11

Multiple D-branes and bound states

In chapter 5, we saw a number of interesting terms arise in the Dp-brane world-volume action which had interpretations as smaller branes. For example, a $U(1)$ flux was a D$(p-2)$-brane fully delocalised in the world-volume, while for the non-Abelian case, we saw a D$(p-4)$-brane arise as an instanton in the world-volume gauge theory. Interestingly, while the latter breaks half of the supersymmetry again, as it ought to, the former is still half BPS, since it is T-dual to a tilted D$(p+1)$-brane.

It is worthwhile trying to understand this better back in the basic description using boundary conditions and open string sectors, and this is the first goal of this chapter. After that, we'll have a closer look at the nature of the BPS bound and the superalgebra, and study various key illustrative examples.

11.1 Dp and Dp' from boundary conditions

Let us consider two D-branes, Dp and Dp', each parallel to the coordinate axes. (We can of course have D-branes at angles[129], but we will not consider this here.) An open string can have both ends on the same D-brane or one on each. The $p-p$ and $p'-p'$ spectra are the same as before, but the $p-p'$ strings are new if $p \neq p'$. Since we are taking the D-branes to be parallel to the coordinate axes, there are four possible sets of boundary conditions for each spatial coordinate X^i of the open string, namely NN (Neumann at both ends), DD, ND, and DN. What really will matter is the number ν of ND plus DN coordinates. A T-duality can switch NN and DD, or ND and DN, but ν is invariant. Of course ν is even because we only have p even or p odd in a given theory in order to have a chance of preserving supersymmetry.

The respective mode expansions are

$$\begin{aligned}
\text{NN:}\quad X^\mu(z,\bar z) &= x^\mu - i\alpha' p^\mu \ln(z\bar z) + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\frac{\alpha^\mu_m}{m}(z^{-m}+\bar z^{-m}),\\
\text{DN, ND:}\quad X^\mu(z,\bar z) &= i\sqrt{\frac{\alpha'}{2}}\sum_{r\in\mathbb{Z}+1/2}\frac{\alpha^\mu_r}{r}(z^{-r}\pm\bar z^{-r}),\\
\text{DD:}\quad X^\mu(z,\bar z) &= -i\frac{\delta X^\mu}{2\pi}\ln(z/\bar z) + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\frac{\alpha^\mu_m}{m}(z^{-m}-\bar z^{-m}).
\end{aligned} \tag{11.1}$$

In particular, the DN and ND coordinates have half-integer moding. The fermions have the same moding in the Ramond sector (by definition) and opposite in the Neveu–Schwarz sector. The string zero point energy is 0 in the R sector as always, and using (2.80) we get:

$$(8-\nu)\left(-\frac{1}{24}-\frac{1}{48}\right)+\nu\left(\frac{1}{48}+\frac{1}{24}\right) = -\frac{1}{2}+\frac{\nu}{8} \tag{11.2}$$

in the NS sector.

The oscillators can raise the level in half-integer units, so only for ν a multiple of four is degeneracy between the R and NS sectors possible. Indeed, it is in this case that the Dp–Dp' system is supersymmetric. We can see this directly. As discussed in sections 8.1.1 and 8.2, a D-brane leaves unbroken the supersymmetries

$$Q_\alpha + P\tilde Q_\alpha, \tag{11.3}$$

where P acts as a reflection in the direction transverse to the D-brane. With a second D-brane, the only unbroken supersymmetries will be those that are also of the form

$$Q_\alpha + P'\tilde Q_\alpha = Q_\alpha + P(P^{-1}P')\tilde Q_\alpha, \tag{11.4}$$

with P' the reflection transverse to the second D-brane. Then the unbroken supersymmetries correspond to the $+1$ eigenvalues of $P^{-1}P'$. In DD and NN directions this is trivial, while in DN and ND directions it is a net parity transformation. Since the number ν of such dimensions is even, we can pair them as we did in section 7.1.1, and write $P^{-1}P'$ as a product of rotations by π,

$$e^{i\pi(J_1+\cdots+J_{\nu/2})}. \tag{11.5}$$

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In a spinor representation, each $e^{i\pi J}$ has eigenvalues $\pm i$, so there will be unbroken supersymmetry only if ν is a multiple of four as found above*.

