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PRIMARY TRANSVERSE CREVASSES*

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ABSTRACT. Measurements of strain-rates on a temperate glacier in a region of initial transverse fracturing indicate a critical strain-rate of $3.5 \pm 0.5 \times 10^{-5}$ d^{-1}, associated with a regional strain-rate gradient of 5×10^{-8} d^{-1} m^{-1}. At only one section of the glacier is the theoretical longitudinal strain-rate (Nye, 1959[c]) in approximate agreement with the value measured at the surface at that point. Corresponding measurements on a polar glacier (temperature -27.9°C at 10 m depth during the summer) indicate that the critical strain-rate is about $0.6 \pm 0.05 \times 10^{-5}$ d^{-1}, which is associated with a gradient of strain rate of about 3×10^{-9} d^{-1} m^{-1}. At one section there is close agreement between the theoretical and measured longitudinal strain-rate. For the temperate glacier crevasse depths ranged from 23.5 to 28 m; in the polar glacier one crevasse was 23.9 ± 0.5 m deep, assuming a wedge form. Only an approximate agreement with the measured values of depth is obtained by using the regional strain-rate values in Nye's crevasse-depth formula.

Over a distance of 1.2 km the temperate glacier transverse crevasse spacings are very variable, ranging from 30 m to 96 m, but initially the spacings range from 55 m to 96 m, and for the first four cases the spacing s varies from 2.7 d to 3.3 d, where d is the crevasse depth. In the cold ice, crevasse spacings are far more uniform, ranging from 57 m to 66 m. A value of $s \approx 2.5\,d$ is obtained in only one case. This greater uniformity of spacing may be explained in terms of the dynamics of flow. Despite large differences in thermal, dimensional and strain-rate parameters between the two glaciers, (1) the crevasse depths are closely similar, and (2) the spacings of crevasses are similar. It has been demonstrated from two lines of evidence that the assumption that the strain on an intercrevasse block is negligible is not correct. The direction of the principal extending strain-rate is, in the most reliable cases, perpendicular to the crevasse traces within 2° to 7°.

RÉSUMÉ. *Crevasses primaires transversales.* Des mesures de la vitesse de déformation d'un glacier tempéré dans la région origine de crevasses transversales donnent une vitesse-limite pour la formation des crevasses de $3{,}5 \pm 0{,}5 \times 10^{-5}$ par jour, jointe à un gradient de vitesse de déformation de 5×10^{-8} par m et jour. En une coupe du glacier seulement, la valeur mesurée à la surface est approximativement en accord avec la vitesse de déformation théorique de Nye. Des mesures sur un glacier polaire à la temperature de $-27{,}9$° et à une profondeur de 10 m donnent une limite de rupture par $0{,}6 \pm 0{,}05$ vitesse de déformation par jour avec un gradient de la vitesse de déformation d'environ 3×10^{-9} par mètre et jour. Dans une section, il y a un bon accord entre la vitesse de déformation longitudinale théorique et mesurée. Les profondeurs des crevasses du glacier tempéré varient de 23,5 à 28 m; une crevasse du glacier polaire en forme de coin était de $23{,}9 \pm 0{,}5$ m. La formule de la profondeur de crevasses de Nye utilisant les valeurs de la vitesse de déformation régionale ne donne qu'un accord approximatif avec les profondeurs mesurées.

Sur le glacier tempéré, sur une distance de 1,2 km les distances entre les crevasses transversales sont très variables, de 30 m à 96 m; mais les distances initiales sont de 55 à 96 m. Pour les quatre premières crevasses, les distances varient entre 2,7 et 3,3 fois la profondeur. Dans la glace froide, les distances sont beaucoup plus uniformes, de 57 à 66 m. Dans un cas seulement la distance est de l'order de 2,5 fois la profoundeur. L'uniformité plus marquée des distances peut être expliquée a l'aide des lois de fluage. En dépit des différences importantes du régime thermique, de la taille et de la vitesse de déformation entre les deux glaciers, la profondeur et la distance des crevasses sont très semblables. Il a été démontré par deux lignes d'évidence qu'on ne peut pas considérer la déformation d'un bloc de glace entre des crevasses comme négligeable. La direction de la vitesse de déformation principale est dans les cas les plus sûrs, perpendiculaire à la direction des crevasses 2° à 7° près.

