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(Non-)fractality on Large Scales

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Abstract. The debate about the possible smoothness of the Universe on large scales as opposed to an unbounded fractal hierarchy has been the subject of increasing interest in recent years. The controversy arises as a consequence of different statistical analyses performed on surveys of galaxy redshifts. I review the observational evidence supporting the idea that a gradual transition occurs in the galaxy distribution: from a fractal regime at small scales to large scale homogeneity.

1. Introduction

Already in the eighteenth century, Emmanuel Kant and Johann Lambert conceived a hierarchical universe of stars clustered into larger systems, which today we call galaxies, and which, in turn, were clustered into larger systems and so on. This hierarchical view of the universe was proposed by John Herschel and Richard Proctor in the nineteenth century as a solution to Olber's paradox and was seriously defended by several astronomers at the beginning of this century: Carl Charlier and Fournier d'Albe (for a historical approach to the hierarchical universe, see Harrison 1981). More recently, Gerard de Vaucouleurs (1970) found observational evidences supporting this idea in the distribution of galaxies in clusters and superclusters.

With the introduction of the fractal concept by Mandelbrot (1982), hierarchical clustering has been reinterpreted in terms of a self-similar or scale-invariant fractal distribution of galaxies. It seems clearly established that, at small distances ($r < 10\,h^{-1}$ Mpc), the galaxy clustering is fractal. If fractality were to extend to larger scales, our cosmological models would be in trouble, because one of their fundamental tenets is the Cosmological Principle introduced originally by Einstein (although the term was coined by E. A. Milne in 1933).

The Cosmological Principle is the assumption that the large-scale universe is spatially homogeneous and isotropic, and according to it, the distribution of galaxies should break scale invariance at a given distance, showing a clear transition to homogeneity at large scales.

The strongest observational evidence supporting the validity of the Cosmological Principle is the isotropy of the Cosmic Microwave Background radiation. Other observations supporting this hypothesis include the angular distribution of radio sources, the analysis of the X-ray background and the distribution of quasars and Lyman-α clouds (for a review, see Wu, Lahav & Rees 1999). The distribution of γ-ray bursts also supports the homogeneity picture (see the poster by Mészáros et al. presented at this meeting). All these tests are essentially two-

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dimensional, because they analyse objects or radiation as seen on the celestial sphere. An additional observational evidence of this kind comes from the analysis of the angular correlation function for the Lick and the APM galaxy surveys. It scales with magnitude as predicted for a homogeneous universe (Peebles 1993).

Although the three-dimensional distribution of galaxies provided by the redshift catalogues show unambiguously a tendency to homogeneity at large scales, a group of physicists and astronomers leaded by L. Pietronero have been defending during the past 12 years that the Universe is fractal up to the largest scales probed by the present generation of redshift surveys. In this contribution I shall discuss some observational evidences ruling out this possibility.

2. A fractal universe

Several fractal constructions have been proposed so far as artificial universes trying to mimic some properties of the real galaxy distribution. Mandelbrot (1975) proposed a model based on a Rayleigh-Lévy flight, where galaxies are placed at the steps of a random walk. The direction of each jump is isotropically chosen at random and its length follows a power-law probability distribution function.

Soneira & Peebles (1978) introduced a model based on the superposition of fractal clumps which was able to reproduce not only the appearance of the projected distribution of galaxies as seen in the Lick maps (Seldner et al. 1977), but also the angular two- and three- point correlation function. Each of this fractal clumps is built as follows: in a sphere of radius R, we randomly place η spheres of radius r/λ, with $\lambda > 1$. Within each of these spheres, we again place η new spheres of radius R/λ^2. The process is repeated L times and the last generation of η^L centres are considered the galaxies of a clustering hierarchy or a bounded fractal with dimension $D = \log \eta / \log \lambda$.