For example, type I theory, besides the D9-branes, will have D1-branes and D5-branes. This is consistent with the fact that the only R–R field strengths are the three-form and its Hodge-dual seven-form. The D5-brane is required to have two Chan–Paton degrees of freedom (which can be thought of as images under Ω) and so an $SU(2)$ gauge group[130, 132].

When $\nu = 0$, $P^{-1}P' = 1$ identically and there is a full ten-dimensional spinor of supersymmetries. This is the same as for the original type I theory, to which it is T-dual. In $D = 4$ units, this is $\mathcal{N} = 4$, or sixteen supercharges. For $\nu = 4$ or $\nu = 8$ there is $D = 4$ $\mathcal{N} = 2$ supersymmetry.

Let us now study the spectrum for $\nu = 4$, saving $\nu = 8$ for later. Sometimes it is useful to draw a quick table showing where the branes are located. Here is one for the (9,5) system, where the D5-brane is pointlike in the x^6, x^7, x^8, x^9 directions and the D9-brane is (of course) extended everywhere.

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D9	—	—	—	—	—	—	—	—	—	—
D5	—	—	—	—	—	—	•	•	•	•

A dash under x^i means that the brane is extended in that direction, while a dot means that it is pointlike there.

Continuing with our analysis, we see that the NS zero-point energy is zero. There are four periodic world-sheet fermions ψ^i, namely those in the ND directions. The four zero modes generate $2^{4/2}$ or four ground states, of which two survive the GSO projection. In the R sector the zero-point energy is also zero; there are four periodic transverse ψ, from the NN and DD directions not counting the directions $\mu = 0, 1$. Again these generate four ground states of which two survive the GSO projection. The full content of the $p - p'$ system is then is half of an $N = 2$ hypermultiplet. The other half comes from the $p' - p$ states, obtained from the orientation reversed strings: these are distinct because for $\nu \neq 0$ the ends are always on different D-branes.

Let us write the action for the bosonic $p - p'$ fields χ^A, starting with $(p, p') = (9, 5)$. Here A is a doublet index under the $SU(2)_{\mathrm{R}}$ of the $N = 2$ algebra. The field χ^A has charges $(+1, -1)$ under the $U(1) \times U(1)$ gauge theories on the branes, since one end leaves, and the other arrives. The

* We will see that there are supersymmetric *bound states* when $\nu = 2$.

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minimally coupled action is then

$$\int d^6\xi\,\left(\sum_{a=0}^{5}\left|(\partial_a + iA_a - iA'_a)\chi\right|^2 + \left(\frac{1}{4g^2_{\mathrm{YM},p}} + \frac{1}{4g^2_{\mathrm{YM},p'}}\right)\sum_{I=1}^{3}(\chi^\dagger\tau^I\chi)^2\right), \tag{11.6}$$

with A_a and A'_a the brane gauge fields, $g_{\mathrm{YM},p}$ and $g_{\mathrm{YM},p'}$ the effective Yang–Mills couplings (8.13), and τ^I the Pauli matrices. The second term is from the $N=2$ D-terms for the two gauge fields. It can also be written as a commutator $\mathrm{Tr}\,[\phi^i,\phi^j]^2$ for appropriately chosen fields ϕ^i, showing that its form is controlled by the dimensional reduction of an F^2 pure Yang–Mills term. See section 13.1 for more on this.

The integral is over the five-brane world-volume, which lies in the nine-brane world-volume. Under T-dualities in any of the ND directions, one obtains $(p,p') = (8,6)$, $(7,7)$, $(6,8)$, or $(5,9)$, but the intersection of the branes remains $(5+1)$-dimensional and the p – p' strings live on the intersection with action (11.6). In the present case the D-term is non-vanishing only for $\chi^A = 0$, though more generally (say when there are several coincident p and p'-branes), there will be additional massless charged fields and flat directions arise.

