INFLUENCE OF MAGNETIC FIELDS ON THE STRUCTURE OF THE SOLAR CORONA*

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ABSTRACT

A current-free approximation to the coronal magnetic field is calculated from measured photospheric magnetic fields (Mt. Wilson) and traced by computer. The calculated field structure is then compared to a white-light photograph of the November 12, 1966 eclipse.

It is generally agreed that solar magnetic fields play a dominant role in determining the gross structure of the corona. However, the nature of the spatial correlation between the magnetic fields which penetrate the corona and such observed coronal features as streamers and rays is uncertain. The purpose of our still incomplete investigation is to determine the correspondence between the magnetic-field configuration and the density structure of the solar corona.

The basic data for our calculation were the daily Mt. Wilson maps of the line-of-sight magnetic field at the solar surface for the period October 29 – November 26, 1966. We divided the solar surface into 24 latitude zones ($\Delta \sin\lambda = 0{\cdot}0833$ in latitude λ) and 27 longitude sectors ($\Delta\phi = 13{\cdot}3°$ in longitude ϕ), thereby creating 648 surface elements of equal area. From the Mt. Wilson data we estimated the average (line-of-sight) magnetic field in each of the surface elements. When magnetic measurements for a given element were available over several days, the assigned field was taken to be the mean of the individual daily observations weighted according to the following scheme:

Distance of Sector from Central Meridian (days)	*Assigned Weight*
0, ± 1	1
± 2	0·9
± 3	0·7
$> \|3\|$	0

The net magnetic flux through the solar surface, as determined by the data, is not necessarily zero because (1) the measurements were taken over a solar rotation rather

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than instantaneously, (2) only the line-of-sight component was measured, and (3) the accuracy was limited to $\sim 0{\cdot}5$ gauss. This difficulty may be overcome by a simple calibration. Let D_{ij} represent the assigned line-of-sight magnetic field obtained from the Mt. Wilson data for region (i, j), where $i = 1, \ldots, 24$ specifies the latitude zone and $j = 1, \ldots, 27$ specifies the longitude sector. Since we require that the net magnetic flux Φ through the solar surface be zero, we let

$$\Phi = \Delta A \sum_{i=1}^{24} \sum_{j=1}^{27} (D_{ij} - \delta) \sec \lambda_i = 0, \qquad (1)$$

where ΔA is the area of the surface element, λ_i is the latitude of the midpoint of the element, and δ is the correction term. Thus,

$$\delta = \frac{\sum_{i=1}^{24} \left(\sum_{j=1}^{27} D_{ij} \right) \sec \lambda_i}{27 \times \sum_{i=1}^{24} \sec \lambda_i}, \qquad (2)$$

which we find to be $\sim 5 \times 10^{-2}$ gauss for the data used. The corrected line-of-sight magnetic field in each region (i, j) is then

$$D'_{ij} = D_{ij} - \delta. \qquad (3)$$

Under the assumption that the corona is current-free, the magnetic field above the solar surface is completely specified by the distribution of the normal component of the photospheric field and the requirement that the field vanish at infinity. Since only the line-of-sight fields are known, we assume that the total field B_{ij} is *normal* to the photosphere and thus is related to the corrected line-of-sight field D'_{ij} by

$$B_{ij} = D'_{ij} \sec \lambda_i. \qquad (4)$$

Moreover, since $\nabla \times \mathbf{B} = 0$ above the photosphere, we can represent $\mathbf{B}$ as the gradient of a scalar potential ψ, so that $\mathbf{B} = -\nabla\psi$. Also $\nabla \cdot \mathbf{B} = 0$, so that $\nabla^2 \psi = 0$. The solution of this Laplacian equation in the region $r \geqslant R_\odot$ is (Chapman and Bartels, 1940)

$$\psi(r, \theta, \phi) = R_\odot \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left[\left(\frac{R_\odot}{r} \right)^{n+1} (g_n^m \cos m\phi + h_n^m \sin m\phi) P_n^m(\theta) \right], \qquad (5)$$

where the P_n^m are the associated Legendre Polynomials, θ is the colatitude, and g_n^m and h_n^m are constants to be determined from the data.

