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# Bounded dissipation predicts finite asymptotic state of near-wall turbulence

**J.C. Klewicki**†

Department of Mechanical Engineering, University of Melbourne, Parkville, VIC 3010, Australia

Enormous efforts have been devoted to the prediction and control of turbulent wall flows. A primary consideration here is the Reynolds number scaling problem in which non-dimensional representations are sought that render the normalized variables of interest unchanged for varying Reynolds number. Relative to this objective and in contrast to the mean velocity, there remains a considerable lack of clarity associated with the apparent failure of inner normalization (i.e. using the friction velocity and kinematic viscosity) when applied to the statistical profiles of fluctuating quantities in the near-wall region. In their present work Chen & Sreenivasan (*J. Fluid Mech.*, vol. 933, 2022, A20) generalize their earlier effort, Chen & Sreenivasan (*J. Fluid Mech.*, vol. 908, 2021, R3), and present a rational framework for characterizing and describing the evolution of turbulence quantities that either attain a near-wall peak or have non-zero wall values. Their analysis enjoys considerable empirical support. Physically, the asymptotic boundedness of the inner-normalized dissipation is used to reason that there is a limiting state of near-wall turbulence at asymptotically large Reynolds numbers. The law of bounded dissipation arguments put forth by Chen and Sreenivasan prescribe the recovery of inner scaling and suggest new possibilities regarding the physics of how wall turbulence matures to its asymptotic state.

**Key words:** turbulent boundary layers, turbulence theory

## 1. Introduction

Characterizing Reynolds number dependence is a long-standing challenge of central importance to the study of turbulent fluid flows along solid surfaces. Here, attention is on the so-called canonical flows that include the flat plate boundary layer and fully developed pipe and channel flows, and the friction Reynolds number of interest is given by $Re_\tau = \delta u_\tau / \nu$. In this expression, $\delta$ is the boundary layer thickness, pipe radius or channel half-height, $\nu$ is the kinematic viscosity and $u_\tau$ ($= \sqrt{\tau_w/\rho}$) is the friction velocity, where

† Email address for correspondence: klewicki@unimelb.edu.au

$\tau_w$ is the mean wall shear stress and $\rho$ is the mass density. For the present purposes, it is useful to conceptualize the boundary layer (for example) as a dynamical machine whose function is to transport free-stream momentum to the wall (Sreenivasan 1989). As it pertains to scaling, the problem of how this momentum transport exerts a net drag force on the surface as a function of Reynolds number comes into focus. Namely, the mechanisms of net drag generation are locally characterized by the mean wall shear stress, at any given streamwise location in the boundary layer or for any given mass flow rate in fully developed pipe or channel flows. Relative to scaling the mean flow, this perspective inherently embraces the observed universality of the inner-normalized mean velocity profile, and the classical law of the wall formulation more generally, e.g. Tennekes & Lumley (1972) and Pope (2000). A law of the wall based formulation for scaling the statistical profiles of fluctuating quantities does not, however, find the same compelling level of empirical support over the range of Reynolds numbers presently available. It is in this regard that the law of bounded dissipation proposed by Chen & Sreenivasan (2022) provides a new and promising characterization of Reynolds number dependence as $Re_\tau \rightarrow \infty$.