Under T-dualities in r NN directions, one obtains $(p,p') = (9-r, 5-r)$. The action becomes

$$\int d^{6-r}\xi\,\left(\sum_{a=0}^{5-r}\left|(\partial_a + iA_a - iA'_a)\chi\right|^2 + \frac{\chi^\dagger\chi}{(2\pi\alpha')^2}\sum_{a=6-r}^{5}(X_a - X'_a)^2\right.$$
$$\left. + \left(\frac{1}{4g^2_{\mathrm{YM},p}} + \frac{1}{4g^2_{\mathrm{YM}p'}}\right)\sum_{i=1}^{3}(\chi^\dagger\tau^I\chi)^2\right). \tag{11.7}$$

The second term, proportional to the separation of the branes, is from the tension of the stretched string.

11.2 The BPS bound for the Dp–Dp' system

The ten dimensional $\mathcal{N} = 2$ supersymmetry algebra (in a Majorana basis) is

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} &= 2(\Gamma^0\Gamma^\mu)_{\alpha\beta}(P_\mu + Q^{\mathrm{NS}}_\mu/2\pi\alpha') \\
\{\tilde{Q}_\alpha, \tilde{Q}_\beta\} &= 2(\Gamma^0\Gamma^\mu)_{\alpha\beta}(P_\mu - Q^{\mathrm{NS}}_\mu/2\pi\alpha') \\
\{Q_\alpha, \tilde{Q}_\beta\} &= 2\sum_p \frac{\tau_p}{p!}(\Gamma^0\Gamma^{m_1}\dots\Gamma^{m_p})_{\alpha\beta}Q^{\mathrm{R}}_{m_1\dots m_p}.
\end{aligned} \tag{11.8}$$

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Here $Q^{\rm NS}$ is the charge to which the NS–NS two-form couples, it is essentially the winding of a fundamental string stretched along $\mathcal{M}_1$:

$$Q^{\rm NS}_\mu \equiv \frac{Q^{\rm NS}}{v_1}\int_{\mathcal{M}_1} dX^\mu, \quad \text{with} \quad Q^{\rm NS} = \frac{1}{{\rm Vol}\, S^7}\int_{S^7} e^{-2\Phi}\, {}^*H^{(3)} \tag{11.9}$$

and the charge $Q^{\rm NS}$ is normalised to one per unit spatial world-volume, $v_1 = L$, the length of the string. It is obtained by integrating over the S^7 which surrounds the string. The $Q^{\rm R}$ are the R–R charges, defined as a generalisation of winding on the space $\mathcal{M}_p$:

$$Q^{\rm R}_{\mu_1\dots\mu_p} \equiv \frac{Q^{\rm R}_p}{v_p}\int_{\mathcal{M}_p} dX^{\mu_1}\wedge\dots dX^{\mu_p}, \quad \text{with} \quad Q^{\rm R}_p = \frac{1}{{\rm Vol}\, S^{8-p}}\int_{S^{8-p}} {}^*G^{(p+2)}. \tag{11.10}$$

The sum in (11.8) runs over all orderings of indices, and we divide by $p!$ Of course, p is even for IIA or odd for IIB. The R–R charges appear in the product of the right- and left-moving supersymmetries, since the corresponding vertex operators are a product of spin fields, while the NS–NS charges appear in right–right and left–left combinations of supercharges.

As an example of how this all works, consider an object of length L, with the charges of p fundamental strings (‘F-strings’, for short) and q D1-branes (‘D-strings’) in the IIB theory, at rest and aligned along the direction X^1. The anticommutator implies

$$\frac{1}{2}\left\{\begin{bmatrix} Q_\alpha \\ \tilde{Q}_\alpha \end{bmatrix}, \begin{bmatrix} Q_\beta & \tilde{Q}_\beta \end{bmatrix}\right\} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} M\delta_{\alpha\beta} + \begin{bmatrix} p & q/g_{\rm s} \\ q/g_{\rm s} & -p \end{bmatrix} \frac{L(\Gamma^0\Gamma^1)_{\alpha\beta}}{2\pi\alpha'}. \tag{11.11}$$

The eigenvalues of $\Gamma^0\Gamma^1$ are ±1 so those of the right hand side are $M \pm L(p^2+q^2/g^2)^{1/2}/2\pi\alpha'$. The left side is a positive matrix, and so we get the ‘BPS bound’ on the tension [133]

$$\frac{M}{L} \geq \frac{\sqrt{p^2+q^2/g_s^2}}{2\pi\alpha'} \equiv \tau_{p,q}. \tag{11.12}$$

Quite pleasingly, this is saturated by the fundamental string, $(p,q) = (1,0)$, and by the D-string, $(p,q) = (0,1)$.

