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SAUL KRIPKE (1940–2022)

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Abstract. Saul Aaron Kripke, the most influential philosopher and logician of his generation, died on September 15, 2022, at the age of 81.

§**1. Introduction.** Saul Aaron Kripke, the most influential philosopher and logician of his generation, died on September 15, 2022, at the age of 81.

Kripke's greatest intellectual contributions were made in logic and logic-related philosophy. In the area of logic, he produced fundamental works on the semantics of modal and intuitionistic logic and the theory of truth.

While still in high school, Saul wrote the paper, *A completeness theorem in modal logic* [K59a], with a detailed exposition in [K63b] and other publications. This work introduced Kripke Semantics, which remains to this day standard in the area of modal logic, nonclassical logics, and their ubiquitous applications.

Kripke's theory of truth, presented in *Outline of a theory of truth* [K76], opened one of the most popular avenues of research in logic and philosophy. In this work, Kripke uses a Knaster–Tarski-style fixed point in a most elegant way.

At different periods, Kripke taught at Harvard, MIT, Rockefeller University, Princeton, and, from 2003, at the City University of New York. He was a recipient of the Rolf Schock Prize in logic and philosophy awarded by the Royal Swedish Academy of Sciences, and a number of honorary degrees. His only formal academic qualification was a BS degree in mathematics from Harvard.

§**2. Modal and intuitionistic logic.** Around the middle of the twentieth century, Kripke revolutionized modal logic which, while an ancient subject, was a backwater when he started out. Quine argued that the very subject was a mistake. Even apart from Quine's objections, modal logic lacked an intuitive, usable semantics. Work was axiomatic, or sometimes algebraic. In

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a series of papers [K59a, K62a, K63a, K63b, K63c, K65] and a remarkable abstract [K59b], Kripke irrevocably changed the field, with possible world semantics, now commonly called Kripke models. Others had various parts of the overall idea, but it was Kripke who saw it all clearly, in its natural simplicity.

The idea of possible worlds goes back to Leibniz. The actual world is chosen from those possible, by God, because it is the best. *Possible world* terminology was available, appealing, and suggestive. Here, briefly, is Kripke's idea. A model has a set of possible worlds, which are just bare points. Each possible world has an assignment of truth values to propositional variables, independent from world to world. Central is a binary relation between possible worlds, commonly read as *accessibility*. For instance, if worlds are time instants, possible tomorrows are accessible from today, but not the other way around. Concrete examples are many, but mathematically a model is simply a directed graph with valuation functions on nodes.

Using a propositional language with modal operators, truth values at each possible world are calculable. Boolean behavior is standard. Most importantly, a formula expressing the necessity of X is true at a possible world provided X is true at every possible world accessible from that world. Loosely, to be necessary is to be true at all *accessible* possible worlds.

When Kripke began, there were a number of known axiomatic modal logics. For many, he showed completeness results of the form: formula X is provable if and only if X is true at every possible world of each of his models for which the accessibility relation meets some specified mathematical conditions, such as reflexivity, transitivity, symmetry, and so on. This linked modal logics to common mathematical structures with simple properties. People now had familiar structures to work with.

Kripke extended his ideas to quantification, showing axiomatic conditions then being debated also corresponded to natural structural conditions on his possible world models. For instance, a move from a possible world to an accessible one must never decrease (never increase, never change) the domain of quantification. Kripke also introduced the first *nonstandard* possible world, one in which truth does not follow his usual evaluation rules for necessity and possibility. This provided a semantics for several common modal logics that were not covered by his previous models. Today nonstandard worlds are common tools.

After Kripke's initial string of papers, he wrote no more on modal semantics. Others provided simpler proofs of his completeness theorems and extended his ideas to logics beyond those his proof methods handled. Modal model theory became an interesting field in its own right. As we noted, basically a Kripke model is a labeled directed graph, and these are common mathematical objects.

Kripke's ideas applied to many areas beyond philosophy. Computer science is one. For instance, take possible worlds to be machine states, and accessibility to represent the execution of a given nondeterministic program. Kripke machinery enables proof of results about program behavior. Or again, we might have agents, each with different information. Agent knowledge can be represented using multiple accessibility relations on states of knowledge, one for each agent. One can study how models change as agent information changes, for instance, through public announcements. Briefly, Kripke-style models turn up in computer science, information theory, economics, linguistics, and so on. Simplicity makes them applicable tools; power makes them useful ones.

Though Kripke published nothing further about modal logic after 1965, there were two descendants of significance, one technical, the other philosophical.