ZUSAMMENFASSUNG. *Primäre Querspalten.* Messungen der Verformungsgeschwindigkeit im Gebiet beginnender Querspaltenbildung eines temperierten Gletschers ergeben einen Grenzwert für Spaltenbildung von $3{,}5 \pm 0{,}5 \times 10^{-5}$ pro Tag einem örtlichen Gradienten der Verformungsgeschwindigkeit von 5×10^{-8} pro Tag und Meter. Nur in einem Querschnitt des Gletschers stimmt die an der Oberfläche gemessene Verformung angenähert mit Nyes theoretischer Längsverformungsgeschwindigkeit überein. Entsprechende Messungen auf einem Polargletscher mit Sommertemperatur $-27{,}9$°C in 10 m Tiefe führen auf eine Grenzgeschwindigkeit der Verformung von etwa $0{,}6 \pm 0{,}05 \times 10^{-5}$ pro Tag, verbunden mit einem Gradienten von 3×10^{-9} pro Tag und Meter. In einem Querschnitt besteht gute Übereinstimmung zwischen theoretischer und gemessener Längsverformungsgeschwindigkeit. Die Spalten auf dem temperierten Gletscher waren 23,5 bis 28 m tief; eine Spalte auf dem Polargletscher war bei keilförmigem Querschnitt $23{,}9 \pm 0{,}5$ m tief. Nyes Formel für die Berechnung der Spaltentiefe aus den örtlichen Verformungsgeschwindigkeiten stimmt nur angenähert mit den Messungen überein.

Auf dem temperierten Gletscher haben die Querspalten über einer Strecke von 1,2 km stark wechselnde Abstände zwischen 30 und 96 m; die ursprünglichen Abstände liegen zwischen 55 und 96 m. In den ersten 4 Fällen beträgt der Abstand das 2,7- bis 3,3-fache der Spaltentiefe. Die Spalten im kalten Gletscher haben weit gleichmässigere Abstände zwischen 57 und 66 m. Nur in einem Fall ist der Abstand ungefähr das 2,5-fache der Tiefe. Diese grössere Einheitlichkeit der Abstände kann mit Hilfe der Fliessdynamik erklärt werden. Trotz grosser Unterschiede in Wärmehaushalt, in den Dimensionen und in den Werten der Verformungsgeschwindigkeit zwischen beiden Gletschern haben ihre Spalten annähernd gleiche Tiefe und ähnliche Abstände. Auf zweifache Weise wurde gezeigt, dass die Annahme, die Verformung der Eismasse zwischen den Spalten dürfe vernachlässigt werden, nicht zulässig ist. Die Richtung der Hauptdehnungsgeschwindigkeit ist in den zuverlässigsten Fällen innerhalb von 2° bis 7° senkrecht zur Spaltenrichtung.