It is not too hard to extend this to a system with the quantum numbers of Dirichlet p and p' branes. The result for ν a multiple of four is

$$M \geq \tau_p v_p + \tau_{p'} v_{p'} \tag{11.13}$$

and for ν even but not a multiple of four it is †

$$M \geq \sqrt{\tau_p^2 v_p^2 + \tau_{p'}^2 v_{p'}^2}. \tag{11.14}$$

† The difference between the two cases comes from the relative sign of $\Gamma^M(\Gamma^{M'})^T$ and $\Gamma^{M'}(\Gamma^M)^T$.

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The branes are wrapped on tori of volumes v_p and v'_p in order to make the masses finite.

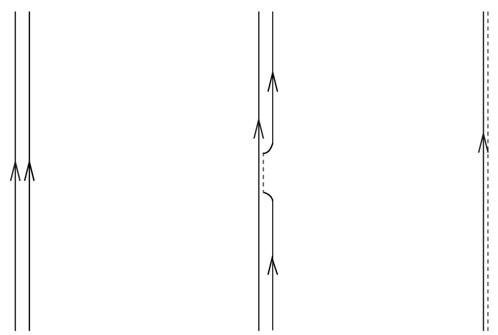
The results (11.13) and (11.14) are consistent with the earlier results on supersymmetry breaking. For ν a multiple of four, a separated p-brane and p'-brane do indeed saturate the bound (11.13). For ν not a multiple of four, they do not saturate the bound (11.14) and cannot be supersymmetric.

11.3 Bound states of fundamental strings and D-strings

Consider a parallel D1-brane (D-string) and a fundamental string (F-string) lying along X^1. The total tension

$$\tau_{D1} + \tau_{F1} = \frac{g_s^{-1} + 1}{2\pi\alpha'} \qquad (11.15)$$

exceeds the BPS bound (11.12) and so this configuration is not supersymmetric. However, it can lower its energy[26] as shown in figure 11.1. The F-string breaks, its endpoints attached to the D-string. The endpoints can then move off to infinity, leaving only the D-string behind. Of course, the D-string must now carry the charge of the F-string as well. This comes about because the F-string endpoints are charged under the D-string gauge field, so a flux runs between them; this flux remains at the end. Thus the final D-string carries both the NS–NS and R–R two-form charges. The flux is of order g_s, its energy density is of order g_s, and so the final tension is $(g_s^{-1} + O(g_s))/2\pi\alpha'$. This is below the tension of the separated strings and of the same form as the BPS bound (11.12) for a $(1,1)$ string. A more detailed calculation shows that the final tension saturates



(*a*) (*b*) (*c*)

Fig. 11.1. (*a*) A parallel D-string and F-string, which is not supersymmetric. (*b*) The F-string breaks, its ends attaching to the D-string, resulting in (*c*) the final supersymmetric state, a D-string with flux.

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the bound[118], so the state is supersymmetric. In effect, the F-string has dissolved into the D-string, leaving flux behind.

We can see quite readily that this is a supersymmetric situation using T-duality. We can choose a gauge in which the electric flux is $F_{01}=\dot{A}_1$. T-dualising along the x^1 direction, we ought to get a D0-brane, which we do, except that it is moving with constant velocity, since we get $\dot{X}^1 = 2\pi\alpha'\dot{A}_1$. This clearly has the same supersymmetry as a stationary D0-brane, having been simply boosted.