Intuitionistic logic was formalized by Heyting in 1930, and classical logicians were deeply interested. Two semantics had been developed, both provably complete using classical mathematical arguments. One was algebraic, and the other, due to Beth, involved tree structures that bore resemblance to Kripke's modal models. The Beth version, while providing more intuition than the algebraic, almost always produced infinite trees and so was awkward and complicated.

Gödel established a connection between intuitionistic logic and modal logic in 1933. Kripke used that connection to create a new semantics for intuitionistic logic, building on his possible world models [K63a]. In this, it is natural to think of the nodes, not as worlds but as possible stages of mathematical exploration. Models can often be finite; indeed the invalidity of excluded middle requires just two states. Kripke intuitionistic models are now a standard tool in the classical exploration of intuitionistic logic and its possible extensions. They may have served as partial motivation for the creation of the Routley–Meyer semantics for relevance logics. Today possible world-style semantics is widespread in the nonclassical logic community.

For philosophers, Kripke is probably most famous for his book *Naming and Necessity* [K80]. This was not written, but spoken as a series of three lectures and later published as [K72]. Without containing a single formula, *Naming and Necessity* is nonetheless clearly based on possible world semantics, which is now central to how philosophers organize their thoughts and their discourse.

§**3. Truth.** Kripke's one publication [K76] on truth and paradox greatly influenced work on the topic despite being only an outline of a lecture. Like Tarski, Kripke concentrates on truth for sentences, not propositions, taking possibility of self-referential sentences to be established beyond question by Gödel. Tarski envisioned mathematical uses for truth when it was in disrepute owing to paradoxes, and sought to rehabilitate the notion. But, for his purposes, only truth predicate T_0 for a language L_0 not itself containing a truth predicate was needed. One can expand L_0 by adding the predicate T_0, obtaining a language L_1, but then a new truth predicate T_1 will be required, and a hierarchy results. Kripke notes that it is tricky to extend the hierarchy into the transfinite and that in natural language we speak of truth without explicit indices. He points to a difficulty even with *implicit* indices in his famous examples where Nixon says 'Everything Dean says about Watergate is true' and Dean says 'Everything Nixon says about Watergate is false'. The highest index of each speaker will have to be greater than that of the other—an impossibility. And whether a paradox arises will depend on empirical facts (about whether everything *else* Dean or Nixon says is true or is false) and cannot be computed from the syntax of the sentences involved.

But if Nixon says 'the sun is shining' when it is raining, that is false, and then if Dean says 'something Nixon says about the weather is false', that is true, and on. Such examples motivate a formal proposal for a language containing a truth predicate T allowed to be *partial*, with T('A') sometimes neither true nor false. Starting from an initial partial assignment of truth values to sentences of form T('A'), one can compute truth values for some other sentences including logical compounds, sentences of form T('T('A')'), and more. Among different schemes for extending assignments of truth values, Kripke focuses on the '3-valued' approach of Kleene and the 'supervaluation' approach of van Fraassen. A common feature of these is *monotonicity*: if S and T are two partial assignments of truth values and S is included in T (i.e., everything given a value by S is given the same value by T), and if S^* and T^* are the extensions of S and T, respectively, according to the scheme, then S^* is included in T^*. Iterated application through transfinite ordinals will by general results on inductive definitions (v. [Mo74]) lead to a *fixed point* after which no further extension occurs. Kripke concentrates on the *minimum* fixed point, though there exist others (and Martin and Woodruff [MW75] had already shown the existence of *maximal* fixed points).

A sentence A getting a value in the minimum fixed point is one that is 'grounded': in checking what the truth-value of A depends on (as that of 'What Nixon said an hour ago is false' depends on that of what Nixon said, which if that was 'What Dean said just now is true' in turn depends on the truth-value what Dean said, and so on), the process eventually bottoms out with sentences without the truth predicate. There are sentences like the *truth-teller* 'this very sentence is true' that are ungrounded but unlike the *liar* 'this very sentence is false' are not paradoxical. Paradoxical sentences have truth values in *no* fixed points; truth-teller sentences are true in some fixed points but not others. There are also ungrounded sentences that have no value in the minimal fixed point but are true in others and false in none. Kripke defines a sentence to be *intrinsically* true if it is true in some fixed point that values no sentence true that is valued false in any other fixed point, and shows that there is a *maximum* intrinsic fixed point. We have now four fixed points of special interest: minimum *vs.* maximum intrinsic and Kleene *vs.* van Fraassen. And there are more.

A theorist who thought the minimum fixed point on the Kleene was a good model of our prereflective notion of truth might be tempted to say of a liar A that *A is not true*. But on the theory as described so far, $\neg\mathrm{T}(`A')$ would have no truth-value in the minimum fixed point either, so what the theorist is tempted to say could only be asserted from a postreflective point of view. 'The ghost of the Tarski hierarchy is still with us', as Kripke puts it. In addition to the many authors who have pursued Kripke's ideas in multiple directions (e.g., new proofs of Gödel's theorem, axiomatic theories of truth, and calculations of complexity of fixed points), there was extensive but unpublished further work by Kripke himself, about which some information is provided second-hand by [Bu11].

