BINARY STATISTICS AND STAR FORMATION*

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Abstract. Binary statistics, in particular the distributions of mass ratios and orbital periods, are reviewed in an attempt to obtain clues to possible star formation and cloud fragmentation processes. Various observational selection effects which hamper the establishment of the true distributions are discussed. Four different theories of binary formation are compared (fission, fragmentation, capture, and the disintegration of small star clusters), none of which can be ruled out. We conclude that there may be many ways to form binary systems. The dominant mode of binary formation could be ring fragmentation or disc fragmentation depending upon whether the distribution of mass ratios is found to decrease or to increase towards small mass ratios. Future speckle interferometric measurements of a sufficiently large sample of close visual binaries are suggested to settle this important observational question. The present paper is special in that it brings together a wealth of useful information, both observational and theoretical, in one place.

Atoms form molecules,
*stars form binaries.***

1. Introduction

1.1. Batten's binary statistics

The total frequency of binary and multiple systems as well as the distribution of the mass ratio among the components and the distribution of orbital periods are important data and constraints for the theory of star formation. It is well known that at least half of the stars in the solar neighborhood belong to binary or multiple systems (cf. Heintz, 1969; Abt, 1979, 1983); in the classical book on the subject (Batten, 1973) the statistics of binary and multiple systems read as follows (approximate figures):

$\frac{1}{2}$ of all stars are binaries or belong to systems of higher multiplicity;
$\frac{1}{3}$ of all binary systems belong to triple systems;
$\frac{1}{4}$ of all triple systems belong to quadruple systems, etc., ...

A binary star which deserves to be mentioned in the present context of star formation is the spectroscopic binary Delta Orionis (actually a triple system, since there is a distant companion) the spectrum of which led to the discovery of the interstellar medium by Hartmann in 1904: in contrast to the stellar lines which were shifting periodically in time, with a period of 5.7 days, its spectrum also showed 'stationary' Ca II (H and K) absorption lines which were correctly interpreted as being due to the calcium ions of an intervening interstellar gas.

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** We quote Su-Shu Huang in IAU-Colloquium No. 33 = Revista Mexicana, Vol. 3 (1977). Many papers related to the present problem can be found there.

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1.2. Basic questions, definitions, and numbers

Given that a substantial fraction of stars formed in pairs, one may, quite generally, raise the following two fundamental questions, neither of which is satisfactorily answered at present:

(1) Do binary systems preferentially occur with components of roughly equal masses or with components of largely unequal masses?

(2) Do binary systems preferentially form close pairs or wide pairs?

Here we define the binary mass ratio as $q = M_B/M_A$, where M_A is the primary and M_B is the secondary ($M_A > M_B$ or $q < 1$ initially; however mass exchange or mass loss may cause $q > 1$ at a later stage). The distinction between close pairs and wide pairs is adopted as usual, i.e.: close pairs are close enough to be able to exchange mass during stellar evolution while wide pairs are not; the dividing line then is at a separation of a few AU for the binary components.

Observationally, there are two main categories of binary stellar systems: those which can be resolved into two components by a telescope, and those which appear to be one object visually, but can be identified as consisting of two stars due to their periodically variable spectrum. The former are called visual binaries (VB), the latter spectroscopic binaries (SB). The spectroscopic binaries are subdivided into two classes: spectroscopic binaries with single lines (SB1's) and with double lines (SB2's). For SB1's it is sufficient to observe a line shifting with time, whereas for SB2's it is necessary to observe a line splitting. SB2's directly allow the determination of the mass ratio of the binary components; SB1's only yield the mass function of the system (i.e., the quantity $Q = M_B^3 \sin^3 i/(M_A + M_B)^2$], so that the additional information about the primary mass and the inclination angle i is required, if the mass ratio is to be inferred. (The mass function must not be confused with the Initial Mass Function (IMF) which describes the relative proportions with which stars of different masses are born. The existence of unresolved binaries in star counts influences the definition of the IMF: see Appendix B.) The threshold in the projected radial velocity of the orbital motion for the detection of spectroscopic binaries is discussed in Appendix A. In the most recent SB-Catalogue (Batten *et al.*, 1978) with almost 1000 systems listed roughly $\frac{1}{3}$ are SB2's and the remaining $\frac{2}{3}$ are SB1's; 15–20% of the SB1's are eclipsing binaries, many of which have an evolved component. There are only a few dozens eclipsing SB2's. It is to them, however, that we owe almost all our knowledge of stellar masses and the calibration of the stellar mass luminosity relation (see Popper, 1967, 1980: Blaauw, 1981).

1.3. The scope of the paper

The main purpose of the present article will be to review some statistical data about binary stellar systems and to analyse them as potential clues to the theory of star formation. Unfortunately, most of the statistical data are severely biased by observational selection effects. Any catalogue of binary stars (e.g. IDS-*Catalogue of Visual Binary Stars* or the 6th and 7th *Catalogue of the Orbital Elements of Spectroscopic Binary Stars*) is bound to represent a very incomplete sample as will become evident in the course of the discussion below. Moreover the catalogue data are not a priori corrected for possible changes in the orbital elements due to interactions between close binary components.

In the past, observations of interacting close binaries have often been used to test the theory of stellar evolution. An evolutionary sequence has emerged (see the review by Trimble, 1983). In the future, observations of non-interacting wide binaries should be used to constrain the theory of star formation. It is hoped that the present article will provide a stimulus for the observers to attack this task.

The paper is divided into two main parts, an observational one on binary statistics (Section 2) and a theoretical one on the origin of binary and multiple stars (Section 3). A final chapter summarises the major conclusions and lists some suggestions for future work. A brief outline of the contents of Sections 2 and 3 is seen from Table I.

TABLE I

Subsections 2.4, 2.5, and 2.6 can be omitted on first reading. Two appendices deal with (A) the detection of binaries and (B) the Initial Mass Function corrected for the existence of binary stars.

Stellar rotation and its relation to star formation will not be discussed in the present paper, since there is an adequate, recent discussion of this by Franco (1983) (see also Woolfson (1978), Wesson (1979), Vogel and Kuhi (1981), Wolff *et al.* (1982), Guthrie (1983), Gray (1982), and Fleck (1982)).

2. Binary Statistics

2.1. The Total Binary Frequency

We shall not discuss the total binary star frequency on the Main Sequence. This has been done very carefully in a recent paper by Gieseking (1983). We wish to quote his summary: "Many authors (especially spectroscopists) tend to overestimate the fraction of detected binaries. But paradoxically, many authors at the same time may underestimate the total binary frequency, because overestimating their detection probability and/or overestimating the volume of the binary parameter space covered by their observations, they tend to underestimate the number of binaries which escaped detection. (A meaningful comparison between published binary frequencies is only possible, if they refer to equivalent volumes of binary parameter space.) The details of a possible variation of the binary frequency along the Main Sequence as well as its true value is not yet well established. There is no observational evidence for the binary frequency to vary strongly along the Main Sequence. There is observational evidence which suggests a nearly 100% frequency of binary and multiple systems among Main-Sequence objects."

Judging from the binary frequency of the immediate solar vicinity (taken at face value), we would infer a smaller fraction of the total binary frequency. For instance, van de Kamp's (1971) list of nearby stars (within a sphere of radius 5.2 pc around the Sun) contains 60 visible stars (including the Sun itself) of which 32 are single stars. The other 28 consist of 11 binary systems and 2 triple systems. However, the statistics of both spectroscopic and visual binaries are certainly not complete for small mass ratios, even for van de Kamp's ultralocal sample of stars. In some cases, there is a suspicion that the single stars are not really single but are circled by an invisible companion. Barnard's star is an example of such a system. McCarthy (1983) reports detection of unseen companions to 15 nearby stars due to infrared speckle observations.

Recently, Poveda *et al.* (1982) obtained a true fraction of visual binaries and multiples among field stars as high as 90%, with most of the companions remaining undetected. In order to produce a relatively uniform and homogeneous group of double and multiple stars free of optical and spurious systems and suited for statistical analysis, they applied a 'filter' to the about 70000 entries of the updated *Index Catalogue of Visual Double Stars* (IDS-Catalogue) resulting in a 'filtered' catalogue with nearly 20000 entries eliminated. It was on the basis of this catalogue that they found the duplicity frequency quoted above. They also found that this catalogue is largely complete for pairs brighter than 10th magnitude and with brightness differences less than 1 magnitude. Interestingly enough, roughly one out of three field stars turns out to be a visual multiple; thus there are far more multiple systems than indicated in Batten's (1973) book to which we referred in the Introduction.

Lastly, we note that the evaluation of true total binary frequencies is not possible without knowing at least the frequency distributions of the mass ratio and the separation of the binary components.