How does this fractal clump look like when projected onto the sky? Peebles (1998) showed a series of projections of the particles lying in concentric shells as seen from an observer situated in a given point close to the centre of the first sphere. A similar set of equal-area Aitoff projections are shown in Figure 1. This is a fractal clump with $D = 2$, ($\eta = 2$, $\lambda \simeq 1.41$ and $L = 18$). We have scaled the fraction of particles plotted as a function of $1/r^2$, being r the distance of a particle to the centre. The width of a given shell is always twice the width of the previous one. It is remarkable the anisotropy of the point distribution provided by this model. The inhomogeneity remains even in the larger shells far away from the centre. Davis (1997) performed the same kind of projection for a real flux-limited redshift survey, the IRAS 1.2Jy catalogue. On the right panels of Figure 1, we show the IRAS galaxies lying in concentric shells of increasing size projected onto the sky. The radial distribution of shells is the same as the one shown for the Soneira & Peebles model (left panels in Figure 1), but it is slightly different of the one shown in Davis (1997). We can appreciate that for the more distant and larger shells the distribution looks more homogeneous. This tendency to homogeneity detected visually in the redshift catalogues will be quantified in next sections.

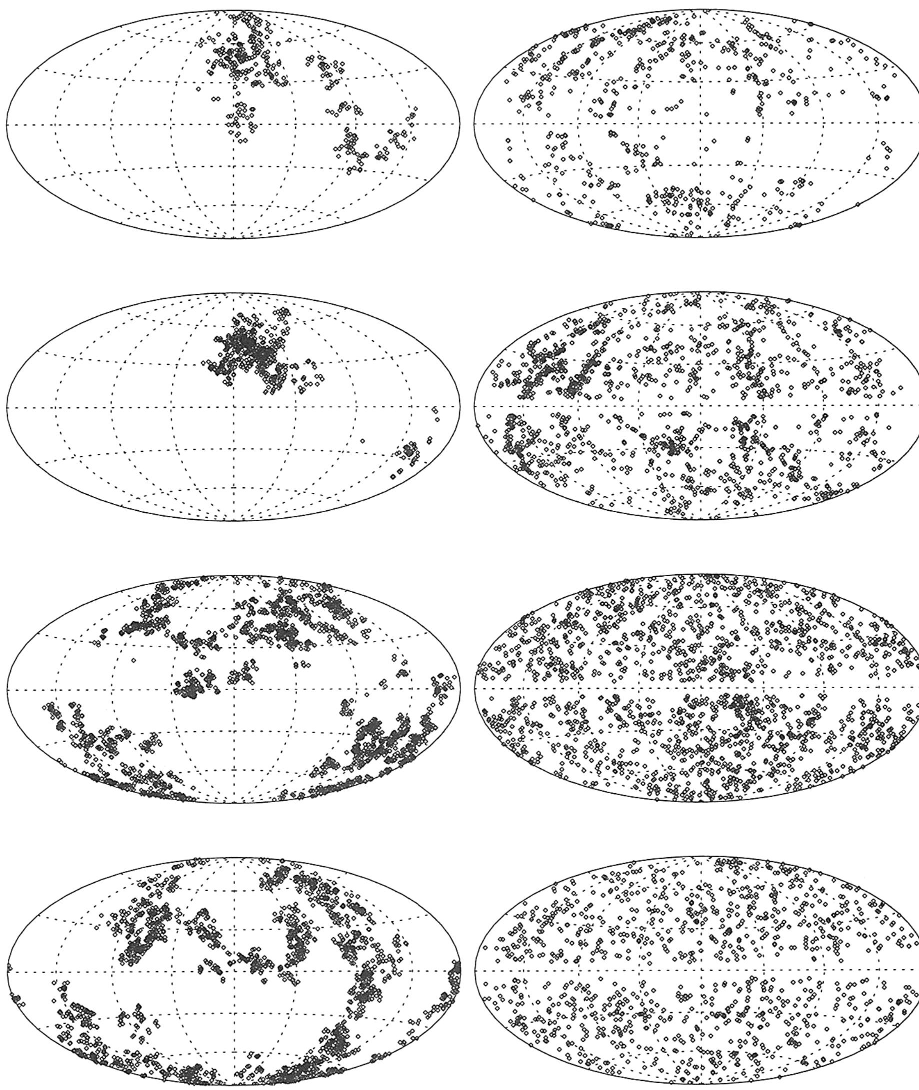

Figure 1. Left panels: equal-area Aitoff projection of a single Soneira & Peebles fractal clump as seen from an observer situated in a point of the model close to the centre of the first sphere. In each panel, from top to bottom, we have projected the points lying in concentric shells of increasing radius and width: $1.25 < r \leq 2.5$, $2.5 < r \leq 5$, $5 < r \leq 10$, $10 < r \leq 20$, in arbitrary units. Right panel: The same projections but for the IRAS 1.2Jy redshift survey. The division in shells is the same as before, but now the units are thousands of km/s in the radial velocity of each galaxy.

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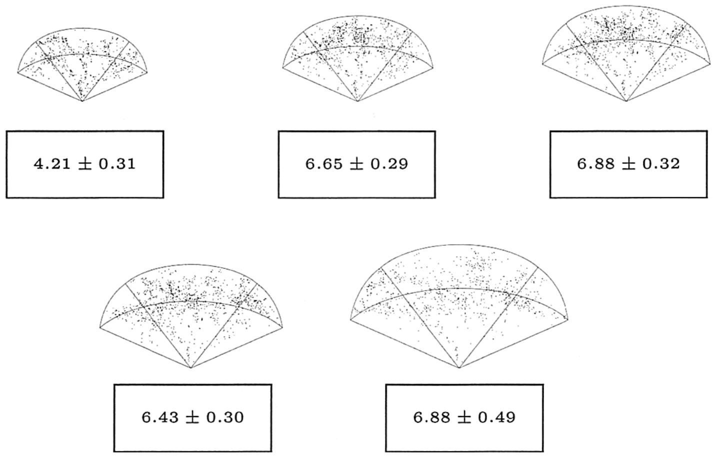