To calculate the number of BPS states we should put the strings in a box of length L to make the spectrum discrete. For the $(1,0)$ F-string, the usual quantisation of the ground state gives eight bosonic and eight fermionic states moving in each direction for $16^2 = 256$ in all. This is the ultrashort representation of supersymmetry: half the 32 generators annihilate the BPS state and the other half generate $2^8 = 256$ states. The same is true of the $(0,1)$ D-string and the $(1,1)$ bound state just found, as will be clear from the later duality discussion of the D-string.

It is worth noting that the $(1,0)$ F-string leaves unbroken half the supersymmetry and the $(0,1)$ D-string leaves unbroken a different half of the supersymmetry. The $(1,1)$ bound state leaves unbroken not the intersection of the two (which is empty), but yet a different half. The unbroken symmetries are linear combinations of the unbroken and broken supersymmetries of the D-string.

All the above extends immediately to p F-strings and one D-string, forming a supersymmetric $(p,1)$ bound state. The more general case of p F-strings and q D-strings is more complicated. The gauge dynamics are now non-Abelian, the interactions are strong in the infrared, and no explicit solution is known. When p and q have a common factor, the BPS bound makes any bound state only neutrally stable against falling apart into subsystems. To avoid this complication let p and q be relatively prime, so any supersymmetric state is discretely below the continuum of separated states. This allows the Hamiltonian to be deformed to a simpler supersymmetric Hamiltonian whose supersymmetric states can be determined explicitly, and again there is one ultrashort representation, 256 states. It is left to the reader to consult the literature[26, 1] for the details.

11.4 The three-string junction

Let us consider further the BPS saturated formula derived and studied in the two previous subsections, and write it as follows:

$$\tau_{p,q} = \sqrt{(p\tau_{1,0})^2 + (q\tau_{0,1})^2}. \tag{11.16}$$

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An obvious solution to this is

$$\tau_{p,q}\sin\alpha = q\tau_{0,1}, \quad \tau_{p,q}\cos\alpha = p\tau_{1,0}. \tag{11.17}$$

with $\tan\alpha = q/(pg_{\mathrm{s}})$. Recall that these are tensions of strings, and therefore we can interpret the equations (11.17) as balance conditions for the components of forces. In fact, it is the required balance for three strings[137, 135], and we draw the case of $p = q = 1$ in figure 11.2.

Is this at all consistent with what we already know? The answer is yes. An F-string is allowed to end on a D-string by definition, and a (1,1) string is produced, due to flux conservation, as we discussed above. The issue here is just how we see that there is bending. The first thing to notice is that the angle α goes to $\pi/2$ in the limit of zero string coupling, and so the D-string appears in that case to run straight. This had better be true, since it is then clear that we simply were allowed to ignore the bending in our previous weakly coupled string analysis. (This study of the bending of branes beyond zero coupling has important consequences for the study of one-loop gauge theory data[139]. We shall study some of this later on.)

Parenthetically, it is nice to see that in the limit of infinite string coupling, α goes to zero. The diagram is better interpreted as a D-string ending on an F-string with no resulting bending. This fits nicely with the fact that the D- and F-strings exchange roles under the strong/weak coupling duality ('S-duality') of the type IIB string theory, as we shall see in chapter 12.

When we wrote the linearised BIon equations in section 5.7, we ignored the 1+1 dimensional case. Let us now include that part of the story here

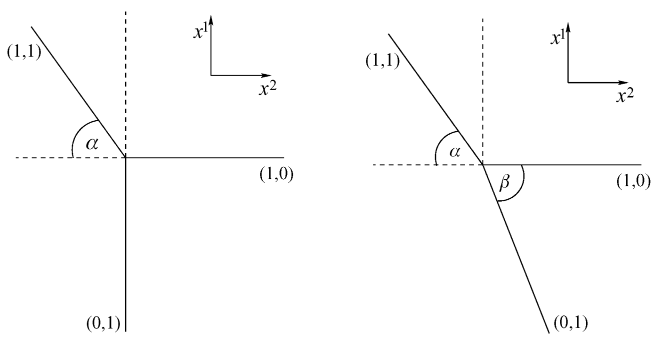

Fig. 11.2. (*a*) When an F-string ends on a D-string it causes it to bend at an angle set by the string coupling. On the other side of the junction is a (1,1) string. This is in fact a BPS state. (*b*) Switching on some amount of the R–R scalar can vary the other angle, as shown.