2.2. Frequency distribution of the mass ratio and the orbital period

2.2.1. *Mass Ratio*

The frequency distribution of mass ratios $f(q)$ was first investigated by Kuiper (1935) who found $f(q) = 2(1 + q)^{-2}$. The distribution is shown in Figure 1 as a challenge for everybody.

Kuiper (1935) believed this result to be true for all (close and wide) binaries. Heintz (1969) favored $f(q) \sim q^{1/2}(1 + q)^{-3}$ for all (spectroscopic and visual) binaries, while Popov (1970) gave $f(q) \sim q^2$ for spectroscopic binaries. Later, Trimble (1974, 1978) analyzed the *Sixth Catalogue of the Orbital Elements of Spectroscopic Binary Stars* and

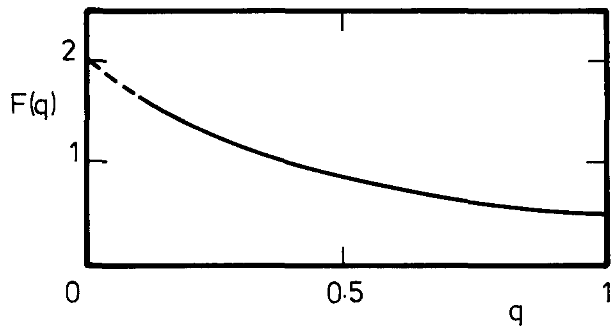

Fig. 1. Possible distribution of initial mass ratios for binary stars (after Kuiper, 1935).

its supplements, and suggested a bimodal frequency distribution of mass ratios with two peaks at $q \sim 0.25$ and $q \sim 1$. Staniucha (1979) analyzed the *Seventh Catalogue of the Orbital Elements of Spectroscopic Binary Stars* (Batten *et al.*, 1978) confirming the bimodal structure of the frequency distribution: the q-distribution of SB1's peaks at $q \sim 0.2$–0.3 that of SB2's at $q \sim 1$. However, as mentioned before, these 'results' are strongly biased by selection effects and evolutionary effects. Evolutionary effects have been discussed by Kruszewski (1967), Paczynski (1971), Thomas (1977), Shu and Lubow (1981), and Giuricin *et al.* (1983). One selection effect which contributes to the difference in the characteristic mass ratio for SB1's and SB2's could be that for a mass ratio near unity (SB2's) the primary and the secondary component of the binary system can move very much relative to the center of mass, while for a moderately small mass ratio (SB1's) only the slow motion of the primary relative to the center of mass can be observed (the fast moving secondary is too faint). The observed q-distribution of SB's is then interpreted in the following way (Gieseking, private communication): The q-distribution is bound to be bimodal, i.e. to have an apparent gap, because of the very low probability of detection of systems with moderate q. From $q = 1$ (for which the velocity amplitude and the probability of detection is largest) towards $q = 0$, the probability of detection starts to decrease due to decreasing double line splitting until there remains only one blended line. For smaller and smaller q the lines of the secondary component begin to disappear so that the lines of the primary component become cleaner and cleaner, eventually allowing the recognition of clear Doppler shifts (second maximum of the SB-detection probability). For $q \to 0$ the probability of detection starts to decrease again, since the velocity amplitude goes to zero. Another selection effect is that observers tend to study SB2's more carefully than SB1's; thus SB2's are overrepresented in Batten *et al.*'s (1978) catalogue.

In a detailed statistical study Lucy and Ricco (1979) claimed that the peak at $q \sim 1$ for SB2's is a real feature. A similar conclusion was reached by Kraicheva *et al.* (1979) who tried to calculate the initial q-distribution from the observed one taking into account models for the evolutionary effects.

Lucy and Ricco (1979)* confined themselves to SB2's with periods less than 25 days and masses in the range $0.5-10 M_\odot$. Of course, they did realize that there is a paramount selection effect for SB2's to have $q \simeq 1$; however, they argued that the decrease to smaller q is too steep to be caused by undetected SB2's alone. Kraicheva *et al.* (1979)* considered spectroscopic binaries with semi-major axes less than 1 AU. Their conclusions may be shaky, because they did not consider the frequency distribution of $\sin^3 i$; they only invoke the statistical mean value (0.68). But, due to the projection effect, even for random inclinations the frequency distribution of the observed inclinations is proportional to $\sin i$. Since this was not taken into account, a possible increase of the frequency distribution of the mass ratio to small mass ratios is underestimated.

Abt and Levy (1976; see also Abt, 1977, 1978) made a study of 76 systems and derived results which are believed to be not seriously affected by selection effects. Their sample of primary components contained essentially all F3–G2 Main-Sequence stars brighter than $V = 5.5$ mag and north of $-20°$ declination. The narrow spectral range corresponds to a stellar mass of $1.2-1.3 M_\odot$. The sample included 36 spectroscopic binaries (4 SB2's, 32 SB1's), 19 visual binaries, and 21 common proper motion (CPM) pairs. For binaries with periods less than 100 yr, they found a different distribution of the mass ratio than for binaries with periods greater than 100 yr. They attribute this difference to the operation of two distinct binary formation mechanisms in the two regimes.

Huang (1977) has noted that the discrepancy between the old distribution of mass ratios derived by Kuiper (1935) and the new distribution derived by Abt and Levy (1976) may be due to the condition imposed by Kuiper that the distribution is a function of $M_2/(M_1 + M_2)$ rather than M_2/M_1.

TABLE II

Distribution of mass ratios according to Abt (1978) for the sample of Abt and Levy (1976). Note that the total number of binary systems is $\Sigma_q N(q) = 88$ (not 76) (12 systems have been added on the basis of incompleteness calculations)

	$q \in (1, \frac{1}{2})$	$q \in (\frac{1}{2}, \frac{1}{4})$	$q \in (\frac{1}{4}, \frac{1}{8})$	$q \in (\frac{1}{8}, \frac{1}{16})$	$q < \frac{1}{16}$	
$N(q)$	13	16	11	8	2	($P < 100$ yr)
$N(q)$	8	11	19	–	–	($P > 100$ yr)

Table II illustrates the distribution of mass ratios (grouped in bins of factors of 2) as inferred from Abt (1978) adding up the numbers for the various period segments.

In our own opinion, the trend in Table III is not clear-cut, and the small numbers involved in the statistics should give rise to caution.

It seems that the number of long-period binaries increases with decreasing mass ratio (much like the distribution proposed by Kuiper, 1935), while the distribution of short-period binaries, if we do not put too much statistical weight on the figure for the lowest mass ratio, stays rather constant (contrary to what is fitted to the data by Abt, 1978, viz. $N \sim q^{1/3}$).

* Both papers are very valuable approaches to the problem in question.

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Ducati and Jaschek (1982) have criticized Abt and Levy's procedure to derive mass ratios from their data (it is not the data that has been criticized). They conclude that the existing data permits one only to assert that SB 1's with small ratios are more frequent than those with large mass ratios (cf. Jaschek and Ferrer, 1972; Jaschek, 1976).

Taken together, the situation for *spectroscopic* binaries is inconclusive as far as their distribution of mass ratios is concerned.

For the 14 (!) high-quality *visual* binaries listed in the review by Popper (1980) the mean value of $M_2/(M_1 + M_2)$ is 0.4, i.e., the mean value of $q = M_2/M_1$ is $\frac{2}{3}$; cf. also the list of Harris *et al.* (1963).

2.2.2. *Orbital Periods*

Frequency distributions of the orbital periods of binary stars have been given by Kuiper (1935, 1955), Brosche (1964), van Albada and Blaauw (1967), Heintz (1969), Kraicheva *et al.* (1979a, b), Staniucha (1979), and by Abt and Levy (1976; see also Abt, 1977, 1978). In principle, the binary period distribution is far easier to establish than the binary mass ratio distribution. In practice there is considerable difficulty in the range from 1 to 10 AU and the corresponding periods (cf. Blaauw, 1981).

We shall first discuss the distribution of the orbital periods for spectroscopic binaries. This distribution has a rather sharp maximum at a period of a few days (Kraicheva *et al.*, 1979a; Staniucha, 1979).

The corresponding distribution of orbital angular momentum per unit mass peaks at $5 \times 10^{18}\ \mathrm{cm^2\ s^{-1}}$ for SB1's and $9 \times 10^{18}\ \mathrm{cm^2\ s^{-1}}$ for SB2's (Kraicheva *et al.*, 1979a). (For comparison, the specific angular momentum of the solar system is $1.5 \times 10^{17}\ \mathrm{cm^2\ s^{-1}}$, 98% of which is in the orbital motion of Jupiter.)