Figure 2. Volume-limited samples extracted from the CfA2 north. From left to right and top to bottom the depth of the samples increases. The size of the diagrams scales with the real volume of the sample. The number of points in each one is, in the same order, 736, 1113, 1159, 1134 and 905. The correlation length r_0 of each sample is shown together with the standard deviation in the weighted log-log linear regression.

3. The correlation length

The probability that a galaxy is observed within an infinitesimal volume element dV at a distance r from a given galaxy is $dP = n(1 + \xi(r))dV$, where n is the mean number density of galaxies and $\xi(r)$ is the so-called two-point correlation function. Therefore $\xi(r)$ measures the clustering in excess $[\xi(r) > 0]$, or in defect $[\xi(r) < 0]$, in comparison with a homogeneous Poisson distribution $[\xi(r) = 0]$.

The correlation length, r_0, is defined as the scale at which the correlation function reaches the value of 1, i.e., $\xi(r_0) = 1$. It is clear from the definition of the correlation function that at a distance r_0 of a given galaxy the density is, on average, twice the mean number density. The correlation length can be interpreted as the scale at which the density fluctuations change from the strongly non-linear regime at short distances to the nearly linear regime at large scales.

One of the strong predictions of a fractal universe is that the correlation length must increase linearly with the radius of the sample (Pietronero 1987; Guzzo 1997). This prediction has been tested over simple fractals as the one introduced in the previous section (Martínez, López–Martí & Pons–Bordería 2001; Paredes, Jones & Martínez 1995). We are going to see whether this prediction is supported by the observed galaxy distribution (see Martínez et al. 2001 for the details). The analysis have been performed on the CfA2 redshift catalogue (Geller & Huchra 1989). From the CfA2 north survey, we have extracted five volume-limited samples enclosed within the angular limits: $8^h \leq \alpha \leq 16^h$ and $8.5° \leq \delta \leq 44.5°$, and with increasing depth: 60, 70, 78, 85 and 101 h^{-1} Mpc, corresponding respectively to absolute magnitude limits of -18.49, -18.85, -19.10, -19.29 and -19.70 (omitting the term $+5\log h$).