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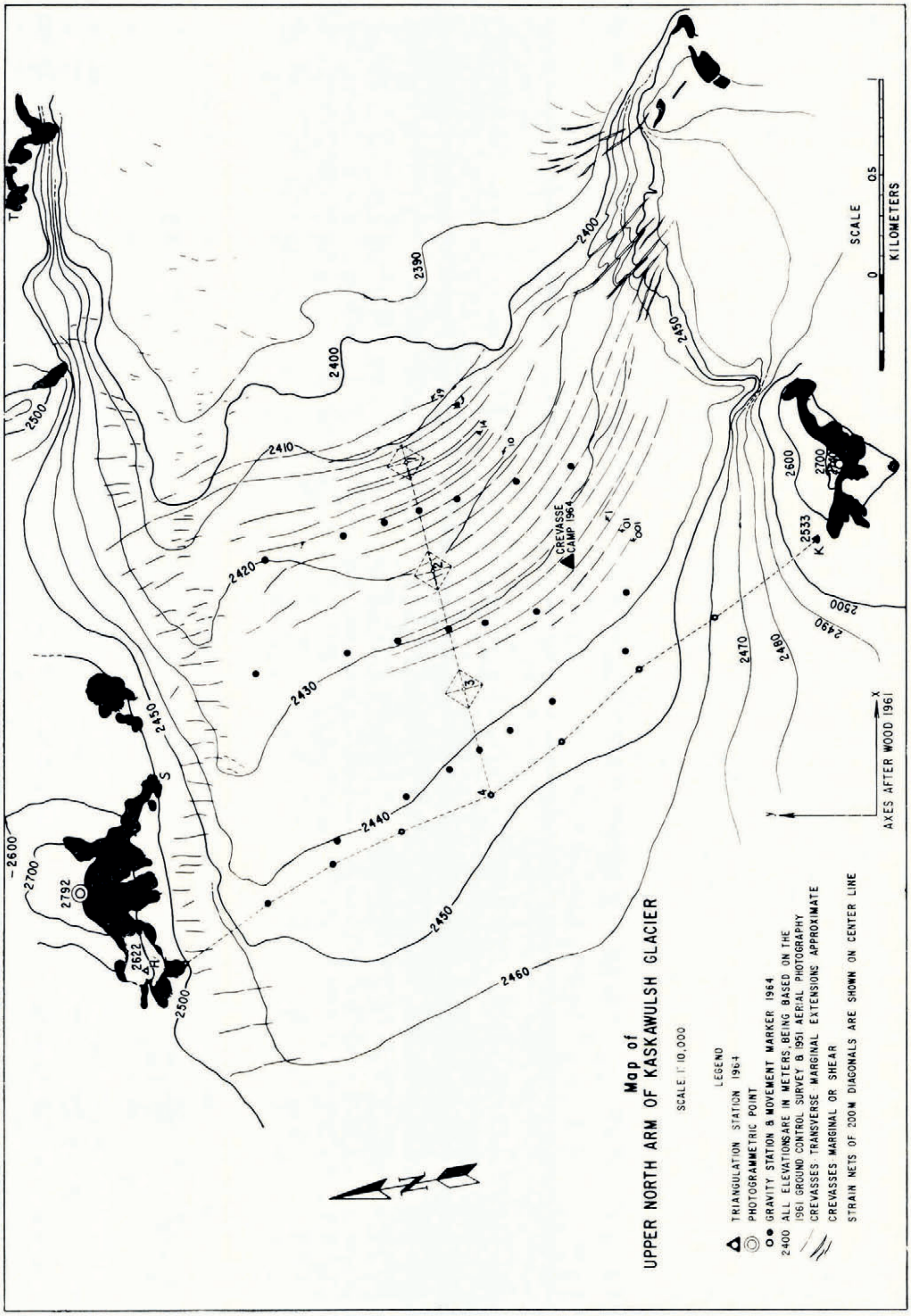

Fig. 1. Geometry of the survey network and the configuration of the transverse crevasse field, Kaskawulsh Glacier.

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Introduction

A previous attempt (Paterson, unpublished) to test the theoretical expression for the longitudinal strain-rate at the center of a valley glacier (Nye, 1951; 1959[c]) has shown, at best, inconclusive results. This has been partly due to the use of only the simplified equation for strain-rate (Nye, 1951) and an apparent omission of testing quantitatively the validity of the curvature term in Equation (1).

It will be shown in this paper that where the curvature term is valid the theoretical value of strain-rate is in fair agreement with the values measured on the surface of two glaciers, one temperate and one polar.

Previous estimates of the strain-rate associated with initial fracture of the glacier surface are given by Meier and others (1957) and Mellor (1964) as 1% year^{-1} (2.8×10^{-5} d^{-1}) and 10^{-9} s^{-1} (8.7×10^{-5} d^{-1}) respectively. The former was for a glacier in which the 10-m temperature was -6.5°C; the temperature of the latter was unspecified. These values were not related with a strain-rate gradient although this is considered to be significant by the present author.

Crevasse depths have been measured with varying degrees of accuracy for many decades but few plausible theoretical analyses have been made and even fewer attempts to test them. The current investigation indicates that the initial formation of crevasses is associated with local strain-rates approximately the same magnitude as the regional values.

Previous analyses of crevasse spacing have been either completely empirical or analytical with doubtful validity, and further work remains to be done.