The apparent characteristic period of a few days reflects the convolution of two effects. The first is that period distribution increases initially for increasing periods or semi-major axes, the second is that the probability of detection decreases with increasing periods or semi-major axes due to the fact that the velocity amplitudes become smaller and smaller. An additional effect is that observations become more and more tedious once the periods become too long.

We now proceed to discuss the period distribution found by Abt and Levy (1976) and given in Abt (1977, 1978). As previously stated, the sample chosen by Abt and Levy is supposed to represent a largely unbiased sample.

Our Figure 2 reproduces Figure 1 from the paper by Abt (1977) and shows the observed histogram of binary periods for the survey done by Abt and Levy (1976).

The most remarkable feature of the histogram is its broad, unimodal shape. The binary periods span a range of at least 6 orders of magnitude from less than a day to more than a thousand years. We also see that there is enough of an overlap between spectroscopic binaries and visual binaries that a bimodal distribution did not develop. A similar unimodal period distribution was also obtained by Heintz (1969) based on binaries of all spectral types, down to apparent magnitude 9. A paper on wide binaries in the solar neighborhood by Retterer and King (1982) gives a nice comparison of both

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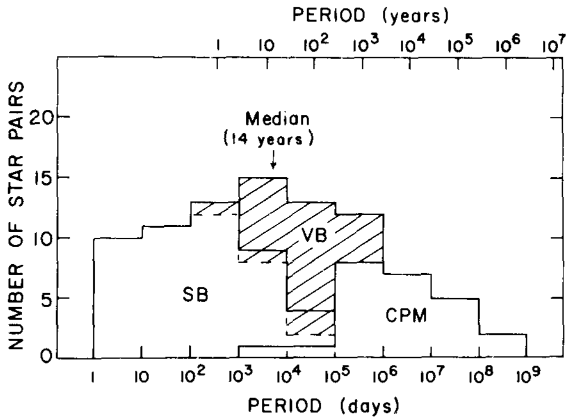

Fig. 2. Best-bet distribution of orbital periods for binary stars (from Abt, 1977).

distributions. Note that, unlike Table II, Figure 2 is not corrected for incompleteness. Such a correction would probably be only a minor affair. It is very unlikely that it would change the unimodal distribution to a bimodal distribution. Earlier work concentrating on the period distribution of B0 to B5 stars (van Albada and Blaauw, 1967) did, however, seem to indicate a bimodal distribution. Abt and Levy (1976) remark that if they studied early-type stars where rotational broadened lines prevent the discovery of many of the spectroscopic binaries with long periods, and the greater distances of most of the stars prevent detection of most of the short period visual pairs, they would expect a bimodal distribution. Further investigation (Gieseking, 1983) shows hat the development of an apparent bimodal period distribution is generally expected and can be interpreted as being due to the low detection probability of periods intermediate between those characteristic for spectroscopic and visual systems. Notice also that the period distribution of SB's in Figure 2 has a maximum at about 1 yr; comparison with the maximum of the period distribution from analysis of the 6th and 7th Catalogue of SB's, being of the order of a few days, clearly shows that many of the longer period SB's have a low detection probability.

One more interesting aspect, common to the Heintz (1969) and the Abt (1978, 1979) survey, should be pointed out: both found that the period distribution depended only weakly on the spectral class of the binary (see, however, Brosche and Hoffmann, 1979).

Given the large spread of binary orbital periods, it is useful to define a physical classification for all binaries in terms of their orbital period (Table III). The nomenclature is chosen to be symmetrical and simple, and the corresponding figures are easy to remember.

A few comments concerning Table III may be in order:

For the extremely close systems ('contact systems') $P < 10^{-3}$ yr pertains to the low mass pairs, whereas $P \lesssim 5 \times 10^{-3}$ yr would pertain to high mass pairs. Of course, there is a range of periods in between. In the binary classification table the values given are for the low mass pairs, because they are by far the more numerous ones. For the

TABLE III

Physical classification of binary systems
(P = orbital period in years)

Extremely close binaries[a]	$P < 10^{-3}$
Very close binaries	$10^{-3} \lesssim P \lesssim 1$
Close binaries	$1 \lesssim P \lesssim 10$
Wide binaries	$10 \lesssim P \lesssim 10^2$
Very wide binaries	$10^2 \lesssim P \lesssim 10^3$
Extremely wide binaries[b]	$P > 10^3$

[a] Contact systems.
[b] CPM-systems.

extremely wide systems ('Common Proper Motion systems') the theoretical upper limit of the semi-major axis is $\sim 10^4$ AU (Retterer and King, 1982) corresponding to orbital periods $\sim 10^6$ yr. Statistical methods are required to separate physical pairs and merely optical pairs (see Poveda *et al.*, 1982).

Whether or not a single formation process is able to explain the huge dispersion of binary periods, all the way from the extremely close to the extremely wide systems, is a major issue of binary star formation theories.

2.2.3. *Correlation between Mass Ratio and Separation*

Having discussed the distribution of the mass ratio and the distribution of the orbital periods (or equivalently the distribution of the separation) of binary systems, it is natural to ask if there are any correlations between these elements. It turns out that small mass ratios clearly tend to go with small separations for the case of the observed spectroscopic binaries (Staniucha, 1979). It is quite obvious, though, that the correlation in part comes from the observational selection which acts against the discovery of binaries with large separation and very small mass ratio*. On the other hand, evolutionary effects such as mass transfer, will change the separation in a systematic way (see Giuricin *et al.*, 1983, for Algol binaries).

On the assumption of mass conservation and conservation of angular momentum the semi-major axis a of a binary changes according to $a \propto (M_A M_B)^{-2}$ (Equation (6) in Paczynski, 1971); thus a $q = 1$ binary would widen its orbit when its mass ratio becomes less than unity after mass exchange (by a factor ~ 2, if it ends up with $M_A : M_B = 1 : 3$).

One might speculate that the same effect operates when a binary system first forms. That would mean that $q = 1$ binaries should have systematically smaller separations than binaries with a small mass ratio but the same total mass.

* It is not obvious, however, how a recent new discovery fits in here; namely, the discovery that the frequency distribution of separations of early (B)-type Main-Sequence stars with visual low mass secondaries does *not* increase all the way towards smaller separations but reaches a maximum and decreases again (Lindroos, 1982).

Another puzzling fact is the discovery that close early (O)-type SB Main-Sequence stars lack mass ratios smaller than $q = 0.3$ (Garmany and Conti, 1980; Abt, 1983).

2.3. Multiple systems

We will not discuss triple systems but concentrate on quadruple systems only. (Dynamically, triple systems have much in common with quadruple systems, anyway)*.

Two distinct types of quadruple systems exist (Batten, 1973):

(a) hierarchical systems (Evans, 1968);

(b) trapezium-like systems (Ambartsumian, 1955).

Hierarchical systems (like Capella) consist of two close pairs being in a wide orbit around each other. Trapezium-like systems (as in Orion) share the property that all the member stars have roughly equal distances from each other. Recent observational work on hierarchical systems has been done by Fekel (1981) with special emphasis on the period ratio between the long and the short period, the question of coplanarity, and the mass ratios of the close pairs. The results were as follows:

For 25 systems with a long period of about 100 yr or less, the mean ratio of long to short period is roughly 3000, a factor ~ 12 higher than given in Batten (1973). A third out of 21 orbital pairs are not coplanar, for the rest coplanarity is a permitted possibility. Of the 25 short-period pairs whose mass ratio are known, 18 have mass ratios greater than 0.6. It is also noteworthy that two substantially different mass ratios can occur in the same system as is the case for the quadruple μ Ori.

Observations of Trapezium-like systems have recently been reported by Salukvadze (1980a, b) concentrating on the young T-associations in Orion and Tau/Aur. It is very interesting that Trapezium-like systems are not only common for massive OB-stars but also for low-mass T-Tauri stars. Of course, Trapezium-like systems have to be extremely young, because these systems are known to be very unstable ($\sim 10^6$ yr, see Allen and Poveda, 1974). Beichman *et al.* (1979) have found multiple compact infrared sources in molecular cluds which seem to be precursors to Trapezium-like systems of OB stars. The mean separation of these sources is 0.17 ± 0.04 pc based on a total of 14 systems which is very similar to the corresponding number for the mean separation of the stars in 31 Trapezium-type OB-clusters (0.12 ± 0.01 pc). Considerably more than half of the compact infrared sources come in double or multiple systems (Wynn-Williams, 1982). It is perhaps not surprising that the galactic distribution of Trapezium-type systems as derived from a total of 915 visual systems is strongly concentrated to the galactic plane (Allen *et al.*, 1977), since these systems are known to be young.