Based on its logical consistency and apparent success for the mean flow, a law of the wall formulation for quantities associated with the turbulent fluctuations was for many years deemed applicable to within the scatter of the data. Thus, for example, the validity of this formulation should encompass the position where the streamwise velocity variance, $\langle u^2 \rangle$, reaches its near-wall peak, e.g. Monin & Yaglom (1971) and Sreenivasan (1989). Note that this quantity presents a useful point of discussion since $\langle u^2 \rangle$ is the easiest velocity fluctuation statistic to measure. With the advancement of facilities and measurement techniques, along with an enhanced focus on sensor resolution, studies in the 1980s and 1990s began to produce credible evidence that inner normalization does not result in an invariant profile across the range of available $Re_\tau$ (Johansson & Alfredsson 1983; Klewicki & Falco 1990). Based upon the critical examination of an increasing number of data sets the emergent result was the existence of a small but detectable increase in the peak value of $\langle u^2 \rangle^+$ with increasing $Re_\tau$. Later measurements from the atmospheric surface layer solidified confidence in this increase (Metzger & Klewicki 2001), but the inherent scatter in the data prevents (to this day) unambiguous support for one particular functional dependence versus another. Initial efforts focused on quantifying the existence and magnitude of the observed Reynolds number variations. Because $Re_\tau$ variations are slow, logarithmic functions are reasonable to employ in the absence any particular theoretical forethought, e.g. see Klewicki (2010) for a range of quantities. Thus, for example, Metzger & Klewicki (2001) used a logarithmic fit of $\sqrt{\langle u^2 \rangle^+}$ to simply characterize the $Re_\tau$ dependence, while I. Marusic and co-workers fit $\langle u^2 \rangle^+$ owing to specific theoretical considerations, e.g. see Marusic, Baars & Hutchins (2017) and references therein and also Pullin, Inoue & Saito (2013). In perhaps the broadest context, the significance of the contribution by Chen & Sreenivasan (2022) is that it presents a physically coherent framework for characterizing and conceptualizing how near-wall turbulence quantities attain their asymptotic state.

## 2. Overview

To best understand the theoretical arguments underpinning what the authors describe as the law of bounded dissipation, one is encouraged to read both Chen & Sreenivasan (2021) and Chen & Sreenivasan (2022). The net result of these arguments is that the weak but apparent $Re_\tau$ increase of an inner-normalized turbulence quantity, $\Phi$, that has a near-wall peak or non-zero wall value follows a power law variation according to

$$\Phi_\infty - \Phi = C_\Phi Re_\tau^{-1/4}. \tag{2.1}$$

In this expression the subscript $\infty$ refers to the asymptotic value of the inner-normalized turbulence quantity, and the $-1/4$ power law is associated with the functional form of the inner-normalized turbulence dissipation rate. According to the analysis, the dissipation rate physically sets limits on the maximum values attained by other turbulence quantities. In contrast to a logarithmic function, that indicates a fixed variation in $\Phi$ for each order of magnitude change in $Re_\tau$, (2.1) indicates that, as the turbulence matures with $Re_\tau$, it approaches a limiting state that effectively becomes invariant for further increases in $Re_\tau$. That is, under this scenario one can, at least in principle, prescribe a sufficiently large Reynolds number such that inner scaling, as embodied in a law of the wall formulation, is satisfied to within a certain accuracy for all larger $Re_\tau$. In the field of wall turbulence this reality would indeed constitute a significant development. For modelling purposes one could imagine developing $Re_\tau$-dependent functions that describe the deviation from the asymptotic value. Relative to flow physics and control, the bounded dissipation perspective would, for example, seem to indicate a limiting strength of the inner/outer interaction, as the authors briefly allude to in their § 5. Such a scenario would likely, for example, pose a limit on the efficacy of drag reduction strategies that target the larger outer region scales of motion.

The bounded dissipation formulation is developed by considering the near-wall balance of terms in the equation for $\langle u^2\rangle$, and involves some notable approximations and assumptions. (Other quantities would presumably rely on a similar formulation.) One element is the reasonable, but reasonably questioned, use of a Taylor series at the wall to obtain a crude (order of magnitude) approximation for the growth rate of the $\langle u^2\rangle$ peak value. Similarly, an important assumption is that the $y^+$ value of the peak in the $\langle u^2\rangle^+$ profile remains invariant with Reynolds number. Relative to clarifying the veracity of these and other elements of the formulation, the good news is that they are empirically testable, and as new data sets at larger $Re_\tau$ become available the theory can be further investigated and refined.

