Geometrical Constructions for Refraction and Reflection in a Prism.

By Dr David Robertson.

(*Read 7th March 1913*).

General Construction for Refracted Ray. The two-circle method of finding the direction of a ray refracted at a plane surface is very old, but seems to be now almost forgotten. It is particularly convenient when the refraction of several rays is to be determined, and its application to the case of a prism is especially elegant and leads to a simple self-contained proof of the condition for minimum deviation.

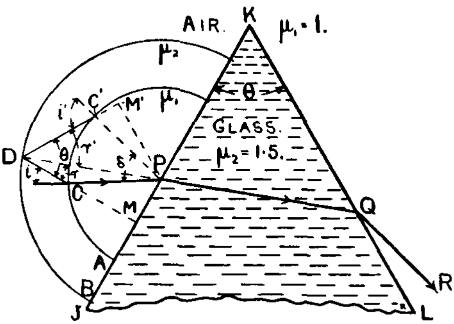

Fig. 1. *General Construction for Refracted Ray.*

In Fig. 1 let JKL be the prism, and P any point on its first surface. Draw two concentric circles whose radii represent to any convenient scale the indices of refraction of the material of the prism and of the surrounding medium, μ_2 and μ_1 respectively. The centre of these circles may with advantage be taken at P. The radius CP parallel to the incident ray cuts the circle for the first medium at C. If CD is drawn normal to the first surface, and cuts the circle for the second medium (prism) at D, then DP is the direction of the internal ray. If DC′ is drawn normal to the second surface, and cuts the circle for the outside medium at C′, then C′P is the direction of the emergent ray.

Proof. Draw PM ⊥ DC.

Then $PD \cdot \sin PDM = PC \cdot \sin PCM$.

Or $$\sin PDM = \frac{PC}{PD} \sin PCM.$$

$$= \frac{\mu_1}{\mu_2} \sin i = \sin r.$$

∴ $\angle PDM = r$, the angle of refraction.

And DP is the direction of refraction.

A similar proof holds for the second refraction.

Deviation. CPC′ is the angle between the incident and emergent rays; that is, it is the deviation δ. CDC′ is the refracting angle of the prism θ, since CD and CD′ are perpendicular to the two faces.

But, $\theta = CDC' = CDP + C'DP = r + r'$.

And $\delta = CPC' = CPD + C'PD$

$$= (i - r) + (i' - r')$$
$$= (i + i') - (r + r')$$
$$= (i + i') - \theta.$$

The angles i and r, or i' and r' are negative when the ray is deviated towards the thin end of the prism.

If $i = i'$, then $r = r'$, and

$$\delta = 2i - \theta$$
$$i = \tfrac{1}{2}(\delta + \theta)$$

and $$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = \frac{\sin \frac{1}{2}(\delta + \theta)}{\sin \frac{1}{2}\theta}.$$

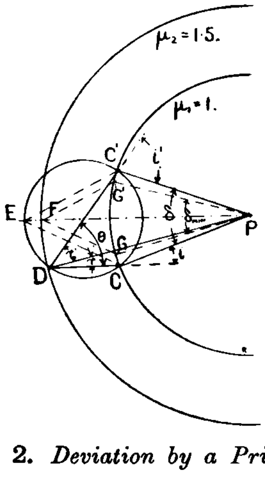

Fig. 2. *Deviation by a Prism.*

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Minimum Deviation. Draw the circle through CDC′ (Fig. 2). Draw PE through the centre of this circle to cut it in E and the D-circle in F. Join EC, EC′. Draw FG ∥ EC and FG′ ∥ EC′, cutting the C-circle in G and G′. Then

$$GFG' = CEC' = CDC' = \theta = \text{constant}.$$

∴ arc GG′ and angle GPG′ are constant, since PF is a fixed length.

If D, E, and F all coincide the incidence and emergence are symmetrical, and the deviation is GPG′. With any other condition, E must lie outside the D-circle, and so C and C′ must lie outside GG′, and the deviation CPC′ must exceed GPG′. Consequently the deviation is least when the incidence and emergence are symmetrical; that is, when the internal ray is perpendicular to the bisector of the refracting angle.

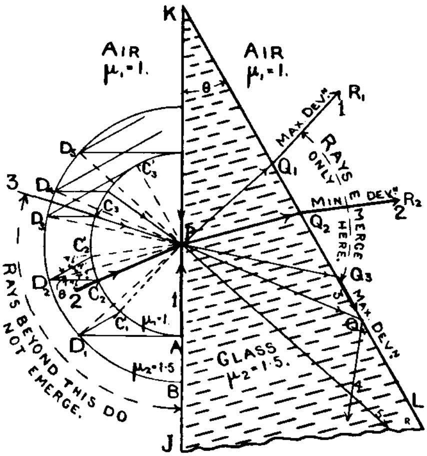

Fig. 3. *Emergent Rays with Fixed Prism.*

Limiting Rays. The ray numbered 1 in Fig. 3 enters the prism at grazing incidence, while 3 leaves it at grazing emergence. The latter is obtained by drawing PC_3' parallel to the second surface, and working the construction backwards from C_3'. It is obvious from the geometry of the diagram that the figures $AD_1C_1'P$ and $C_3'D_3C_3P$ could be made to coincide by suitably folding the diagram over, and that consequently the two limiting deviations are alike.

Rays beyond No. 3 do not emerge, but are totally reflected at the second surface.

Fig. 3 shows the emergent rays for a number of rays incident at the same point on a fixed prism.

Experimental Application. The following simple method of determining the index of refraction of a prism (due to Professor Poynting ? *) gives good results, and is not generally known. Draw a straight line on paper, and place the prism over it in such a way

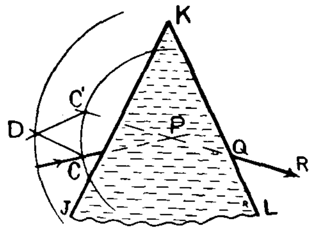

Fig. 4. *Experimental Determination of* μ.

it can be seen through the prism. Draw JKL (Fig. 4) the trace of the prism, and QR coinciding with the apparent direction of the line as seen through the prism. Remove the prism and produce RQ beyond P where it cuts the original line. With P as centre and any radius (conveniently a simple number of mms. or inches) draw an arc to cut the original line in C, and second line, produced backwards, in C′. Draw CD perpendicular to the surface of incidence JK. Draw C′D perpendicular to the surface of emergence KL, and intersecting CD in D. Then $\mu = DP \div CP$.

If a second circle, concentric with the first, be drawn through D, it will at once be recognised that the diagram is part of the two-circle construction already considered, and that it consequently gives the desired result. The only difference is that the centre of the circles has been moved to the point of intersection of the incident and emergent rays.

* The author is indebted to Dr R. A. Houstoun for drawing his attention to this method.