as a 1+1 dimensional gauge theory discussion. There is a flux F_{01} on the world-volume, and the end of the F-string is an electric source. Given that there is only one spatial dimension, the F-string creates a discontinuity on the flux, such that e.g.[140, 60]:

$$F_{01} = \begin{cases} g_s, & x_1 > 0 \\ 0, & x_1 < 0 \end{cases}, \tag{11.18}$$

so we can choose a gauge such that

$$A_0 = \begin{cases} g_s x^1, & x_1 > 0 \\ 0, & x_1 < 0 \end{cases}. \tag{11.19}$$

Just as in section 5.7, this is BPS if one of the eight scalars Φ^m is also switched on so that

$$\Phi^2(x^1) = A_0. \tag{11.20}$$

How do we interpret this? Since $(2\pi\alpha')\Phi^2$ represents the x^2 position of the D-string, we see that for $x^1 < 0$ the D-string is lying along the x^1 axis, while for $x^1 > 0$, it lies on a line forming an angle $\tan^{-1}(1/g_s)$ with the x^1, axis.

Recall the T_1-dual picture we mentioned in the previous section, where we saw that the flux on the D-string (making the (1,1) string) is equivalent to a D0-brane moving with velocity $(2\pi\alpha')F_{01}$. Now we see that the D0-brane loses its velocity at $x^1 = 0$. This is fine, since the apparent impulse is accounted for by the momentum carried by the F-string in the T-dual picture. (One has to tilt the diagram in order to T-dualise along the (1,1) string in order to see that there is F-string momentum.)

Since we have seen many times that the presence of flux on the world-volume of a Dp-brane is equivalent to having a dissolved D$(p-2)$-brane, i.e. non-zero $C_{(p-1)}$ source, we can modify the flux on the $x^1 < 0$ part of the string this way by turning on the R–R scalar C_0. This means that $\Phi^2(x^1)$ will be linear there too, and so the angle β between the D- and F-strings can be varied too (see figure 11.2(b)). It is interesting to derive the balance conditions from this, and then convert it into a modified tension formula, but we will not do that here[140].

It is not hard to imagine that given the presence we have already deduced of a general (p, q) string in the theory that there are three-string junctions to be made out of any three strings such that the (p, q)-charges add up correctly, giving a condition on the angles at which they can meet. This is harder to do in the full non-Abelian gauge theories on their world-volumes, but in fact a complete formula can be derived using the underlying $SL(2, \mathbb{Z})$ symmetry of the type IIB string theory. We will have more to say about this symmetry later.

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General three-string junctions have been shown to be important in a number of applications, and there is a large literature on the subject which we are unfortunately not able to review here.

11.5 Aspects of D-brane bound states

Bound states of p-branes and p'-branes have many applications. Some of them will appear in our later lectures, and so it is worth listing some of the results here. Here we focus on $p' = 0$, since we can always reach it from a general (p, p') using T-duality.

11.5.1 0–0 bound states

The BPS bound for the quantum numbers of n 0-branes is $n\tau_0$, so any bound state will be at the edge of the continuum. What we would like to know is if there is actually a true bound state wave function, i.e. a wavefunction which is normalisable. To make the bound state counting well defined, compactify one direction and give the system momentum m/R with m and n relatively prime[141]. The bound state now lies discretely below the continuum, because the momentum cannot be shared evenly among unbound subsystems.

This bound state problem is T-dual to the one considered in section 11.3. Taking the T-dual, the n D0-branes become D1-branes, while the momentum becomes winding number, corresponding to m F-strings. There is therefore one ultrashort multiplet of supersymmetric states when m and n are relatively prime[141]. This bound state should still be present back in infinite volume, since one can take R to be large compared to the size of the bound state. There is a danger that the size of the wavefunction we have just implicitly found might simply grow with R such that as $R \to \infty$ it becomes non-normalisable again. More careful analysis is needed to show this. It is sufficient to say here that the bound states for arbitrary numbers of D0-branes are needed for the consistency of string duality, so this is an important problem. Some strong arguments have been presented in the literature ($n = 2$ is proven), but the general case is not yet proven[142].