Geographical and Physical Descriptions of the Two Glaciers Investigated

(I) The Kaskawulsh Glacier lies within the Icefield Ranges, St Elias Mountains, Yukon Territory, Canada. The area investigated covers 4 km^2 of glacier surface located approximately at lat. 60° 47′ N., long. 139° 28′ W., on the north arm of the glacier above an ice fall (Fig. 1).

The area lies in the accumulation zone, between 2 390 m and 2 450 m a.s.l., the balance line being at about 2 150 m a.s.l. Here the glacier is about 4 km wide and more than 600 m deep. Late in the summer the ice is at 0°C to a depth of at least 24 m so that the glacier here may be regarded as temperate. The mean central surface ice flow is about 130 m year^{-1} horizontally. Surface slopes are about 2°. Ice flow is convergent as well as extending in nature.

At about 10 km from the ice divide where the north arm originates (2 650 m) the first transverse fractures were detected at about 2 435 m a.s.l. There is strong marginal crevassing, which extends up-glacier beyond the first transverse fractures. The influence of the former on the latter has been neglected. The field observations were made between 4 July and 15 August 1964.

(II) The Meserve Glacier (Fig. 2) occupies an elongated cirque on the south side of Wright Valley, Antarctica. The area investigated covers less than 1 km^2 of glacier surface located at approximately lat. 77° 35′ S., long. 162° 42′ E. at an elevation of 1 200 m, in the firn area above an ice fall. The ice is up to 200 m thick and about 1.3 km wide. The temperature at 10 m depth is -27.9°C. Mean central surface velocities are 8.8 mm d^{-1} (3.21 m year^{-1}) and ice flow is convergent as well as extending in nature.

Transverse crevasses are strongly developed but are difficult to detect, so that the position of crevasse I (Fig. 3) is only approximate. On the western margin shear crevasses distort the ends of the transverse crevasses, but this is not the case on the east side. The observations reported here were made between November 1966 and January 1967.

Theoretical Considerations

Longitudinal strain-rates

Nye (1951, 1952, 1957, 1959[a], [b], [c]) has developed an equation for the longitudinal strain-rate in an ice sheet or at the center of a valley glacier whose width is much greater than its depth. The terms of the equation are measurable parameters.

