

Contents

<i>Preface</i>	<i>page</i> xiii
1 Introduction	1
1.1 A Note on Notation	3
2 Quantum Mechanics and the Path Integral	4
2.1 Schrödinger Equation and Probability	4
2.2 Position and Momentum Eigenstates	4
2.3 Energy Eigenstates and Semi-Classical States	6
2.4 Time Evolution and Transition Amplitudes	7
2.5 The Euclidean Path Integral	8
3 The Symmetric Double Well	13
3.1 Classical Critical Points	13
3.2 Analysis of the Euclidean Path Integral	15
3.3 Tunnelling Amplitudes and the Instanton	16
3.4 The Instanton Contribution to the Path Integral	19
3.4.1 Translational Invariance Zero Mode	19
3.4.2 Multi-instanton Contribution	21
3.4.3 Two-dimensional Integral Paradigm	24
3.5 Evaluation of the Determinant	25
3.5.1 Calculation of the Free Determinant	29
3.5.2 Evaluation of K	31
3.6 Extracting the Lowest Energy Levels	35
3.7 Tunnelling in Periodic Potentials	36
4 Decay of a Meta-stable State	41
4.1 Decay Amplitude and Bounce Instantons	41
4.2 Calculating the Determinant	44
4.3 Negative Mode	44
4.4 Defining the Analytic Continuation	46
4.4.1 An Explicit Example	46

4.5	Extracting the Imaginary Part	50
4.5.1	A Little Complex Analysis	50
4.6	Analysis for the General Case	54
5	Quantum Field Theory and the Path Integral	59
5.1	Preliminaries	59
5.2	Canonical Quantization	61
5.2.1	Canonical Quantization of Particle Mechanics	61
5.2.2	Canonical Quantization of Fields	61
5.3	Quantization via the Path Integral	63
5.3.1	The Gaussian Functional Integral	64
5.3.2	The Propagator	67
5.3.3	Analytic Continuation to Euclidean Time	68
6	Decay of the False Vacuum	71
6.1	The Bounce Instanton Solution	72
6.2	The Thin-Wall Approximation	75
6.3	The Fluctuation Determinant	77
6.4	The Fate of the False Vacuum Continued	79
6.4.1	Minkowski Evolution After the Tunnelling	80
6.4.2	Energetics	83
6.5	Technical Details	84
6.5.1	Exactly One Negative Mode	84
6.5.2	Fluctuation Determinant and Renormalization	86
6.6	Gravitational Corrections: Coleman–De Luccia	90
6.6.1	Gravitational Bounce	92
6.7	Induced Vacuum Decay	100
6.7.1	Cosmic String Decay	100
6.7.2	Energetics and Dynamics of the Thin, False String	102
6.7.3	Instantons and the Bulge	104
6.7.4	Tunnelling Amplitude	107
7	Large Orders in Perturbation Theory	111
7.1	Generalities	111
7.2	Particle Mechanics	112
7.3	Generalization to Field Theory	117
7.4	Instantons and Quantum Spin Tunnelling	118
7.5	Spin-Coherent States and the Path Integral for Spin Systems	118
7.6	Coordinate-Independent Formalism	121
7.6.1	Coordinate-Dependent Analysis	121
7.6.2	Coordinate-Independent Analysis	124

7.7	Instantons in the Spin Exchange Model	128
7.8	The Haldane-like Spin Chain and Instantons	135
7.8.1	Even Number of Sites and Spin-Coherent State Path Integral	137
7.8.2	Odd Spin Chain, Frustration and Solitons	139
8	Quantum Electrodynamics in 1+1 Dimensions	143
8.1	The Abelian Higgs Model	143
8.2	The Euclidean Theory and Finite Action	145
8.2.1	Topological Homotopy Classes	146
8.2.2	Nielsen–Olesen Vortices	147
8.3	Tunnelling Transitions	151
8.4	The Wilson Loop	152
8.4.1	Expectation Value of the Wilson Loop Operator	154
9	The Polyakov Proof of Confinement	157
9.1	Georgi–Glashow model	157
9.2	Euclidean Theory	159
9.2.1	Topological Homotopy Classes	161
9.2.2	Magnetic Monopole Solutions	162
9.3	Monopole Ansatz with Maximal Symmetry	166
9.3.1	Monopole Equations	167
9.4	Non-Abelian Gauge Field Theories	167
9.4.1	Classical Non-Abelian Gauge Invariance	168
9.4.2	The Field Strength	169
9.5	Quantizing Gauge Field Configurations	173
9.5.1	The Faddeev–Popov Determinant	174
9.6	Monopoles in the Functional Integral	177
9.6.1	The Classical Action	177
9.6.2	Monopole Contribution: Zero Modes	178
9.6.3	Defining the Integration Measure	180
9.7	Coulomb Gas and Debye Screening	183
10	Monopole Pair Production	185
10.1	't Hooft–Polyakov Magnetic Monopoles	185
10.2	The Euclidean Equations of Motion	185
10.3	The Point Monopole Approximation	187
10.4	The Euclidean Action	188
10.5	The Coulomb Energy	190
10.6	The Fluctuation Determinant	193
10.7	The Final Amplitude for Decay	199

11 Quantum Chromodynamics (QCD)	201
11.1 Definition of QCD	201
11.1.1 The Quark Model and Chiral Symmetry	202
11.1.2 Problems with Chiral Symmetry	204
11.1.3 The Lagrangian of QCD	207
11.2 Topology of the Gauge Fields	209
11.2.1 Topological Winding Number	211
11.3 The Yang–Mills Functional Integral	213
11.3.1 Finite Action Gauge Fields in a Box	214
11.3.2 The Theta Vacua	220
11.3.3 The Yang–Mills Instantons	222
11.4 Theta Vacua in QCD	224
11.4.1 Instantons: Specifics	227
11.4.2 Transitions Between Vacua	230
11.5 Instantons and Confinement	231
11.6 Quarks in QCD	237
11.6.1 Quantum Fermi Fields	240
11.6.2 Fermionic Functional Integral	241
11.6.3 The Axial Anomaly	246
11.6.4 The $U(1)$ Problem	249
11.6.5 Why is there no Goldstone Boson?	253
12 Instantons, Supersymmetry and Morse Theory	259
12.1 A Little Differential Geometry	259
12.1.1 Riemannian Manifolds	259
12.1.2 The Tangent Space, Cotangent Space and Tensors	260
12.2 The de Rham Cohomology	261
12.2.1 The Exterior Algebra	261
12.2.2 Exterior Derivative	262
12.2.3 Integration	263
12.2.4 The Laplacian and the Hodge Decomposition	264
12.2.5 Homology	265
12.2.6 De Rham Cohomology	266
12.3 Supersymmetric Quantum Mechanics	267
12.3.1 The Supersymmetry Algebra	267
12.3.2 Supersymmetric Cohomology	269
12.3.3 1-d Supersymmetric Quantum Mechanics	271
12.3.4 A Useful Deformation	274
12.4 Morse Theory	277
12.4.1 Supersymmetry and the Exterior Algebra	279
12.4.2 The Witten Deformation	280

<i>Contents</i>	xi
12.4.3 The Weak Morse Inequalities	282
12.4.4 Polynomial Morse Inequalities	285
12.4.5 Witten's Coboundary Operator	288
12.4.6 Supersymmetric Sigma Model	289
12.4.7 The Instanton Calculation	294
Appendix A An Aside on $O(4)$	297
Appendix B Asymptotic Analysis	299
<i>Bibliography</i>	301
<i>Index</i>	308