In Figure 2, we show the three-dimensional diagrams of these samples. We have measured the correlation function of all these samples and estimated the value of the correlation length of each one by fitting a power-law within the range $[3-10]\,h^{-1}$ Mpc, weighting with Poisson errors. The values of the correlation length are given in the same Figure, (see also Table 1 in Martínez et al. 2001). These numbers show that the expected behaviour for a fractal pattern is not observed; on the contrary, all samples present rather similar values of r_0, except the closest and smallest one. The nearly constant value of $r_0 \simeq 6.7\,h^{-1}$ Mpc in redshift space argues strongly against the unbounded fractal interpretation of galaxy clustering. A similar result was found by Cappi et al. (1998) analysing the Southern Sky Redshift Survey 2. This result is also supported by the correlation length obtained for the deepest available redshift surveys analysed up to now: the Stromlo-APM, the Las Campanas redshift survey and the ESP redshift survey. All of them provide a value $r_0 \simeq 6\,h^{-1}$ Mpc in redshift space (Loveday et al. 1995; Tucker et al. 1997; Guzzo et al. 2000). Moreover, recent studies of other deep samples have shown that the correlation length in real space is also very stable with redshift (Gladders & Yee 2000; see also the contributions by R. Carlberg and C. Frenk to this meeting).

4. The correlation dimension at large scales

The best available catalogues for the statistical analysis of the possible fractal nature of the galaxy distribution at large scales are: the Stromlo-APM, the Las Campanas, the ESP and the IRAS PSCz redshift surveys. In Figure 3 (left panel) we show the first slice of the CfA2 catalogue, together with the region encompassing three slices of the Las Campanas survey, which is four times deeper. It is remarkable that the structures (clusters, filaments and voids) are of similar size in both samples. Moreover, the largest structures observed are clearly much smaller than the size of the Las Campanas survey. In a fractal pattern we should probably find larger voids and structures in the deepest sample. This is illustrated in Figure 3 (right panel) where we show a slice of the Soneira & Pebbles fractal introduced in section 2.

One of the most useful statistical measures to study the fractality at large scales is the integral of the correlation function:

$$K(r) = \int_0^r 4\pi s^2(1+\xi(s))ds. \tag{1}$$

For a homogeneous Poisson process this function is just $K_{\rm Pois}(r) = 4\pi r^3/3$. The number of neighbours on average that a given galaxy has within a distance r is then $nK(r) = N(<r)$. A point pattern is said to be fractal with correlation dimension D_2 when

$$N(<r) \propto r^{D_2} \tag{2}$$

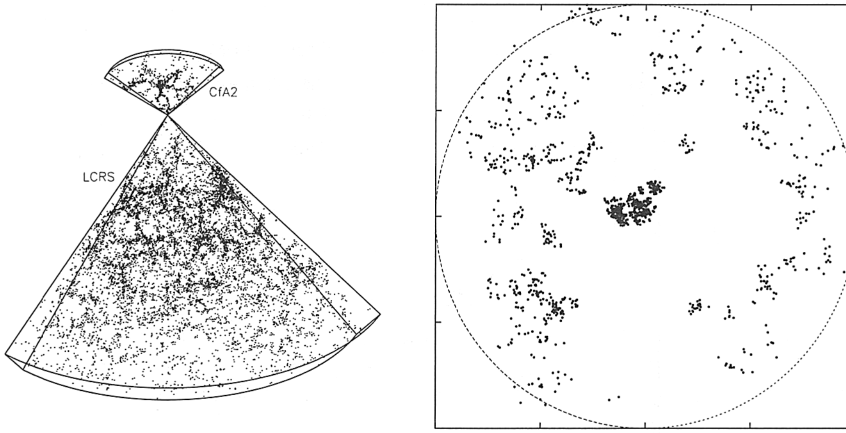

Figure 3. Left panel: The first slice of the CfA2 catalogue together with the region of the Las Campanas redshift survey (LCRS), four times deeper, in the south Galactic cap. The decrease of the number of galaxies at large redshifts is a consequence the catalogues being flux limited. Right panel: A slice of the Soneira & Peebles fractal model with $D = 2$. We can see that the size of the structures and voids increases with the radius of the sample for the fractal pattern, but this property is not sheared by the galaxy distribution, which on the contrary, comes close to homogeneity on large scales.