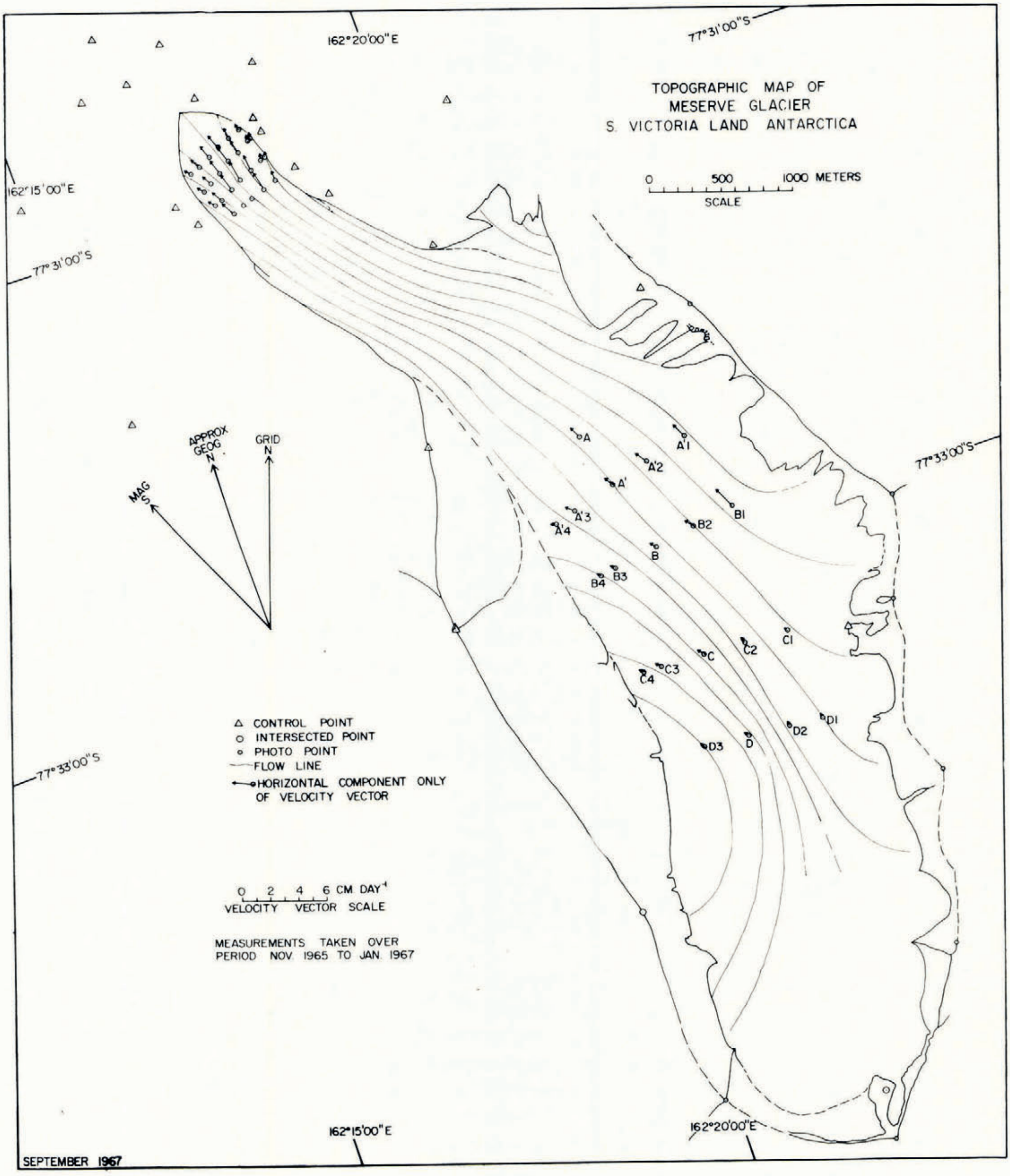

Fig. 2. Topographic map of Meserve Glacier.

Ice is assumed to be both homogeneous and isotropic. Allowance is made for the fact that the shear stress on the bed is not constant.

From the results of Nye (1959[a], p. 401; 1959[c], p. 506) the longitudinal strain-rate $\dot{\epsilon}_x$ in a model glacier can be expressed as:

$$\dot{\epsilon}_x = \frac{mV_b}{mV_b+U_s}\left(\frac{a}{h}+U_s\,\kappa_\alpha \cot\alpha-\frac{1}{h}\frac{\partial h}{\partial t}-\dot{\epsilon}_y+\tfrac{1}{2}h\bar{V}\frac{\partial \kappa}{\partial x}\right) \tag{1}$$

where m = empirical constant $\approx \frac{1}{2}(n+1)$ where $n \approx 3$ is given in the flow law,
V_b = basal sliding speed (m d^{-1}),
$\bar{V}$ = mean ice speed over the depth and width at a given cross-section of the glacier,
U_s = central surface velocity component at a point parallel to the glacier surface (m d^{-1}),
a = rate of accumulation or ablation at the surface of the glacier expressed as equivalent ice thickness (m d^{-1}),
h = ice thickness (m),
κ_α = curvature of surface,
α = slope of surface,
t = time,
$\dot{\epsilon}_y$ = transverse strain-rate (d^{-1}),
x, y = coordinate axes where x is the direction down-glacier along the central flow line, and y is transverse.

Other terms used in this paper are:

B = empirical factor in flow law (bar^{-n} year^{-1}): $\dot{\epsilon} = B\sigma^n$ where $\dot{\epsilon}$ is the effective strain-rate; σ is the effective stress. The value of B is temperature dependent, but also is influenced by the geometric and crystallographic orientation as well as the density of the ice,
B_0 = value of B at 0°K,
k = empirical constant,
θ = temperature (°C),
T = temperature (°K),
$\bar{\rho}$ = mean density of surface layers of firn and ice (g cm^{-3}),
Q = ultimate tensile strength of surface ice (bars),
s = crevasse spacing (m),
$\dot{\epsilon}_c$ = critical strain-rate for fracture (d^{-1}),
$\dot{\epsilon}_K$ = strain-rate across a crevasse (d^{-1}),
w = width of glacier at a given cross-section (m),
W_c = crevasse width (m),
U = surface ice speed (in a horizontal direction) due to plastic deformation only; $U = U_s - V_b$,
$\bar{U}_s$ = mean central surface velocity component parallel to the glacier surface over a given length of center line (m d^{-1}),
$\bar{V}_c$ = mean ice speed over the depth h, on the center flow-line,
β = bed slope,
κ_β = curvature of the bed,
σ_c = critical stress for fracture (bars).