* In order to be stable, triple systems like quadruples have to be hierarchical, i.e. the separation of the third star C to the center of mass of the binary AB has to be large enough (see the discussion and the reference in Szebehely, 1977; Fekel, 1981).

A classical example for a stable triple system is Algol = βPersei at a distance of 30 pc. The long-period system Algol AB-C ($P = 1.862$ yr) is nearly coplanar with the eclipsing binary Algol AB ($P = 2.867$ days). Masses newly determined by speckle-interferometric observations (Bonneau, 1979) are: $M_A = 0.73 \pm 0.12 M_\odot$, $M_B = 3.4 \pm 0.6 M_\odot$, and $M_C = 1.7 \pm 0.2 M_\odot$.

2.4. Young double stars (Pop. I)

2.4.1. *Protostars and pre-Main-Sequence Objects*

With the advent of modern observing techniques – such as infrared speckle interferometry – it has become possible to catch a couple of double stars still associated with their mother molecular cloud which gave birth to them. We have listed them in Table IV including tentative parameters as far as these are known. Note that the famous BN-object in the Orion molecular cloud appears to be a single star (Foy *et al.*, 1979), while the most powerful embedded Orion source IRc2 is probably double with a separation of 350 AU (Chelli *et al.*, 1983).

TABLE IV

Young objects discovered to be double

Name	Distance	Mass	Separation	Ref.	Comments
T-Tau	150 pc	$3M_\odot$	145 AU	1, 2	$\Delta m_{3.8\mu} = 1.47$
MonR2 IRS3	950 pc	$8M_\odot$	830 AU	3, 5, 6	triple?
W3 IRS5	2300 pc	$25M_\odot$	3000 AU	3, 4, 6	member of a small cluster

Key to the references:
1 = Dyck *et al.* (1982a).
2 = Hanson *et al.* (1983).
3 = Dyck and Howell (1982).
4 = Howell *et al.* (19881).
5 = McCarthy (1982).
6 = Wynn-Williams (1982).

In addition to the young objects listed in Table IV, the first pre-Main-Sequence spectroscopic binary (named X-ray 1 or E0429 + 1755) has recently been discovered in the Taurus-Auriga star formation complex (Mundt *et al.*, 1983). It is a non-eclipsing SB2 with mass ratio unity and period 4 days.

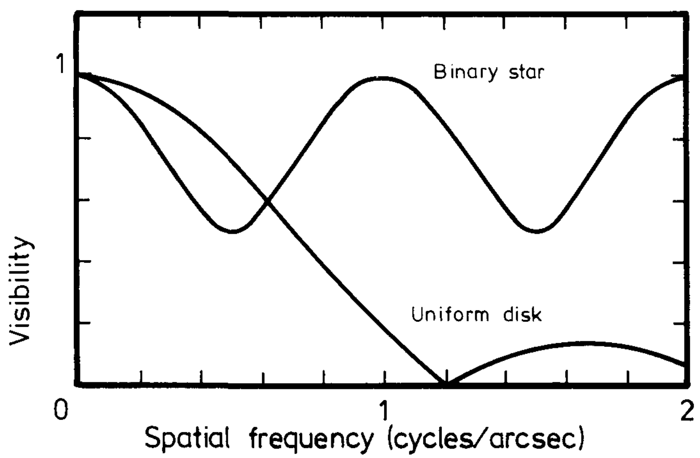

Fig. 3. Typical result of an infrared speckle observation: square root of object power spectrum (visibility I) as a function of spatial frequency, for a binary system and a uniform disk. The visibility function is defined in the usual way that $(I_{max} - I_{min})/(I_{max} + I_{min})$ equals the brightness ratio of the binary components (1 : 3 here). The separation of the binary components is given by the inverse of the period of the spatial frequency (1″ here). (This figure has been kindly provided by M. Dyck.)

In the southern hemisphere S CrA has long been known to be a visual pair of pre-Main-Sequence stars (separation 1″, period ~ 1000 yr).

Finally, we draw attention to the list of pre-Main-Sequence stars compiled by Cohen and Kuhi (1979). A fair fraction of their optical pairs may well be physical pairs, i.e. visual binaries.

The infrared speckle technique allows are to distinguish a young binary system from a protostellar disk (see Figure 3). It also allows the determination of the binary's angular separation as well as the brightness ratio. Therefore, this new observational method holds great promise for the extension of binary statistics to star formation regions.

2.4.2. *Open Clusters*

It is clearly very important to ask whether binary statistics in open clusters are different from the binary statistics of field stars addressed so far in the previous sections. The general impression that prevails in the literature is that there is no striking difference between the total binary frequency in open clusters as compared to the field. For example, Jaschek (1976) gives 35% as the minimum average percentage of binaries for 4 well-studied clusters (Pleiades, Praesepe, Coma Berenices, and α Persei). For the Hyades cluster, the corresponding number is 40% (Carney, 1982). This fraction is slightly higher, presumably because the photometric identification was extended from the optical to the near infrared yielding a few binary candidates among the cluster dwarfs. Possible cluster-to-cluster variations in binary content have been discussed by Abt and Sanders (1973) who also claimed the existence of an anticorrelation between the axial rotational velocity of cluster stars and the percentage of cluster binaries (see also Abt, 1979; Levato and Morrell, 1983). A particular controversial case is the cluster IC 4665 (Abt and Snowden, 1964; Crampton *et al.*, 1976).

Trimble and Ostriker (1978) examined the question of whether the mass ratio in binary systems in open clusters is significantly different from the mass-ratio in binary systems for field stars. The answer depends very sensitively upon where the Main Sequence is drawn for each cluster. Not only a companion, but also stellar rotation can lift the position of a Main-Sequence star in the HR-diagram. Therefore, a Main Sequence fitted to rotating single stars will suppress the binarity of some objects, and hence serve to distort the statistics*. On the basis of the existing data, the above question cannot be answered with any certainty.

The number and semi-major axis distribution of the visual binaries in the Pleiades was the objective of a statistical study by Brosche and Hoffmann (1979). They obtained results that were in agreement with the frequencies of very wide binaries in the solar neighborhood, except for one deviation: there is a deficiency of binary systems with faint components ($6.5 < M_{vis} \leq 9.2$) in the Pleiades as compared with the nearby stars. This might constitute the first observable deviation of the magnitude independency of double star statistics.

* There is also the danger of confusion between single pre-Main-Sequence objects and Main-Sequence binaries (see Stauffer, 1982, for the Pleiades cluster).

Another major issue connected with binary stars in open clusters is the following: Do the orbital planes of cluster binaries exhibit a preferential orientation or do they not? Kraft (1965) investigating the Hyades and the Coma cluster came up with the conclusion that the orbital planes are randomly distributed, although he also stated that one cannot exclude a weak correlation. Note that Huang and Wade (1966) could not find a preferred orientation of the orbital planes of field star eclipsing binaries with the galactic plane. Finally, knowing that some clusters tend to have their fastest rotating stars towards the projected cluster center (Abt, 1970), it would be of great interest to get information about the spatial distribution of binary systems inside the clusters. Extensive speckle observations of clusters members may be a suitable way to approach this problem.

2.5. Old double stars (Pop. II)

Was the dynamics of star formation different in the early epoch when the galactic disk was not yet formed? One approach to deal with this issue is to investigate the binary frequency among the metal-poor high-velocity stars which belong to the halo population (Pop. II). If the binary frequency among those stars turns out to be significantly different from the binary frequency among the low-velocity disk stars, then we have some evidence how important the large-scale dynamics of the collapse of the proto-Galaxy and perhaps metallicity were to the small-scale dynamics of star formation. Searches for halo binaries have been undertaken in both domains in which halo stars are found: in the field and in globular clusters. The 'primordial' binary fraction in globular clusters is difficult to determine, because of continuing binary disruption and formation (Hut, 1983), but the binary fraction plays a crucial role in the dynamical evolution of a globular cluster (Dokuchaev and Ozernoy, 1978; Spitzer and Mathieu, 1980). Trimble (1980) reviewed the interplay between theory and observations of binaries in globular clusters, and we will not discuss it further, except to say that the common belief that there are no close binaries in the globular clusters is almost certainly going to be challenged in the course of future observations (see Alexander and Budding, 1979, for a discussion of the selection effects).

Among the field halo stars, results of systematic searches for radial velocity variability have been published by Abt and Levy (1969) and by Crampton and Hartwick (1972). Abt and Levy concluded that the frequency of short-period spectroscopic binaries among Pop. II stars was lower than among stars of Pop. I. This conclusion was confirmed by Crampton and Hartwick in an enlarged sample of extremely metal-deficient subdwarfs. The Abt and Levy sample included 68 F- and G-type high-velocity dwarfs, and it was compared with a sample of 42 low-velocity dwarfs of about the same spectral types.