Other related integral quantities used in the literature are $\Gamma^*(r)$ (Coleman & Pietronero 1992) and $\hat{g}(r)$ (Amendola & Palladino 1999):

$$\Gamma^*(r) = \frac{nK(r)}{K_{\mathrm{Pois}}(r)} = \frac{3N(<r)}{4\pi r^3} = n\hat{g}(r) \tag{3}$$

The correlation dimension is evaluated as:

$$D_2 = \frac{d(\log N(<r))}{d(\log r)} = 3 + \frac{d(\log \hat{g}(r))}{d(\log r)} \tag{4}$$

The two-point correlation function for a fractal pattern should vary as $1+\xi(r) \propto r^{D_2-3}$. In Figure 4, we show the function $1+\xi(r)$ for several deep galaxy redshift surveys (Martínez 1999). The fractal behaviour at small scales disappears at larger distances, providing evidence for a gradual transition to homogeneity. It is remarkable that the break of the scale-invariant power-law appears at the same scale, approximately $10 - 15\,h^{-1}$ Mpc, for the four samples.

Martínez et al. (1998) have studied the correlation dimension by means of Equation 4 for the Stromlo-APM survey. They have found a scale dependent behaviour of D_2. This quantity reaches a value of 2.8 at the largest scales probed by the sample. Similar results have been obtained by Hatton (1999) analysing the same sample by means of $\Gamma^*(r)$. A scale dependent correlation dimension was also found by Amendola & Palladino (1999) applying the function $\hat{g}(r)$ on the Las Campanas survey. These authors have developed an interesting method to reliably measure $\hat{g}(r)$ at large distances based on radial cells, reporting values of D_2 around the homogeneity value 3 for the largest analysed scales. In a

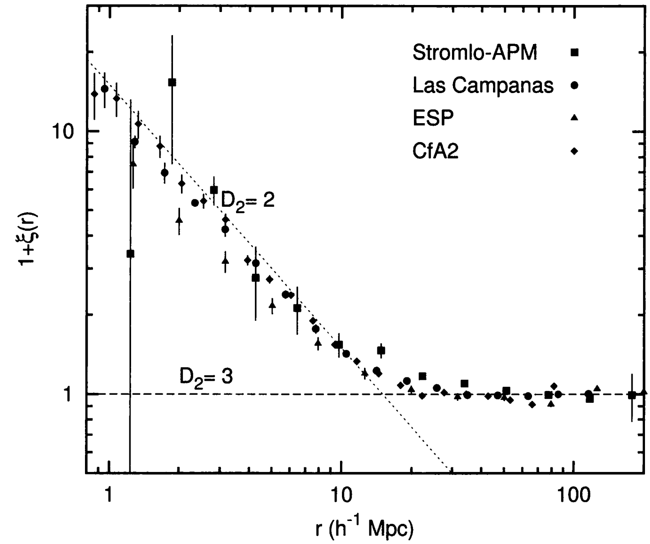

Figure 4. The function $1+\xi(r)$ for several redshift surveys (see the legend). Two reference lines have been plotted, one corresponding to a fractal with dimension $D_2 = 2$ and the second one corresponding to an homogeneous distribution ($D_2 = 3$). We can appreciate how all data show a gradual transition from the fractal regime at short scales to a more homogeneous distribution at large scales.

recent paper, Pan & Coles (2000) have studied the PSCz catalogue by means of the multifractal algorithms. They have obtained a value of $D_2 = 2.99$ at scales larger than $30\,h^{-1}$ Mpc, while for r below $\sim 10\,h^{-1}$ Mpc, they have got $D_2 = 2.16$, a value comparable with the one obtained by Martínez & Coles (1994) for the QDOT-IRAS galaxy redshift survey. The transition to homogeneity is also apparent in the plots of $\Gamma^*(r)$ obtained by the proponents of the unbounded fractal universe. In Sylos-Labini, Montuori & Pietronero (1998), the plots of $\Gamma^*(r)$ corresponding to samples such as the Stromlo-APM or the IRAS 2Jy show an unambiguous deviation of the power-law at large scales. In spite of the fact that the authors interpreted these results in a different way, they could likely be just the fingerprint of the transition to homogeneity.