At the solar surface, the radial component of magnetic field

$$B_r(r = R_\odot, \theta, \phi) = -\left.\frac{\partial \psi}{\partial r}\right|_{r = R_\odot} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} (n+1)(g_n^m \cos m\phi + h_n^m \sin m\phi)\, P_n^m(\theta) \qquad (6)$$

corresponds to B_{ij} of Equation (4) and is therefore known. The coefficients g_n^m and h_n^m can then be calculated from

$$\begin{Bmatrix} g_n^m \\ h_n^m \end{Bmatrix} = \frac{2n+1}{4\pi(n+1)} \int_0^{\pi} \int_0^{2\pi} B_r(R_\odot, \theta, \phi)\, P_n^m(\theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \sin\theta \, d\theta \, d\phi \,, \tag{7}$$

in which we have used the Schmidt normalization (Chapman and Bartels, 1940). In practice, the double integration of Equation (7) is replaced by a summation over the 648 elementary regions, and $B_r(R_\odot, \theta, \phi) \sin\theta$ in the integrand is replaced by D'_{ij} of Equation (3). The coefficients g_n^m, h_n^m together with Equation (5), then determine the field at any point (r, θ, ϕ) where $r \geqslant R_\odot$ by means of

$$B_r = -\frac{\partial\psi}{\partial r}; \quad B_\theta = -\frac{1}{r}\frac{\partial\psi}{\partial\theta}; \quad B_\phi = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}. \tag{8}$$

Since Equation (5) is a rapidly converging polynomial expansion, the series may be restricted to order $n=5$ with little compromise in accuracy.

Using this technique, we have traced those magnetic-field lines which originate at the centres of the elementary surface areas. The magnetic field out to a distance of $3R_\odot$ from the centre of the Sun was calculated in this manner. For comparison with the observed corona, the calculated field lines were then projected against the plane of the sky (Figure 1a) for a given longitude ϕ_0 of the centre of the solar disk.

Several comments regarding this presentation of the magnetic-field lines in the corona are in order. First, the density of field lines is in no way related to the strength of the magnetic field (as in the usual representation), since the foot points have been chosen *geometrically*. Second, the three-dimensional structure of the coronal magnetic field cannot be determined from a simple projection of the field lines, say for some single central longitude ϕ_0. To achieve tri-dimensionality, we may examine the projections for two different central longitudes in a stereo-viewer or may produce a motion picture in which the central longitude changes with time. As displayed at this meeting, the resultant computer-drawn motion picture provides the three-dimensional structure of the field and reveals the presence of hitherto unnoticed arcades of magnetic loops elongated over 60° to 100° of the solar surface. Last, the presence of a solar wind of velocity v and density ρ invalidates our assumption of zero current in the outer corona, so that the magnetic field there cannot be calculated from potential theory. In our simple model we assume that wherever $B \geqslant B_c$, where

$$\frac{B_c^2}{8\pi} = \tfrac{1}{2}\rho v^2 \,, \tag{9}$$

the magnetic lines are accurately approximated by potential fields, and we draw the lines (Figure 1a) as solid. Wherever $B < B_c$ we draw the field lines as dashed to indicate that they are probably completely distorted by the solar wind. The value of B_c was