As far as the long-period or visual binaries are concerned, the frequency among Population II dwarfs may be similar to the frequency among Population I dwarfs. Observational data on this problem comes from the stars within 20 pc of the Sun (Gliese's catalogue). These stars can be separated into two categories according to their space velocities, and the binary fraction in the high-velocity Pop. II and the low-velocity Pop. I can be compared. In making such a comparison, Partridge (1967) was the first to address the question of binarity among the halo field population. His result was: 18% out of 127 stars with high (> 70 km s^{-1}) total space velocity are binaries, while out of 275 stars with low (< 40 km s^{-1}) total space velocity 23% are binaries, ~7% of which are spectroscopic binaries. The validity of these percentages depends, of course, on the absence of strong selection effects. On one hand, there may be an observational bias against the detection of Pop. II binaries, since both members of such a pair will necessarily be faint. Moreover, many Pop. II binary systems are expected to contain a white dwarf, making the detection even more difficult. On the other hand, there is the possibility that two nearby stars moving with the same high space velocity would be more readily identified as members of a binary system than two stars with lower space velocites.

Worley (1969) has discussed the duplicity characteristics of high velocity subdwarfs (mostly spectral type F or G) finding at least 15 of 127 systems which were positively identified as subdwarfs to be double, some of them close visual or spectroscopic pairs. Remembering the observational selection effects, Worley concludes that it has not been proved that binaries are any less frequent among Pop. II stars than amng younger objects (see also Gehren *et al.*, 1981).

A fresh attempt to search for binaries in the halo dwarf stars is reported by Carney (1983). The technique is based on *uvbyUBVRIJHK* photometry and involves the color excess method ($B - V$ vs $V - K$ colors). The sample includes 71 stars and is free from post-Main-Sequence objects. A binary can be identified via its flux distribution. As a result, Carney estimates that the halo dwarf binary frequency may be as high as 20–25%.

New radial velocity work is also in progress. There is an ongoing radial velocity survey of Lowell proper motion stars, about 500 with $7 \leq V \leq 13$ so far, with an extension planned to $V \sim 15$. Repeated radial velocity measurements (accuracy < 1 km s^{-1}) have been made of all stars with radial velocities exceeding 100 km s^{-1}. It appears that the halo binary frequency is some 10% at least, but the analysis is still preliminary (Latham, Stefanik, and Carney, private communication).

If the apparent lack of close binaries in the halo (in the field as well as in globular clusters) is largely due to observational selection, the star formation processes in the early stages of the Galaxy need not be different from those acting at present in the galactic disk. However, Abt (1979) conjectured that metal poor stars are deficient in close binaries, because for low metallicity a contracting protostar will have a greater difficulty in radiating away excess energy and therefore will contract more slowly – slow enough probably that it may be able to shed the excess angular momentum without bifurcating (in this context see also Barry (1977) who studied binarity as a function of metallicity for disk stars in the solar neighborhood).

Since wide binaries are as frequent for halo field stars as for disk field stars, there is no reason to claim that halo stars form a low-angular momentum population (as stated in the literature now and then). If anything, a possible deficiency of close binaries, together with the normal incidence of wide binaries may be taken to indicate a high-angular momentum population (presumably due to the highly turbulent velocity field generated during an irregular protogalactic collapse).

2.6. Binaries in other galaxies

Almost nothing is known about binaries in other galaxies. One thing we do know (Kopal, private communication) is the lack of bright eclipsing binaries in the spiral arms of M31: the absolute magnitude of the brightest eclipsing binaries in M31 is about 2 magnitudes fainter than in our own Galaxy. Another fact is the discovery of an eclipsing binary in Ursa Minor, a metal-poor dwarf elliptical galaxy in the Local Group (Webbink, 1980).

3. On the Origin of Binary and Multiple Systems

In this section we attempt to interpret the present statistical data in an effort to provide a framework for future observations related to binary statistics. Firstly, we introduce the angular momentum problem in star formation; secondly we will contrast four existing theories of binary formation with each other. Thirdly, we will consider the implications of present and future data for attempts to discriminate between the various binary formation theories.

3.1. The angular momentum problem in star formation

3.1.1. *An Example*

Stars are formed mainly, though perhaps not entirely, in molecular clouds. Molecular clouds tend to be clumpy on all scales that have been resolved up to now (see, for instance, Walmsley's 1982 good overview of molecular clouds and star formation). Clumps having masses of the order of a solar mass corresponding to typical sizes ~ 0.1 pc in which the ambient gas density is $\sim 10^{-19}$ g cm^{-3} seem to be quite common. Although those appear to be rather quiescent (in the sense that their molecular linewidths are nearly thermal, i.e. 0.1 km s^{-1} at typical temperatures ~ 10 K), we estimate that the specific angular momentum of such a clump – a fair fraction of the product of the linewidth times the size – amounts to as much as $\sim 10^{21}$ cm^2 s^{-1}, a factor 10^4 higher than the specific angular momentum of the solar system. If this clump is to form a binary system with two $0.5 M_{\odot}$ components orbiting each other, and without losing angular momentum, their separation would have to be 5×10^3 AU, extremely wide indeed. The calculated separation is also in agreement with the expected shrinkage of a uniformly rotating sphere whose ratio r = rot. energy/grav. energy in our case is $\approx 6\%$ (shrinkage $= 3r$). The example that we have chosen here is realistic enough to demonstrate the angular momentum problem in star formation. Asking how it might be possible to obtain a binary system with a smaller separation from the above initial conditions is asking how to solve or to circumvent the angular momentum problem. The former implies transport of angular momentum (either local or global transport) while the clump contracts, the latter implies segregation of angular momentum, i.e. the specific angular momentum of each fluid element is conserved (at least up to the stage of fragmentation) yet only the low angular momentum material near the rotation axis finally ends up in a pair of protostars. Thus depending on transport or conservation of angular momentum the outcome of the collapse may be a binary with component masses $\sim 0.5 M_\odot$ or $\sim 0.1 M_\odot$ each. In either case, conversion of spin to orbital angular momentum is required to form a binary. Moreover, even after conversion, the spin of the fragments orbiting each other may still be too large to allow for further dynamical collapse of the fragments towards the high density which is characteristic for the stellar interior (~ 1 g cm^{-3}). Hence, the fragments would remain fragments and could not become stars unless again transport of angular momentum is operative. Thus, strictly speaking, transport of angular momentum is absolutely essential for star formation, even for binary stars for which much of the angular momentum is stored in orbital motion. Hierarchical fragmentation without any transport of angular momentum would lead to substellar masses (Mestel, 1965; von Hoerner, 1968).

3.1.2. *The Collapse of a Rotating Cloud – General Results*

Let us follow the collapse of a rotating cloud in more detail in order to get an idea of how a binary system may actually be forming. Although fragmentation is fundamentally a three-dimensional problem, many of the results that we have come from two dimensional, i.e. axisymmetric numerical collapse calculations (for a recent review of numerical collapse calculations with emphasis on the effects of rotation see Bodenheimer (1981)). We shall distinguish between calculations based on conservation and transport of angular momentum, respectively.

(i) *Conservation of Angular Momentum*

Provided that the distribution of specific angular momentum is normal, people now agree that the axisymmetric collapse of a rotating cloud results in a toroidal density maximum ('ring structure') around the cloud center rather than in a central condensation (e.g. Tscharnuter, 1980). The reality of this feature was a matter of strong debate in the past. It was believed that artificial angular momentum transport towards the cloud center (caused by the numerical method) could be the reason for the appearance of the ring. Meanwhile comparative studies of different numerical codes and a higher order numerical scheme have confirmed the physical nature of the ring; in addition, convincing analytical arguments have been given (Tohline, 1980; Norman, 1980; Boss, 1980) showing that ring formation results from the dynamical competition between centrifugal and gravitational forces: while the collapse proceeds the fluid elements near the cloud center overshoot the centrifugal barrier and rebound in the radial direction colliding with the still infalling outer layers. This excites a toroidal density wave (cf. Bodenheimer, 1981, p. 24). The initial distribution of specific angular momentum within the cloud as well as the initial cloud density profile affect the position and the size of the ring (Tohline, 1980). Once the ring is formed, not only does it accrete mass but it also accretes angular momentum. Therefore the ring-like density maximum moves outward away from the rotation axis (the ring diameter grows). When the ring mass grows, the gravitational potential minimum moves into the ring, and the ring approaches a stage of hydrostatic gravo-centrifugal equilibrium (cf. Ostriker, 1964). At that point the ring mass (M), the ring diameter (D), and the specific angular momentum of the ring (J/M) are related approximately as $D \propto M$ and $(J/M) \propto M$.