5. Estimators of the statistical quantities

Different estimators have been used in order to apply the statistical measures described above to the real galaxy distribution. Although it might be not very evident, the nub of the controversy lies in this matter. The problem arises because these estimates are affected by edge effects, which result from the inability to count neighbours outside the sample boundaries. Pietronero and co-workers avoid the edge correction by using the minus estimators, that can be applied only up to the radius of the largest sphere enclosed within the sample volume. These estimators do not include as centres for counting neighbours at a given

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scale r those galaxies lying at a distance less than r of the sample boundary. The estimators used by most of the cosmologists assume implicitly that the point process is homogeneous (invariant under translations) and isotropic (invariant under rotations) and an edge correction is applied according to this assumption. For example, Ripley's estimator for the $K(r)$ function reads (Baddeley et al. 1993):

$$\hat{K}(r) = \frac{V}{N^2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{\theta(r - |\mathbf{x}_i - \mathbf{x}_j|)}{\omega_{ij}} \tag{5}$$

where V is the volume of the sample and N the total number of points. The count of the number of neighbours that galaxy i have within a sphere of radius r is weighted by an edge correction in the denominator ω_{ij}. This correction is equal to the conditional probability that the jth point is observed given that it is at a distance r from the ith point. For an homogeneous and isotropic process it is rather straightforward to calculate this probability by geometrical methods. Similar corrections are applied to estimators of the correlation function $\xi(r)$, although typically, instead of geometrical corrections, the edge effects are taken into account by normalizing the number of galaxy pairs at a given distance with the corresponding quantities measured on Poisson samples. These samples are generated within the same volumes as the real ones and mimic their selection properties.

It has been shown, however, that on small scales all estimators provide comparable results (Kerscher 1999, Pons–Bordería et al. 1999), but at large scales the minus estimator provides very poor performance (Kerscher, Szapudi & Szalay 2000). Moreover, several authors (Martínez et al. 1990; Lemson & Sanders 1991; Provenzale, Guzzo & Murante 1994) have simulated fractal point distributions generated within finite boundaries mocking the shape of the real galaxy catalogues. When applying the standard estimators for the correlation function to these fractal sets, these authors recover the expected theoretical power-law decaying behavior without finding the plateau observed in Figure 4. This result indicates that the artificial homogenization supposedly introduced by the estimators is not playing any important role. Although this is a good test, strictly speaking, only implies that if the galaxy distribution resembles such kind of fractals, Figure 4 should not present the observed flattening. Obviously, this does not solve the issue unambiguously. Therefore, if we hope to reach consensus about this matter, it seems necessary an agreement on the kind of analysis to be performed on the forthcoming large redshift surveys.

6. Conclusions

Let us summarize the main results presented in this paper:

1. While the distribution of galaxies shows fractal patterns at small scales, these disappear at larger scales. The deepest available galaxy samples show "the end of greatness" (Kirshner 1996), because the structures observed in these samples are of the same size as the ones observed in shallower galaxy surveys.

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2. The correlation length does not increase linearly with the size of the sample as should happen for a fractal.

3. The correlation function and the correlation integral show clearly a gradual transition to homogeneity. In particular the correlation dimension, D_2, is a scale dependent quantity approaching 3 at large scales.

4. Statistical measures or estimators which implicitly assume isotropy and homogeneity should be applied with caution (Lahav 1999) in order that the results be meaningful.

5. It seems to me that the conclusions above are rather firm, but if some doubt remains, the next generation of wide and deep redshift surveys (SDSS, 2dF) will likely provide the final word to this issue.

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