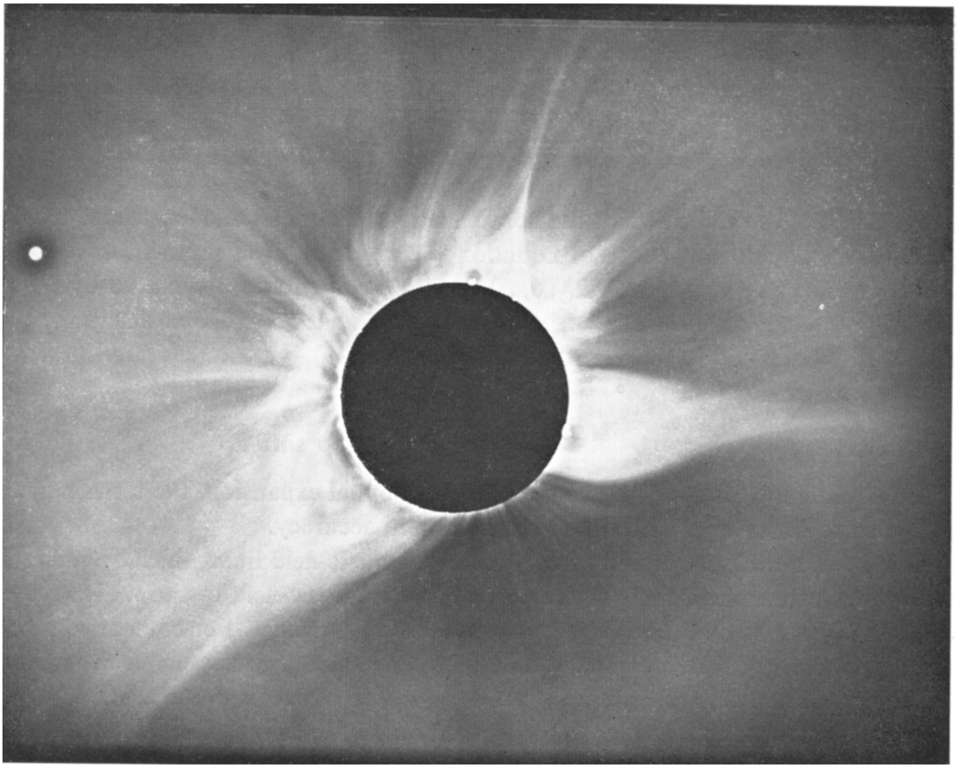

FIG. 1b. *The solar corona of November 12, 1966 photographed with a radially symmetric, neutral density filter in the focal plane of the camera to compensate for the steep decline of coronal radiance with increasing distance. The latitude of the print is further extended by use of the Fluor-o-dodge process. The overposed image of Venus appears in the NE quadrant.*

chosen empirically by comparing the calculated magnetic-loop configurations with similar loops actually visible in the corona at the same location (at the bases of the SE and SW streamers). To produce the transition from closed (solid) to distorted (dashed) structures at the observed height of about one solar radius above the limb requires

$$B_c \sim 4{\cdot}7 \times 10^{-2} \text{ gauss}.$$

With a streamer density $N_e \sim 10^7 \text{ cm}^{-3}$ at $2R_\odot$ (Malitson and Erickson, 1966) Equation (9) gives for the velocity of the solar wind

$$v \sim 28 \text{ km/sec}.$$

Such a velocity roughly agrees with the few empirical estimates available (Newkirk, 1967), but is considerably higher than that predicted by the theoretical models (e.g. Meyer and Schmidt, 1966; Whang *et al.*, 1966).

Since our determination of the three-dimensional locations of the various coronal structures is incomplete, we can, for the present, compare only the appearance of the projected magnetic-field lines and the corona. Such a comparison is, however, suggestive. Of particular note are the following correspondences:

(1) The SW and SE streamers appear to be located over arcades of magnetic loops.

(2) The bush-like structure at the NE limb appears to be associated with magnetic loops passing over the limb. Similar loops appear within the corona.

(3) The outer corona above the Northern hemisphere displays a complex of gently curved rays whose shapes match those of the magnetic-field lines.

(4) The coronal condensation in the NW quadrant appears at a location where many field lines converge, although other similar convergences show nothing remarkable in the corona.

Our preliminary analysis suggests that the potential magnetic fields below $2R_{\odot}$ yield a reasonable approximation to the true fields present in the corona and that such fields are, in fact, a most important agent for determining the structure of the lower corona.

Acknowledgments

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Note added in proof: In the method presented here, the Legendre coefficients are calculated in Equation (7) on the assumption of a completely radial surface field. Since this talk was presented, the Legendre coefficients have been recalculated by a least-mean-square fit with respect to the line-of-sight magnetic field. This new method calculates the best possible potential magnetic field for the data available. The qualitative results for $n=5$, however, are essentially the same as presented here in Figure 1a. When the Legendre Polynomial is expanded to $n=9$ or higher order, even more striking similarities arise between the calculated magnetic field and the observed coronal structure. These new results will be the basis of a forthcoming paper.

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DISCUSSION

H. U. Schmidt: Why do you choose the ratio between undisturbed magnetic and total material energy density as a parameter to describe the break-up point of fieldlines, i.e. the lower end of current sheets separating zones of opposite polarity in the solar wind and overlying such zones in the solar photosphere? I think one should rather choose the ratio between magnetic and solar wind energy density only. This parameter has quite a different dependence on the height than your parameter, and I think it can be determined from the Parker model with a sufficient accuracy.

Newkirk: I agree that the comparison with the energy density of the solar wind would have been a better criterion. However, we wished to use a condition which could also tell us whether the field could support low-elevation density structures as well. (**Note added in proof:** The final manuscript makes just this comparison.)

Davis: Your very beautifully presented calculations omit one factor which I feel to be important. It is a reasonable approximation to use potential fields, but not to apply boundary conditions on the flux at the solar surface only. Over the equatorial regions at least, and presumably over the entire Sun, the solar wind sweeps a substantial fraction of the total flux out to infinity. This will substantially modify the calculated field and, in particular, is likely to substantially reduce the number of large arches.

Newkirk: Our criterion of comparing the energy density of the material and in the magnetic field is admittedly only a crude way of indicating what portions of the field lines are valid. Our pictures of the dashed lines indicate field lines which are probably strung out by the solar wind. At present we cannot solve the complete problem and calculate the form of the lower portion of the lines as modified by the flow above.