As more mass is added to the ring, the ring may start collapsing on itself. With non-axisymmetric azimuthal density perturbations imposed on the ring ($\cos(m\phi)$-perturbations, $m = 2, 3, 4, \ldots$), the ring will fragment so that two or more orbiting fragments will emerge (see Norman and Wilson, 1978, for isothermal rings; Cook and Harlow, 1978, for adiabatic rings; see also Lucy, 1981, whose method avoids the adoption of an initial perturbation mode). Since $(J/M) \propto M$ and M, the ring mass, is typically an order of magnitude less than the cloud mass, only 10% of the original specific spin angular momentum of the cloud show up in the orbital plus spin angular momentum of the fragments (the spin of each fragment is in turn of the order of 10% of the orbital angular momentum). If a binary system is formed, the mass ratio could be $q = 1$ but there is no a priori reason against a smaller mass ratio (Lucy, 1981). If a triple system is formed (see Boss, 1982), the final configuration might be a binary with the lightest member being ejected. In this case we expect the mass ratio to be closer to unity.

(ii) *Transport of Angular Momentum*

– *Local transport.* Turbulent friction has long been known to cause redistribution of angular momentum in rotating disks (von Weizsäcker, 1948; Lüst, 1952; see also Lynden-Bell and Pringle, 1974). Angular momentum flows to the outer parts of the disk and this enables mass to flow to the inner parts and to form a central condensation. In recent times, axisymmetric 2D numerical collapse calculations including turbulent friction (Regev and Shaviv, 1981; Tscharnuter, 1981) have shown that even small amounts of turbulent friction prevent toroidal structures from forming, but lead to a central stellar object surrounded by a disk in approximately centrifugal equilibrium. The bulge-to-disk mass ratio after the dynamical phase depends on the efficiency of the redistribution of angular momentum. In 2D, most of the disk gas will be accreted onto the single, central object but, in 3D, the possibility of disk fragmentation remains, especially for cold disks (Quirk, 1973; Genkin and Safronov, 1975; Schmitz, 1983). For dynamical and geometrical reasons it appears likely that the mass ratio in binaries resulting from disk fragmentation would tend to be rather small, although this is only a guess which calls for confirmation by detailed calculations. In 3D, gravitational torques from a central bar-like or triaxial structure (Wood, 1981) or from spiral density waves (Larson, 1983) may allow much of the mass to fall to the center, and the formation of binary or multiple systems may be due to independent condensation of the components, similar to what Larson (1978) finds in his numerical calculations of cloud fragmentation.

– *Global transport.* The magnetic field permeating a rotating fragment can carry a substantial amount of the fragment's angular momentum to the external cloud medium, provided that the magnetic field in the fragment is connected with the external medium (see Mestel, 1965). The reason for such efficient 'magnetic braking' lies in the fact that the moment of inertia of the external medium is large. Matter in the external medium near the fragment is pulled by the field lines in the direction of motion of the fragment;

a torque is imparted to it at the expense of the rotational motion of the fragment which therefore most rotate slower than before (Mouschovias, 1981; review paper). In the case of magnetic field lines perpendicular to the axis of rotation the braking is more efficient (typically by an order of magnitude) than for the aligned rotator where magnetic field and rotation vectors are parallel to each other. The perpendicular case can even result in retrograde rotation of the fragment (Mouschovias and Paleologou, 1980; Dorfi, 1982). Ambipolar diffusion, in other words the gradual separation between the ionised matter component of the fragment and its neutral matter component at high gas density (Black and Scott, 1982) ultimately limits the efficiency of this angular momentum transport mechanism. For a discussion of the time scales of ambipolar diffusion and of magnetic braking (more or less of the order of the free-fall time) we refer again to Mouschovias (1981). In summary, as stated by Mouschovias and Paleologou (1980), the magnetic field seems to remain frozen in the matter long enough to resolve much of the angular momentum problem, and decouples from the matter rapidly enough for solar-type stars to form within about 3×10^7 yr, as evidently required by observations of the spatial separation of young stars and the apparent location of a spiral shock wave.

3.2. Theories of the Formation of Binary Stars

There are four rival theories of the formation of binary stars in the literature which, in turn, may be subdivided into two groups roughly related to the observational classification into spectroscopic binaries (SB) and visual binaries (VB). These basic binary formation mechanisms are displayed in Table V.

TABLE V

The 4 basic binary formation mechanisms

SB (close pairs)	VB (wide pairs)
Fragmentation	Capture (indep. condens.)
Fission	Disintegration (small clusters)

The physical difference between fragmentation and fission is explained in the following way (cf. Lucy, 1981): Fragmentation results from the break-up of a rotating protostellar cloud into two or more pieces during or immediately following a phase of dynamical collapse. Fission results from the bifurcation of a rotating protostar during its quasi-static pre-Main-Sequence Kelvin–Helmholtz contraction, if the ratio of the rotational to the gravitational energy density exceeds 0.25 (dynamical instability) or 0.14 (secular instability) according to Ostriker and Bodenheimer (1973).

– *Fragmentation.* The current idea of fragmentation differs from the former picture in which fragments spontaneously appear when their masses are greater than the local Jeans mass (Hoyle, 1953; Hunter, 1962). Density perturbations can, in fact, damp initially due to pressure effects, unless the perturbation amplitude is high enough. After about one initial free-fall time, when the cloud collapse is slowed down by pressure effects parallel to the rotation axis and primarily rotational effects perpendicular to the axis, the fragmentation begins. Fragmentation can occur either directly as a consequence of the initial perturbation imposed on the cloud, or through an intermediate ring stage depending on the thermal energy of the cloud measured in units of the gravitational energy. The dominant mode of fragmentation seems to be the binary mode but in many calculations this mode is imposed on the cloud from the beginning. A low thermal energy favors the formation of multiple systems. A comprehensive summary of all these results about fragmentation of an isothermal cloud is given in a paper by Bodenheimer *et al.* (1980). The properties of the fragments in the isothermal case are such that they are unstable to further collapse. The fragments form in the innermost part of the cloud which has a lower angular momentum per unit mass than the average for the cloud. This effect, combined with the conversion of spin to orbital motion, results in a reduction of spin angular momentum per unit mass by a factor 10 to 20 from that of the initial cloud. Thus, after a series of several collapses and fragmentations the specific angular momentum as well as the fragment masses can be reduced by considerable factors. Bodenheimer (1978) extrapolated that such a hierarchical process could result in direct evolution from a massive interstellar cloud to Main-Sequence binary and multiple systems within the observed range of masses* and orbital angular momenta.

Larson (1978) approached the fragmentation problem with a different numerical method, simulating hydrodynamics by a coarse fluid particle code. This method implies efficient transport of angular momentum, and fragmentation turns out to be quite direct rather than hierarchical with several steps. Multiple systems or small star clusters seem to be the outcome for clouds comprising several Jeans masses (i.e. clouds with low thermal energy at the onset of their collapse). It is also noteworthy that the fraction of cloud mass ending up in protostellar objects is higher by far in Larson's scheme compared with Bodenheimer's which leads to a fraction less than 1%.

– *Fission.* The outcome of numerical fission calculations for optically thick, rotating protostars is currently uncertain. Following the pioneering 3D numerical approach to the fission problem (Lucy, 1977, for apolytropic index 0.5 and a uniformly rotating initial model), further similar 3D numerical experiments of differentially rotating polytropes (Gingold and Monaghan, 1978, for polytropic indices 0.5 and 1.5) have been performed. The results of these calculations were diverse: fission accompanied by mass shedding (40%), fission without mass loss, and no fission but mass shedding (25%), respectively, were found. According to Lucy (1977) and Lucy and Ricco (1979) fission results from the instability of the third harmonic of an elongated triaxial rotating ellipsoid yielding a mass ratio of the components of about one third. In contrast with this result, Hachisu and Eriguchi (1982) obtained fission with mass ratio unity for the case of incompressible polytropes (i.e. zero polytropic index). They investigated the dumbbell- or bone-shaped equilibrium sequence of rigidly rotating polytropes. Durison and Tohline (1981) also

* In fact, such a *multiplicative* star formation process is able to explain the shape and the dispersion of the Miller/Scalo-IMF (Zinnecker, 1981).

studied the fission hypothesis for rotating polytropes of polytropic index 1.5 whose angular momentum distribution was that of a rigidly rotating uniform sphere. They imposed density perturbations which evolved into a dumbbell configuration for very rapid rotation. This configuration develops inside corotation while outside corotation material is ejected in the form of two spiral arms which wrap due to differential rotation, emerge into a detached disk, and eventually narrow into a radially expanding ring. The ring contains 16% of the mass but more than half the angular momentum. A similar process had been suggested earlier by Drobyshevski (1974). His idea was as follows: After collapse, convection is set up quickly in the protostar. Convection will make rotation uniform, even if it was initially differential, so that the balance between centrifugal and gravitational forces in the outer layers will be destroyed. These layers will then be thrown off forming a ring around the star. The mass loss results in an incrase of the convective zone in extent so that transfer of matter to the ring will proceed until all the convective envelope is lost. The ring is unstable and will probably form a second component. The mass ratio of the new system is therefore determined by the mass ratio of the convective zone and the stable core of the protostar. However, irrespective of the initial mechanism of separation of the components, equalisation of their masses may occur in the course of subsequent disc accretion of matter onto the system (cf. McCrea, 1956).

