligned} h_1 v_{mn} &= m v_{mn}, \quad & x_1 v_{mn} &= v_{m+1,n}, \quad & y_1 v_{mn} &= v_{m-1,n}, \\ h_2 v_{mn} &= n v_{mn}, \quad & x_2 v_{mn} &= v_{m,n+1}, \quad & y_2 v_{mn} &= v_{m,n-1}, \end{aligned}$$

for $m, n = -1, 0, 1$.

Lemma 3.4. *The recurrence relations*

(a) $\theta(p+1, q, r, s) = \theta(p, q, r, s) + \binom{r}{2}\theta(p, q, r-1, s)$ *if* $p \le r$,

(b) $\theta(p, q+1, r, s) = \theta(p, q, r, s) + \binom{s}{2}\theta(p, q, r, s-1)$ *if* $q \le s$,

(c) $\theta(p, q, r+1, s) = \theta(p, q, r, s) + \binom{p}{2}\theta(p-1, q, r, s)$ *if* $r \le p$,

(d) $\theta(p, q, r, s+1) = \theta(p, q, r, s) + \binom{q}{2}\theta(p, q-1, r, s)$ *if* $s \le q$,

together with $\theta(1, 1, 1, 1) = 2$ *and* $\theta(p, q, r, s) = 0$ *if* $|p-r| > 1$ *or* $|q-s| > 1$ *or if any of* p, q, r, s *is* 0, *define a function* $\theta\colon \mathbb{N}^4 \to \mathbb{Z}$.

Proof. It is clear that reductions using (a) or (c) commute with reductions using (b) or (d). We fix q, s denote $\theta(p, q, r, s)$ by $\theta(p, r)$ and prove that $\theta(p, r)$ is well-defined (assuming $\theta(1, 1) = \theta(1, q, 1, s)$ is defined). We suppose that $\theta(p, r)$ is well-defined for all $p, r \le k$ and satisfies

(i) $\theta(p, r) = \theta(r, p)$,

(ii) $\theta(r, r) = r\theta(r, r-1)$.

We have two ways of calculating $\theta(k+1, k)$ which we must show give the same result. The rule (c) requires $\theta(k+1, k)$ to be

$$\theta(k+1, k-1) + \binom{k+1}{2}\theta(k, k-1) = \binom{k+1}{2}\theta(k, k-1)$$

since $\theta(k+1, k-1) = 0$, while the rule (a) gives the value

$$\theta(k, k) + \binom{k}{2}\theta(k, k-1) = k\theta(k, k-1) + \binom{k}{2}\theta(k, k-1)$$

by (ii). As these are equal, $\theta(k+1, k)$ is well-defined and we have

(iii) $\theta(k+1, k) = \binom{k+1}{2}\theta(k, k-1)$.

Similarly we have $\theta(k, k+1) = \binom{k+1}{2}\theta(k-1, k)$. Since $\theta(k, k-1) = \theta(k-1, k)$, it follows that $\theta(k+1, k) = \theta(k, k+1)$.

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We have two ways of calculating $\theta(k+1, k+1)$ which, by symmetry, give the same result. Thus $\theta(k+1, k+1)$ is well-defined, and

$$\begin{aligned}\theta(k+1, k+1) &= \theta(k+1, k) + \binom{k+1}{2}\theta(k, k) && \text{(by (c))}\\ &= \theta(k+1, k) + k\binom{k+1}{2}\theta(k, k-1) && \text{(by (ii))}\\ &= \theta(k+1, k) + k\theta(k+1, k) && \text{(by (iii))}\\ &= (k+1)\theta(k+1, k),\end{aligned}$$

which completes the induction.

We put

$$\begin{aligned} z(x_i, y_j) &= \delta_{ij} v_{\alpha_i - \alpha_j}, & z(x_{pq}, y_{rs}) &= \theta(p, q, r, s) v_{p-r, q-s},\\ z(x_1, y_{rs}) &= -\delta_{1s} v_{1-r, -s}, & z(x_2, y_{rs}) &= \delta_{1r} v_{-r, 1-s},\\ z(x_{pq}, y_1) &= -\delta_{1q} v_{p-1, q}, & z(x_{pq}, y_2) &= \delta_{1p} v_{p, q-1}.\end{aligned}$$

Together with $(*)$ and the requirement that z be bilinear and alternating, this defines $z: L \times L \to V$.