Crevasse depths

Nye (1957) derives a depth formula of the type

$$d = \frac{2}{\bar{\rho}g}\left(\frac{\dot{\epsilon}}{B}\right)^{1/n}(3\sin^2\alpha+1)^{-1/2} \tag{2}$$

which is obtained by replacing the yield stress in tension by the equivalent strain-rate for extending flow. The appropriate values of B and n are presumably given by the experimental results of Glen (1955). The appropriate strain-rate which has not been clearly defined, has been previously taken by the author and Meier and others (1957) as $\dot{W}_c/W_c$. It will be shown that the regional value of $\dot{\epsilon}$ is more appropriate, and physically justifiable.

Crevasse spacing

There is a paucity of general information bearing on the problem of crevasse spacing. Approaches hitherto have been either empirical or analytical of questionable validity. Meier and others (1957) suggest a mean spacing of about 4 d for certain unclassified crevasses in the Blue Ice Valley, Greenland. Holdsworth (1965) found that a mean spacing s is given by $s = 2.8\,d$ with a range of 2.7 d to 3.3 d for true transverse crevasses. Nielsen (1958, p. 47) claims to provide an explanation for the apparent uniformity of crevasse spacing. He considers a flowing cantilever ice slab, which, on attaining an unsupported length s, fractures in elastic bending. If the ice thickness is h (constant) and the tensile strength of the upper layers Q (assumed approximately equal to the bending strength), then it may be shown that the spacing

$$s = \left(\frac{Qh}{3\bar{\rho}g}\right). \tag{3}$$

Nielsen's expression can be modified by inserting the surface strain-rate, assuming that bending and direct extension have essentially the same effect on the surface layers of the glacier. Thus the spacing formula becomes

$$s^2 = \frac{h}{3\bar{\rho}g}\left(\frac{\dot{\epsilon}_c}{B}\right)^{1/n}. \tag{4}$$

This implies that for small variations in h and $\dot{\epsilon}_c$, s is approximately a constant for a given glacier.

If plastic theory is used, the result obtained is not substantially different; it will not be given here. Nielsen's concept suffers from the fact that the bending moment, which is a function of h, cannot be produced by cantilever action. In addition, the existence of adjacent fractures is not considered whereas the few data collected suggest a reasonable correlation between the depth of primary transverse fractures and the fracture spacing.

By considering only the distribution of surface stress built up behind the last crevasse, and assuming this to be roughly linear, we could write

$$s = \sigma_c\left(\frac{\partial \sigma_x}{\partial x}\right)^{-1} \tag{5}$$

where s is the distance from the existing crevasse, where $\sigma_x = 0$, to the position of the next incipient fracture, and $\partial\sigma_x/\partial x$ = gradient of stress up-glacier.

Equation (5) may be rewritten as

$$s = \dot{\epsilon}_c^{1/n}\left(\frac{\partial \dot{\epsilon}_x^{1/n}}{\partial x}\right)^{-1}. \tag{6}$$

Therefore, by knowing a value of $\dot{\epsilon}_c$ and being able to measure $\dot{\epsilon}_x$ at points up-glacier, a value of the next spacing s could be estimated. This has not been tested because of the lack of accurate closely-spaced strain values in the area concerned.

A fundamental approach to the problem would be to investigate in a suitable model study the stress distribution around a crevasse, using photo-plastic techniques (Frocht and Thomson, c1958).