Finally, concerning fission, we would like to refer to an earlier review paper of Ostriker (1970).

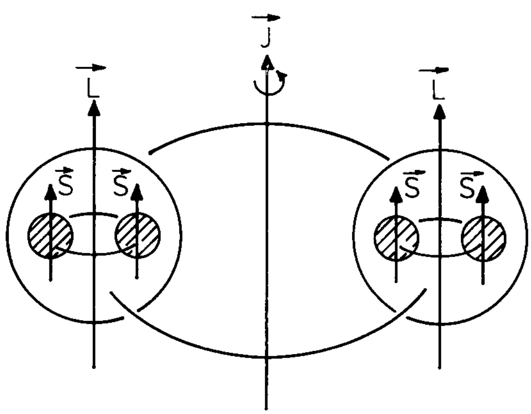

Fig. 4a. Bodenheimer's scheme of hierarchical ring fragmentation.

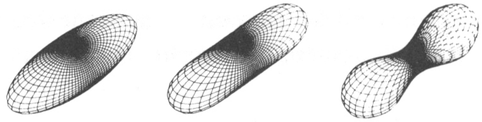

Fig. 4b. Dumbell-equilibrium sequence and fission of a protostar (from Hachisu and Eriguchi, 1982).

The fragmentation and the fission process are illustrated in Figures 4a and 4b, respectively. After such a long discussion of fragmentation and fission theory (warranted because it is the close binaries whose origin is the big issue) the two theories for the origin of wide binaries (capture theory and cluster disintegration theory) will be dealt with more briefly.

– *Capture.* Capture theory assumes stars to originate from single independent nearby condensations which subsequently get bound into a wide binary when fragments (protostellar cores with extended accretion envelopes) collide with non-zero impact parameters (greater than the sum of the core sizes), and viscous interaction dissipates the kinetic energy of the relative motion of the fragments (Silk, 1978; Silk and Takahashi, 1979). Capture theory implies random pairing in a protocluster cloud. The chances of capture through three-body encounters of galactic field stars are very small, although this kind of capture does occur in dense star clusters like globular clusters. Random pairing during star formation according to a stellar mass spectrum increasing towards low masses implies that the distribution of the mass ratio increases towards low mass ratios. A Salpeter-type mass spectrum would, however, be too steep at the low mass end to be consistent with the distribution of the mass ratio given in Figure 2 (Warner, 1962a, b). A Miller/Scalo-type mass spectrum which increases only moderately towards lower masses is likely to be consistent (cf. Abt, 1978). The distribution of orbital periods expected from capture theory may explain the observed period distribution for periods exceeding 100 yr (see Silk, 1978). An intriguing test of the capture theory for wide binaries would be to observe the spin vectors of the binary component (via the method proposed in Strittmatter, 1981). If the spin vectors are poorly or even randomly orientated to each other, this might be taken as evidence in favor of capture theory. Similarly the absence of any correlation between spin and orbital angular momentum would support capture theory.

– *Cluster disintegration.* Cluster disintegration theory assumes stars to form in small bound clusters (see e.g. Figure 4 in Herbig, 1977) which subsequently lose most of their members by close two-body encounters leaving behind a wide binary which absorbs and carries the initial binding energy of the whole cluster (van Albada, 1968a, b). The left over binary most likely consists of the two heaviest members of the cluster, so the statistical distribution of the mass ratio should increase towards unity if this case prevails. If the cluster initial had n stellar members with mean separation $\bar{a}$ (0.01 pc), then, for equal masses, from the principle of energy conservation we have $\bar{a} = a_{**}n(n-1)/2$, a_{**} being the separation of the remaining binary (e.g. $a_{**} = 0.1\bar{a}$ for $n = 5$ or $a_{**} = 0.02\bar{a}$ for $n = 11$). Since during the condensation of the cluster stars neighboring protostars had a larger separation, tidal disruption during the formation stage was less of a problem than it would be if the binary components formed with the closer separation right away (Kumar, 1972). The disadvantage with the cluster disintegration theory is that it is not a very prolific process of binary formation. However, cluster disintegration can obviously account for the formation of multiple systems (Aarseth, 1977) including Trapezium-type systems (Allen and Poveda, 1974).

3.3. Implications from Binary Statistics

3.3.1. *Mass Ratios*

The all important question is whether the true frequency distribution of the mass ratio in close binaries increases or decreases towards small mass ratios. A steady increase towards small mass ratios would neither be consistent with fragmentation theory nor with fission theory. These theories predict component masses of comparable size. While ring fragmentation might prefer $q = 1$ for $m = 2$ perturbations, ring fragmentation cannot be said to lead to $q = 1$ for the most general perturbations (Lucy, 1981). On the other hand, if ring fragmentation leads to $q \neq 1$, subsequent disk accretion onto the binary may equalise the component masses. The same is true for fission if it results in $q \neq 1$ in the first place. Mass exchange during the contact phase might change the fission mass ratio to $q = 1$ (Lucy, 1977). However, the fission process itself has not been studied extensively enough to claim that it does not produce $q = 1$ in many cases. The conclusion from these considerations is that the present theory for the formation of close binaries is in trouble if future observations do indeed reveal that the frequency of the mass ratio in close binaries is not peaked near $q = 1$ but increases for $q \to 0$. In that case the dominant formation process could be the fragmentation of a gaseous disk surrounding the central condensation. As far as the mass ratio in wide binaries is concerned, capture theory better accounts for the observed frequency of the mass ratio than does cluster disintegration theory. This frequency increases towards small mass ratios, while cluster disintegration theory would predict the opposite trend. This, however, does not render cluster disintegration theory obsolete. The existence of very wide binaries, triple systems with a distant faint third body, and Trapezium systems may well require the disintegration process.

3.3.2. *Orbital Periods*

The crucial question here is whether a single process of star formation can explain the broad unimodal (roughly log-normal) frequency distribution. The wide spread in binary periods or equivalently in orbital angular momentum per unit mass could either be attributed to various mechanisms that change the separation after the formation of binaries by a single process (Kuiper, 1955) or to several formation processes (like those discussed previously in Section 3.2). (An incisive discussion of this problem is given by Huang, 1977.) Moreover, the wide spread of binary orbital angular momentum could have been imposed on the protostellar clouds before cloud collapse and fragmentation into a binary got underway, for example by the process of magnetic braking (Mouschovias, 1977), by the processes of hydrodynamical turbulence (Woolfson, 1978; Larson, 1981), or due to cloud-cloud collisions (Horedt, 1982).

Hierarchical fragmentation à la Bodenheimer (1978) can, in principle, serve as a mechanism to explain the distribution of orbital angular momentum if the number of steps in the hierarchy follow a statistical distribution centered on a most likely number of steps equal to 3 within a range 3 ± 2. We recall that each step corresponds to an order of magnitude in orbital angular momentum per unit mass (see the fragmentation table in Bodenheimer, 1978). The problem is how a very wide binary having gone through only 1 or 2 steps can achieve its very low ratio of spin to orbital angular momentum, while for very close binaries the same ratio is relatively high. This shows that for wide binaries additional processes must be operating to brake the spin of the two subcondensations. Magnetic braking is a good candidate process, since it is efficient primarily during the earlier stages of a contracting protostellar cloud. However, at every step in the hierarchy there is a chance to redistribute angular momentum, and that is why we have invoked a statistical distribution in the number of steps (if there is no redistribution at any step, the hierarchical fragmentation requires the full number of five steps as in Bodenheimer (1978)).