LEMMA 3.5. *z is a cocycle.*

PROOF. We have to show that

$$\delta z(a, b, c) = az(b, c) - bz(a, c) + cz(a, b) - z(ab, c) + z(ac, b) - z(bc, a)$$

vanishes for all triples a, b, c chosen from the $h_i, x_i, y_i, x_{pq}, y_{rs}$. Non-zero terms appear only if at least one of a, b, c is chosen from each of X, Y. Suppose $b \in X_\beta$ and $c \in Y_\gamma$. Then $z(b, c) \in V_{\beta+\gamma}$ and

$$\begin{aligned}\delta z(h, b, c) &= hz(b, c) - z(hb - c) - z(b, hc)\\ &= (\beta + \gamma)(h) z(b, c) - z(\beta(h)b, c) - z(b, \gamma(h)c) = 0\end{aligned}$$

for all $h \in H$. By symmetry, we need only consider cases with $a, b \in X$ and $c \in Y$. One easily verifies that $\delta z(x_1, x_2, y_j) = 0$. Using $(**)$ we obtain

$$\delta z(x_1, x_2, y_{rs}) = \left(\delta_{1r} + \delta_{1s} - \theta(1, 1, r, s) + \binom{r}{2} + \binom{s}{2}\right) v_{1-r, 1-s}.$$

We need only consider the cases $r, s = 1, 2$. We then have $\binom{r}{2} = \delta_{2r}$ and as $\theta(1, 1, r, s) = \theta(1, 1, 1, 1) = 2$, it follows that $\delta z(x_1, x_2, y_{rs}) = 0$.

All terms of $\delta z(x_i, x_{pq}, y_j)$ are zero unless $i = j$. That $\delta z(x_1, x_{pq}, y_1) = 0$ can easily be verified. All terms of $\delta z(x_{pq}, x_{rs}, y_j)$ and of $\delta z(x_{pq}, x_{rs}, y_{ij})$

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are zero. There remains

$$\begin{aligned}\delta z(x_1, x_{pq}, y_{rs}) &= x_1 z(x_{pq}, y_{rs}) - z(x_{p+1,q} y_{rs}) + z(x_1 y_{rs}, x_{pq}) \\ &= \Big(\theta(p, q, r, s) - \theta(p+1, q, r, s) \\ &\qquad + \binom{r}{2}\theta(p, q, r-1, s)\Big) v_{p-r+1, q-s} = 0\end{aligned}$$

by 3.4(a).

LEMMA 3.6. *z is not a coboundary.*

PROOF. Suppose $z = \delta f$. Then

$$f(h_1) = \sum_{i,j} a_{ij} v_{ij} \quad \text{and} \quad f(h_2) = \sum_{i,j} b_{ij} v_{ij}$$

for some $a_{ij}, b_{ij} \in F$. By replacing f by $f - \delta(\sum_{i,j} i a_{ij} v_{ij})$, we may suppose $a_{ij} = 0$ for $i \neq 0$. Now

$$0 = z(h_1, h_2) = \sum_{i,j} (i b_{ij} - j a_{ij}) v_{ij}.$$

Thus $b_{ij} = 0$ for $i \neq 0$ and $a_{0j} = 0$ for $j \neq 0$. By replacing f by $f - \delta(\sum_j j b_{0j} v_{0j})$, we may further suppose $b_{ij} = 0$ if $j \neq 0$. We then have $f(h_i) = a_i v_{00}$ for some $a_1, a_2 \in F$. Now

$$0 = z(h_1, x_2) = h_1 f(x_2) - x_2 f(h_1) = h_1 f(x_2) - a_1 v_{01}.$$

But $v_{01} \notin \operatorname{im}\rho(h_1)$. Therefore $a_1 = 0$. Similarly $a_2 = 0$. For any $h \in H$ and $x \in X_\alpha$,

$$0 = z(h, x) = hf(x) - xf(h) - f(hx) = hf(x) - \alpha(h) f(x).$$

Thus $f(x) \in V_\alpha$, so $f(x_i) = \lambda_i v_{\alpha_i}$ and similarly $f(y_i) = \mu_i v_{-\alpha_i}$ for some $\lambda_i, \mu_i \in F$. But then

$$z(x_i, y_j) = (\mu_j - \lambda_i) v_{\alpha_i - \alpha_j}$$

and we require $\mu_j - \lambda_i = \delta_{ij}$ for $i, j = 1, 2$. These equations have no solution.

This completes the demonstration that Example 3.3 has the claimed properties.

EXAMPLES 3.7. Let E be the algebra over a field of characteristic $p \neq 0$ constructed in 3.2 as the split extension of the X-module A. Let $L = E/A_1$. Then L is isomorphic to E, so L is AFS. Now A_1 is an irreducible L-module and $A_1^L = 0$. But E is a non-split extension of A_1 by L. Thus $H^2(L, A_1) \neq 0$.

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