

## SOME CRITICAL REMARKS ON THE INFLATIONARY UNIVERSE CONCEPT

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The basic idea of inflation in cosmology is very simple: It is the assumption that the expansion factor  $R(t)$  of a Friedmann-Lemaître cosmological model grows exponentially during a brief time interval in the very early universe. The phase of exponential growth is followed by a thermalization stage and a subsequent "normal" evolution  $R(t) \sim t$ . This "inflationary expansion" can help to solve cosmological puzzles inherent in the standard model – such as the large-scale flatness, the horizon structure, the numerical value of the entropy in a comoving volume [for a review see Brandenberger 1985]. To turn this romantic idea of inflation into a quantitative model requires still a lot of work: The simple change in the thermal history of the universe must be derived from a fundamental particle theory. The models proposed so far do not inspire much confidence. In the following a few difficulties of the Higgs field idea, especially the Coleman-Weinberg formalism will be pointed out (section 1). In section 2 some problems connected with the investigation of initially strongly anisotropic or inhomogeneous cosmological models will be mentioned.

### 1. Problems in the Particle Physics Input

There is no generally accepted model for a unified theory of elementary particles, but quite generally the concept of a large local gauge symmetry  $G$  is usually supplemented in grand unified theories (GUT) by the introduction of selfinteracting scalar fields which serve to give a mass to some of the gauge bosons. The self-interaction of the Higgs field  $\phi$

$$V(|\phi|) = \lambda |\phi|^4 - \sigma |\phi|^2 \quad (1)$$

$(\lambda > 0)$

has a maximum (depending on the parameters) at  $|\phi|^2 = \sigma/2\lambda$  (Higgs phase), with  $V''(|\phi|) > 0$ . Usually a representative state from the

ground state orbit is chosen to define the ground state in the symmetric ( $|\phi| = 0$ ) and the Higgs phase ( $|\phi| = \sqrt{\sigma/2\lambda}$ ).

A semiclassical picture is often employed which assumes spontaneous symmetry breaking, i.e. a nonzero expectation value  $\langle\phi(x)\rangle \neq 0$  in the Higgs phase. Inflationary models use this idea to describe a phase transition of the universe field with Higgs fields with the expectation value  $\langle\phi(x)\rangle$  acting as an order parameter. The classical potential energy  $V(|\phi_c|)$  then has a nonzero "vacuum energy density"  $|V(0) - V(\sigma/2\lambda)|$  which can appear like a constant energy density in the Friedmann equation. Inflation occurs when this constant energy density becomes dominant.

This semiclassical picture may be a reasonable description in lowest order perturbation theory, but to test its reliability it should be contrasted to the exact results from model theories.

Studies of Abelian Higgs models on a lattice have yielded mixed results:

- i) Without fixing the gauge there is no spontaneous symmetry breaking, i.e.  $\langle\phi(x)\rangle = 0$  everywhere [see e.g. Borgs, Nill 1986].
- ii) The gauge can be fixed by requiring the gauge transformations to be unity in a fixed direction (axial gauge) [Fröhlich, Morchio, Strocchi 1981] then also  $\langle\phi\rangle = 0$ .
- iii) The so-called  $\alpha$ -gauges consist in adding a term  $1/2\alpha \Sigma (\partial_\mu A_\mu)^2$  to the (euclidean) action. Then for  $\alpha \approx 0$  there is no spontaneous symmetry breaking,  $\langle\phi\rangle = 0$ , in dimensions  $d \leq 4$ . The reasons lie in spinwave contributions  $\sim d^d k / (k^2)^2$  at  $k \approx 0$  [Kennedy, King 1985; Borgs, Nill 1986].
- iv) For  $\alpha > 0$ ,  $G = U(1)$ , and the coupling  $g^2 \gg 1$ , again  $\langle\phi\rangle = 0$ .

The exception is the noncompact case  $G = \mathbb{R}$  ( $\sigma \ll -1$ ,  $g^2 \ll 1$ ,  $\lambda \ll 1$ ), where indeed  $\langle\phi\rangle \neq 0$  [cf. Nill 1987; Borgs, Nill 1986; Kennedy, King 1985].

The semi-classical picture loses 1:3, but perhaps it wins in cosmology.

In a perturbation approach to quantum field theory the effective potential  $V_{\text{eff}}(|\phi|)$  - interpreted as the thermodynamic Helmholtz free energy for the fields  $\phi$  - is computed in "loop" approximations which are basically an expansion in orders of  $\hbar$ , of the Euclidean action  $S_E/\hbar$ . The "1-loop" approximation of the Coleman-Weinberg type gives a double-hump potential with a relative maximum at  $|\phi_c| = 0$ . This is used in inflationary models to describe the time-evolution of the classical scalar field  $\phi_c$ , according to

$$\square_g \phi_c = V'_{\text{eff}}(\phi_c). \quad (2)$$

The effective potential  $V_{\text{eff}}$  must, however, be strictly convex, and the 1-loop expansion can therefore not be used to describe a time evolution

of the scalar field [e.g. Börner, Sellar 1984].

These criticisms apply only to specific versions of the inflationary model, but it remains to be seen whether other suggestions, such as "chaotic inflation" [Linde 1985], can survive a more precise scrutiny.

## 2. Problems with the Input from General Relativity

Most models start already in a homogeneous and isotropic FL universe at  $t < 10^{-35}$  sec. But the inflationary concept is of value only, if it works in more general initial conditions. A few more general cases have been investigated.

- I) It is found that in anisotropic and homogeneous models the anisotropy is strongly reduced by an inflationary phase (Rothman & Ellis 1986). Inhomogeneous and anisotropic cosmologies give rise to a stable state  $\langle \phi \rangle = 0$  if the initial anisotropy is too large, only for reasonably small values does the universe recenter a FL-like stage (Barrow & Turner 1982; Börner & Götz 1987).
- II) There are many choices in deSitter space for a time direction. How then can the choice of a spatially homogeneous time direction be guaranteed during the transition from a vacuum-energy dominated deSitter space to a radiation-dominated FL universe?

### References:

- Barrow J. D., Turner M. S., 1982, *Nature* **292**, 35  
 Börner G., Götz G., 1987 (in prep.)  
 Börner G., Sellar E., 1984, *Adv. Space Research* **3**, 441  
 Borgs C., Nill F., 1986, *Phys. Lett.* **B171**, 289.  
   *Nucl. Phys.* **B270**, 92  
 Brandenberger R. H., 1985, *Rev. Mod. Phys.* **57**, 1  
 Ellis G. F. R., Rothman T., 1986, preprint-Univ. Capetown, SA  
 Fröhlich J., Morchio G., Strocchi F., 1981, *Nucl. Phys.* **B190**, 553  
 Kennedy T., King C., 1985, *Phys. Rev. Lett.* **55**, 776  
 Linde A. D., 1985, *Comments Astrophys.* **16**, 229  
 Nill F., 1987, Ph.D. Thesis Munich Univ. (unpublished)