The observed period ratios between the long and the short period in hierarchical quadruple systems (of the order of 1000) is naturally explained in terms of two steps of the hierarchical ring fragmentation scheme (see again the fragmentation table in Bodenheimer, 1978), although hierarchical quadruple systems could also result from the dynamical decay of Trapezium-type quadruple systems. Trapezium-type quadruple systems themselves may either be the result of cluster disintegration (see above) or fragmentation through a $m = 4$ perturbation for very cold rings (Norman and Wilson, 1978; Rozyczka *et al.*, 1980). The coplanar orbits in some of the observed systems allow us to conclude that rotational hierarchical fragmentation does in fact occur, rarely perhaps, but at least in some favorable cases.

4. Conclusions and Suggestions

4.1. Conclusions

(1) Various binary formation mechanisms exist. These include:
  (a) fragmentation of a collapsing protostellar cloud;
  (b) fission during pre-Main Sequence contraction;
  (c) capture after independent condensation;
  (d) disintegration of small star clusters.

(2) The initial frequency distribution of the mass ratio, once it is established from observations free from selection effects, contains the outstanding information about the dominant binary formation mechanism(s). If largely unequal component masses where the rule rather than the exception for close binaries (an unpopular view at present), a new additional formation mechanism would be required ('the fifth mechanism'; possibly the fragmentation of a gaseous disk surrounding a centrally condensed protostar).

(3) The four basic mechanisms (a) through (d) are able to provide complementary coverage of a wide range of binary separations. The multimodal origin of binaries coupled with a mixture of initial conditions and other processes such as transport of angular momentum is likely to account for the broad, unimodal frequency distribution of the orbital periods. The period ratio P(long)/P(short) in hierarchical

quadruple systems is well explained by hierarchical ring fragmentation during the rotating collapse.

(4) Transport of angular momentum is vital not only for single star formation but also for the formation of binary stars. Magnetic braking is an efficient transport mechanism up to a moderately high gas density.

(5) Binary statistics in open clusters is probably not different from that of Pop. I field stars, implying that star formation processes are not altogether different in both categories. The similar statistics may also imply that single stars are not predominantly the ejecta of unstable triple systems.

(6) More and more Pop. II field stars (high velocity subdwarfs) are now found to be close binaries, challenging previously reported results. Selection effects may operate against the detection of eclipsing binaries in globular clusters. Thus it appears premature to claim that Pop. II forms a low angular momentum population.

4.2. Suggestions

(a) *Concerning theory:*

(1) Develop a fluid particle code with $N > 10^4$ particles for a Cray-1 computer to simulate the 3D hydrodynamics of fission and fragmentation starting with random initial conditions.

(2) Try to model $q = 1$ ring fragmentation analytically. Calculate how the specific ring angular momentum is split into orbital and spin angular momentum. Check whether the fragments so formed rotate slowly enough to undergo substantial contraction.

(b) *Concerning observation:*

(1) Free the SB-catalogues from eclipsing variables and evaluate the data anew.

(2) Check whether newer data reduce the dispersion around the $q = 1$ peak of the frequency distribution of the mass ratio in Lucy and Ricco's work on SB2's.

(3) Check the SB2-data with respect to a possible correlation between the mass ratio and the period for a given total mass of the system (see Section 2.2.3).

(4) Make maximum use of the exciting possibilities of speckle interferometry in observational binary star research. Discover nearby ($\lesssim 100$ pc) double stars down to separations of about 0.025 arc sec (the diffraction limit of a 4 m optical telescope); this would correspond to linear separations less than 2.5 AU, i.e. to close binaries (visual rather than spectroscopic). By measuring the intensity ratio of the components at several wavelengths it may be possible to infer the mass ratio. In that way we could approach the real frequency distribution of the mass ratio for close binaries.

The speckle method should also be used to study the separations in the difficult range from 1 to 40 AU for a well-defined sample such as the members of nearby open star clusters (e.g. the Hyades and the Pleiades).

(c) *The future in binary star research:*

It is our belief that the future in binary star research belongs to speckle observations (optical and infrared), although competitive developments are also in progress (better radial velocity spectrometers, fuller exploitation of lunar occultations). Since the speckle technique can in principle distinguish between binary stars and single stars surrounded by circumstellar disks, the improvement of this technique may ultimately result in an understanding of the bifurcation between the formation of a binary system and the formation of a planetary system associated with a single star. Meanwhile the answer to that problem is up to theory.

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Appendix A: The Detection of Binary Systems

(a) *Visual binaries*

Visual binaries typically have mean separations of 3–5 arc sec depending on luminosity. These separations translate into semi-major axes ranging from 5 AU (for KM dwarf pairs) up to 100 AU (for OB star pairs). The difference arises, because the intrinsically fainter binaries are statistically less distant from the Sun than the intrinsically bright binaries. The recent development and application of the speckle interferometric technique represent a major break-through in visual binary star research (see McAllister, 1977; Morgan *et al.*, 1978; Bonneau *et al.*, 1980; Bonneau and Foy, 1980). An angular resolution of the order of $0\overset{''}{.}1$ (or even less) can be achieved and the data are just about to incrase by a great deal (McAllister *et al.*, 1983). Interestingly enough, the speckle masking method (Weigelt and Wirnitzer, 1983) is capable of measuring the intensity ratio of SB components to an accuracy of 25% (at 5000 Å).

(b) *Spectroscopic binaries*

The threshold in the projected radial velocities of the orbital motion about the center of mass of a binary system for detection of SB1's is typically 3 km s^{-1} (or somewhat less), whereas that for detection of SB2's is typically 10 km s^{-1} (or somewhat more). These numbers hold for intermediate spectral types (A0–K0). For very early spectral types (OB) and very late spectral types (KM) the velocity threshold is higher. For

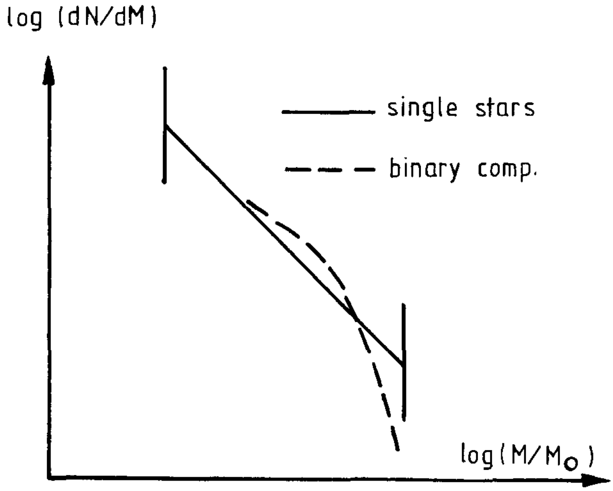

Fig. 5. Model stellar mass spectrum ($M_{min} \leq M \leq M_{max}$) illustrating the effect of the presence of binary systems.

OB-binaries this is due to rotational broadening of the lines caused by the rapid rotation of the individual components about their spin axes; for KM-binaries it is due to the appearance of molecular bands.

By virtue of Kepler's 3rd law the above thresholds for intermediate spectral types correspond to semi-major axes typically less than 1 AU for SB2's and typically less than 10 AU for SB1's. As a rule of thumb, SB1's are binary systems in which the brightness difference between the two components is more than 1 mag while it is less than 1 mag for SB2's. It is important to keep this in mind.

Appendix B: The Problem of Correcting the IMF for Unresolved Binary Systems

In the definition of the Initial Mass Function (IMF) it is implicitly assumed that the fraction of stars which are double is independent of mass ('random pairing'). Otherwise one would have to distinguish between the IMF of single stars (IMF_0) and the IMF of (1) the primaries of binary stars (IMF_1) and (2) the secondaries of binary stars (IMF_2). This problem which is usually ignored (Hartmann, 1970) may be investigated qualitatively by a simple thought experiment: Suppose there would be only single stars with a mass distribution which we may denote IMF_0' (e.g. a power law). Now, let a fraction of single stars split into binary stars with the mass of the primary component being M_1 and the mass of the secondary component being M_2 ($M_0 = M_1 + M_2$). If there is preferential splitting of the higher mass stars (a likely situation, because these are faster rotators, as is known), the effect will be a depletion of the high mass end of the total IMF ($IMF_{tot} = IMF_0 + IMF_1 + IMF_2$) compared to IMF_0', i.e. a steepening.

By mass conservation the intermediate masses will be more frequently populated in the distribution function IMF_{tot} (see Figure 5). We suggest to call IMF_{tot} the true IMF.

Since the observed IMF should correspond to a superposition of IMF_0 and IMF_1 ($IMF_{obs} = IMF_0 + IMF_1$) it is obvious that the observed IMF is not really the true IMF, since it is not corrected for the fainter unresolved companions of the primaries of binary systems. We conclude that the true IMF is somewhat steeper at the high mass end than the observed IMF, if the binary fraction is larger for the higher mass stars (cf. Vanbeveren, 1982).

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