The authors conduct a careful analysis that relies significantly on direct numerical simulations (DNS). This approach is rational since it makes use of the DNS capacity to generate high precision data that are difficult to obtain via measurements. In fact, channel DNS now meet or exceed the Reynolds numbers accessible to physical experiments, while the $Re_\tau$ range for pipe and boundary layer experiments continue to exceed those of DNS. Figure 1 presents streamwise and spanwise wall velocity gradient variances. These are effectively equivalent to the wall values of the $x$ and $z$ shear stress variances, the $z$ and $x$ wall vorticity variances and through the $\langle u^2\rangle$ and $\langle w^2\rangle$ transport equations the wall values of the diffusion term as well. Overall, it is apparent that the $-1/4$ power law curves predicted by the authors' analysis adhere closely to the bulk of the data. In the case of the $x$ gradient (figure 1*a*), however, it appears that a subset of the data seem to better follow a logarithmic variation, while for the $z$ data the logarithmic and $-1/4$ power fits exhibit nearly equivalent agreement with the data over the upper range of $Re_\tau$ investigated. Thus, while the existing data provide encouraging support for the authors' proposition, quality measurements at larger $Re_\tau$ are clearly required to confirm/refute the veracity of (2.1).

A later section of Chen & Sreenivasan (2022) considers the behaviour of the increases in the maximum values in the near-wall high-order moment profiles. Here, they present convincing evidence that the peak values of the $u$ and $w$ moment profiles scale linearly with the order of the moment, $q$. They then show that the observed linear $q$-dependence

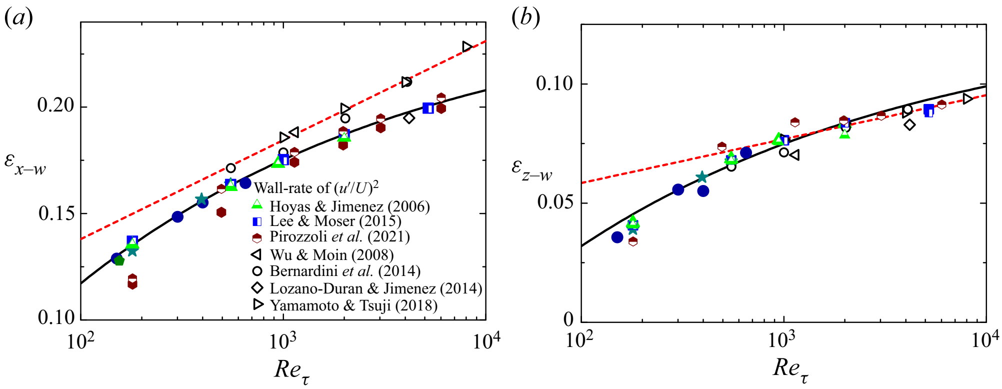


Figure 1. Inner-normalized wall velocity gradient variances: (*a*) $\langle \partial u/\partial y\rangle^{+2}$ (streamwise), (*b*) $\langle \partial w/\partial y\rangle^{+2}$ (spanwise). Figure 7 of Chen & Sreenivasan (2022).

can result from a Gaussian random variable, and in connection with this draw a potentially significant physical association between the non-zero value of the $q = 0$ intercept of the linear fit with the outer (larger scale) influence of Townsend's inactive motions (Townsend 1976).

## 3. Summary and broader issues

The bounded dissipation theory put forth by Chen & Sreenivasan (2022) asymptotically rescues inner scaling for near-wall turbulence. This theory is intuitively reasonable and attractive in its analytical simplicity. Physically, the theory indicates a final state for the turbulence versus an endless variation, and thus encourages one to reconsider existing empirical findings in the context of a new pathway to the asymptotic state. This is an exciting development. Perhaps not surprisingly, this theory also provides a clear impetus to obtain high quality data at large $Re_\tau$ from physical experiments and DNS. In this regard, it is apparent that the data discrepancies in figure 1 cannot be solely attributed to the use of a Taylor approximation at the wall. Thus, as DNS are pushed to higher $Re_\tau$ it is perhaps useful to keep in mind that the spatial resolution required to accurately create a Navier–Stokes realization via DNS is distinct from the sensor resolution needed to make an accurate measurement in an existing physical realization. As studies such as Chen & Sreenivasan (2022) rely on DNS to clarify increasingly delicate theoretical questions, the absolute fidelity requirements of DNS will become increasingly stringent.

**Declaration of interests.** The authors report no conflict of interest.

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