11.5.2 0–2 bound states

Now the BPS bound (expression (11.14)) puts any bound state discretely below the continuum. One can see a hint of a bound state forming by noticing that for a coincident D0-brane and D2-brane the NS 0–2 string has a negative zero-point energy (11.2) and so a tachyon (which survives

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the GSO projection), indicating an instability towards forming something else. In fact the bound state (one short representation) is easily described: the D0-brane dissolves in the D2-brane, leaving flux, as we have seen numerous times. The brane R–R action (expression (9.9)) contains the coupling $C_{(1)}F$, so with the flux the D2-brane also carries the D0-brane charge[143]. There is also one short multiplet for n D0-branes. This same bound state is always present when $\nu = 2$.

11.5.3 0–4 bound states

The BPS bound (11.13) makes any bound state marginally stable, so the problem is made well-defined as in the 0–0 case by compactifying and adding momentum[144]. The interactions in the action (11.7) are relevant in the infrared so this is again a hard problem, but as before it can be deformed into a solvable supersymmetric system. Again there is one multiplet of bound states[144]. Now, though, the bound state is invariant only under $\frac{1}{4}$ of the original supersymmetry, the intersection of the supersymmetries of the D0-brane and of the D4-brane. The bound states then lie in a short (but not ultrashort) multiplet of 2^{12} states.

For two D0-branes and one D4-brane, one gets the correct count as follows[145]. Think of the case that the volume of the D4-brane is large. The 16 supersymmetries broken by the D4-brane generate 256 states that are delocalised on the D4-brane. The eight supersymmetries unbroken by the D4-brane and broken by the D0-brane generate 16 states (half bosonic and half fermionic), localised on the D0-brane. The total number is the product 2^{12}. Now count the number of ways two D0-branes can be put into their 16 states on the D4-brane: there are eight states with both D0-branes in the same (bosonic) state and $\frac{1}{2}16\cdot 15$ states with the D-branes in different states, for a total of $8\cdot 16$ states. But in addition, the two-branes can bind, and there are again 16 states where the bound state binds to the D4-brane. The total, tensoring again with the D4-brane ground states, is $9\cdot 16\cdot 256$.

For n D0-branes and one D4-brane, the degeneracy D_n is given by the generating functional[145] (see insert 3.4, p. 92):

$$\sum_{n=0}^{\infty} q^n D_n \;=\; 256 \prod_{k=1}^{\infty} \left(\frac{1+q^k}{1-q^k}\right)^8 , \tag{11.21}$$

where the term k in the product comes from bound states of k D0-branes then bound to the D4-brane. Some discussion of the D0–D4 bound state, and related issues, can be found in the references[146].

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11.5.4 0–6 bound states

The relevant bound is (11.14) and again any bound state would be below the continuum. The NS zero-point energy for 0–6 strings is positive, so there is no sign of decay. One can give D0-brane charge to the D6-brane by turning on flux, but there is no way to do this and saturate the BPS bound. So it appears that there are *no* supersymmetric bound states. Notice that, unlike the 0–2 case, the 0–6 interaction is repulsive, both at short distance and at long.

11.5.5 0–8 bound states

The case of the D8-brane is special, since it is rather big. It is a domain wall, because there is only one spatial dimension transverse to it. In fact, the D8-brane on its own is not really a consistent object. Trying to put it into type IIA runs into trouble, since the string coupling blows up a finite distance from it on either side because of the nature of its coupling to the dilaton. To stop this happening, one has to introduce a pair of O8-planes, one on each side, because they (for SO groups) have negative charge (-8 times that of the D8-brane) and can soak up the dilaton. We therefore should have 16 D8-branes for consistency, and so we end up in the type I' theory, the T-dual of type I. The bound state problem is now quite different, and certain details of it pertain to the strong coupling limit of certain string theories, and their 'matrix'[157] formulation[147, 148]. We shall revisit this in section 12.5.