§**4. Philosophy of language, mind, and metaphysics.** Through a handful of publications in philosophy of language, mind, and metaphysics, densely packed with keen arguments and insights, Kripke irrevocably changed the trajectory of these fields and had a profound impact on philosophy as a whole.

In his widely celebrated masterpiece, *Naming and Necessity* [K80], Kripke transformed our understanding of the semantics and meta-semantics of names and natural kind terms; the nature of and relations between metaphysical necessity and *a priori* knowledge; the intelligibility of *de re* modality; the relation between mind and body; and more. The influence of this book continues to be felt throughout philosophy, and beyond.

In [K80], Kripke argues forcefully against the once prevalent view that names have a descriptive meaning and refer to whatever it is that satisfies the associated description. In its place, Kripke puts forth a picture of how names get their meaning through causal-historical relations to the individuals they name ([K80], 96–97) and extends this account to natural kind terms, such as 'heat'. These contributions revolutionized our understanding of the semantics and meta-semantics of proper names and natural kind terms and have been influential across a broad swathe of the philosophical landscape.

One of Kripke's most important contributions is the observation that names and definite descriptions behave differently in modal contexts, since names designate rigidly—they denote the same object in all possible worlds in which the object exists—while definite descriptions are nonrigid, denoting different objects in different possible worlds ([K80], pp. 3–15). The thesis that names are rigid designators figures in Kripke's disarmingly intuitive and powerful defense of *de re* modality against Quinean skepticism about the intelligibility of the notion ([K80], pp. 41–47); his rehabilitation of the distinction between essential and accidental properties of an object; and his argument that we can evaluate a modal sentence such as 'Humphrey might have won the election' without having to locate Humphrey's counterpart in nonactual possible worlds ([K80], p. 45, fn. 13). Kripke's discussion of these issues, along with his logical and philosophical work on modality, was instrumental to the revival of metaphysics as a legitimate domain of inquiry, which fundamentally altered the course of late twentieth-century philosophy.

The distinction between rigid and nonrigid designators figures again in Kripke's revolutionary argument that some statements express truths that are both metaphysically necessary and knowable *a posteriori*, alongside the theorem of modal predicate logic: $(x)(y)(x = y \rightarrow \Box(x = y))$, which Kripke proves ([K71]), yet regards as self-evident ([K80], pp. 3–4). Kripke argues that if 'a' and 'b' are rigid designators, then if '$a = b$' expresses a truth, it expresses a necessary truth, which in some cases is known a posteriori. For instance, since the names 'Hesperus' and 'Phosphorus' both rigidly designate Venus, 'Hesperus is Phosphorus' expresses a necessary truth. Since it was an empirical discovery that Hesperus is Phosphorus, this truth is known *a posteriori*. This point can be extended to scientific identifications, such as that of heat and molecular motion. Kripke's powerful case for the necessary *a posteriori* opened the floodgates to *a posteriori* materialism across a range of areas, including the philosophy of mind and meta-ethics.

Though Kripke's arguments have been taken up by materialists, he expressed antipathy towards the view. In the last few pages of [K80], Kripke takes issue with the materialist thesis that mental states, such as pain, are identical to physical states, such as the brain state, B. By appealing to the necessity of identity, Kripke argues that if pain $= B$, then necessarily pain $= B$. He then argues that since pain just is a certain kind of sensation, and since it is possible for B to fail to be associated with that sensation, pain is not identical to B. In response to the rejoinder from one who claims that the identification of mental and physical states is necessary but *a posteriori*, Kripke argues that the analogy breaks down. In the case of the successful theoretical identification of heat and molecular motion, we fixed the reference of 'heat' by way of a contingent property of it: its giving rise to the sensation of heat. In contrast, the reference of 'pain' is fixed by an essential feature of it: its hurts. Kripke's discussion of these issues had a profound impact on debates about the mind–body problem.

In his highly acclaimed book, *Wittgenstein on Rules and Private Language* [K82], Kripke develops further objections to materialism. Here,

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Kripke asks what makes it the case that an arbitrary expression has the meaning that it does, rather than some other meaning or none at all. His objections, not least to materialism, revolved around the insight that meaning is both general and normative. Ultimately, Kripke finds no account to be satisfactory and draws the paradoxical conclusion that there is no fact of the matter what any speaker means by any word ([K82], p. 55). He offers a 'skeptical solution' to this paradox, which attempts to legitimize our ascriptions of meaning in the face of the skeptical conclusion—though this part seems more Wittgensteinian than Kripkean ([K82], p. ix). [K82] generated lively discussion of the interpretation of Wittgenstein; the normativity of meaning and content; the prospects for naturalizing meaning and content; and of providing deflationary or otherwise metaphysically lightweight accounts of semantic and intentional ascriptions. [K82] contains powerful objections to materialism about meaning and intentionality—objections which have hitherto never been met.

§**5. Mathematical logic.** While not as influential as his philosophical works, Saul Kripke's contributions in mathematical logic proper are fundamental.

Being still an undergraduate at Harvard, Saul wrote his first paper on traditional mathematical logic [K62b]. He offered an alternative proof of Gödel's First Incompleteness Theorem using what he called 'flexible' predicates in arithmetic, displaying an extraordinary 'tour de force' in mathematical logic. This technique was later developed further by other logicians, including [HL22].

Another of his earlier contributions was *Transfinite recursion on admissible ordinals* [K64] which—together with [P66]—led to Kripke–Platek set theory *KP*. To this day, *KP* plays a prominent role in the proof-theoretical analysis of higher mathematical principles and impredicativity. Its foundational importance stems from the fact that it is a meaningful intermediate theory between second-order arithmetic and set theories such as *ZF*. For users, *KP* is a theory of admissible sets and ordinals which play a major role in higher computability theory (cf. [S90]), and in descriptive set theory (cf. [Ba75]).

In *Deduction-preserving 'recursive isomorphisms' between theories* [PK67], Kripke and Pour-El showed that all Lindenbaum algebras for consistent recursively enumerable arithmetical theories are recursively isomorphic. This simple but fundamental observation became the basis of numerous further developments, e.g., the rich theory of provability algebras, cf. a comprehensive survey [Be05].

Kripke was also a versatile pioneer in less expected areas. In a short paper [K67], he used Boolean-valued forcing, new at that time, to settle a problem of Sikorsky: it is not the case that every complete Boolean algebra is isomorphic to a direct product of homogeneous complete Boolean algebras. In doing so, he illustrated that forcing was a more general tool than one might have suspected, with applications to 'straight' mathematics and not just to problems of logical foundations and set theory. In what became typical Kripke style, the paper was never really published, but only appeared in conference proceedings.

During his visit to Oxford in 1978, Kripke gave a lecture entitled, *A model-theoretic proof of Gödel incompleteness*. While he went on to present this lecture a number of times, he never published it. In this work, Kripke offered a highly original model-theoretic incompleteness proof based on a new notion of 'fulfillment'. This was a breakthrough which Kripke himself characterized in [KK82] as leading 'to model-theoretic proofs of many theorems (such as Gödel's and Rosser's theorems) usually proved proof-theoretically and to other applications of the model theory and proof theory of arithmetic'. These new ideas were used in the paper *Nonstandard models of Peano Arithmetic* [KK82] in which Kochen and Kripke provided an alternative proof of the Paris–Harrington theorem [PH77].

In his 2014 paper, *The Road to Gödel* [K14], which was first delivered as a lecture in 1999, Kripke put forth the general point that the First Gödel Incompleteness theorem is a natural and 'almost the inevitable result of a historic line of thought'. Indeed, Gödel's proof is a natural formal version of the fundamental liar paradox representable in first-order logic. Kripke also offered a generic nonconstructive incompleteness argument and then produced a new constructive incompleteness proof using Grelling's 'heterological paradox'.

Kripke's final mathematical logic contribution was *The collapse of the Hilbert program: variation on the Gödelian theme* [K22]. This paper was also based on a lecture, first given in 2007. An abstract of a version of the talk appeared in [K09]. In his paper [K22], Kripke examined the Hilbert Program, understood as a realization of the epsilon-substitution method for Peano Arithmetic *PA* and stronger theories. Kripke made the elegant and quite fundamental observation that the failure of this version of the Hilbert Program was apparent even without reference to Gödel's incompleteness theorems. A simple direct argument, using just Gödel numbering, suffices to show the impossibility of realization of the Hilbert epsilon-substitution effort.

§**6. Conclusion.** Saul Kripke left a monumental legacy of philosophical and logical work that has had a profound and expansive influence well beyond the confines of any one discipline. His genius combines common sense and clarity, audacity, and pursuit of core problems of the area. He is an inspiring model of excellence and of foundational thinking for generations to come.

Having laid the groundwork for a variety of flourishing research programs—and having set the agenda in core areas of philosophical and logical inquiry—his work will have a lasting influence.

Many of Kripke's contributions exist to this day only as lecture notes and recordings. The Saul Kripke Center at CUNY has been working to systematically convert these works into conventional publications.

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