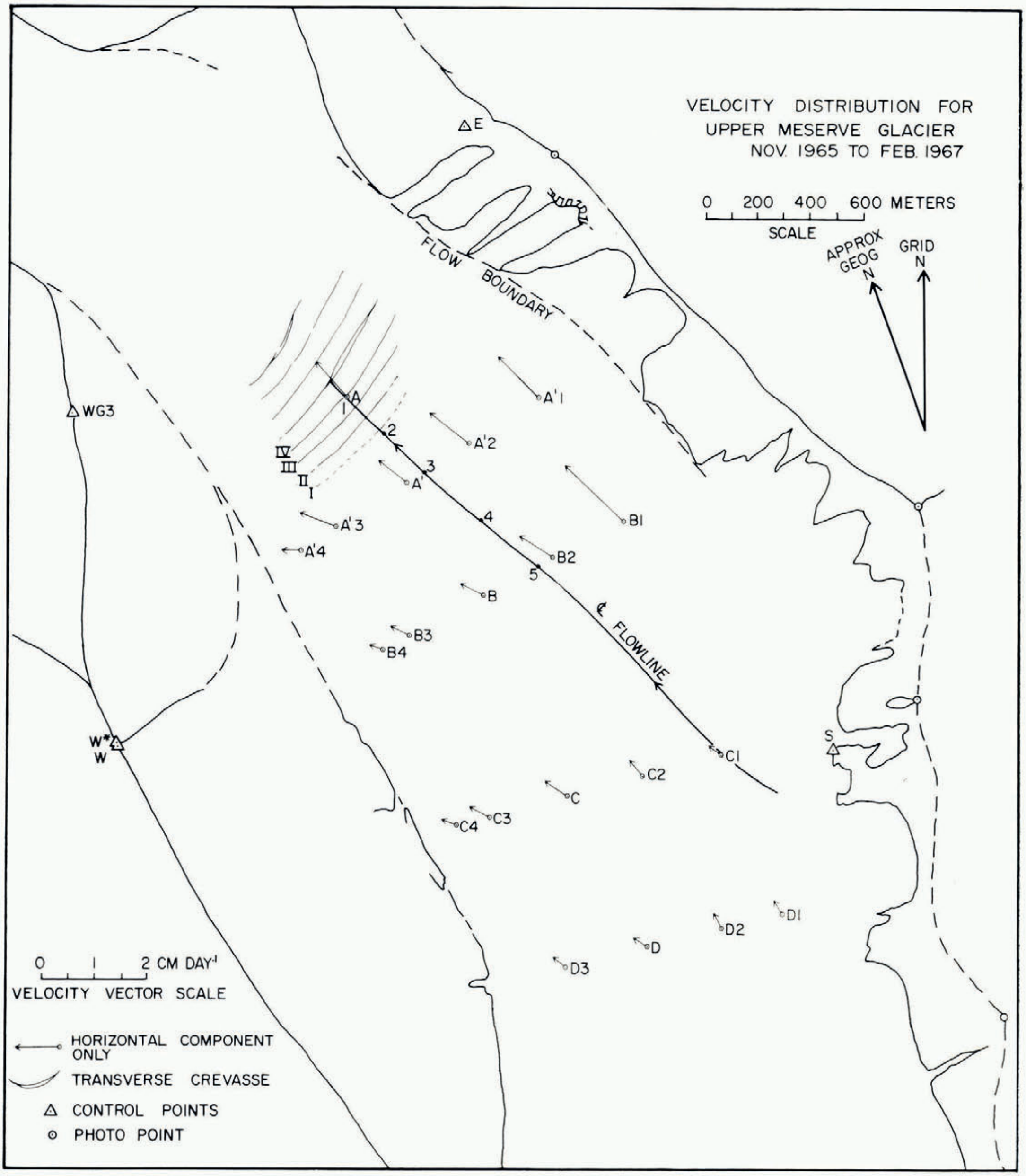

Fig. 3. Velocity distribution for upper Meserve Glacier.

Field Measurements and Results

(I) *Kaskawulsh Glacier*

Velocity determination. Surface velocity measurements in the area of the crevasse investigations for the one-year period 1962–63 have been made by Sharni (unpublished), and for the year 1963–64 by Brecher (1966), who also attempted to determine short-term variations of surface velocity during the summer of 1964. The configuration of velocity vectors is shown in

Figure 4. Any variations of velocity were obscured by the standard error of measurement, which was about 10%. Figure 5 shows the surface velocity distribution (horizontal component) for July to August 1964.*

Strain-rate determination. Standard strain diamonds provided values of central strain-rate for the period from 1 July to 11 August 1964. Procedures of data reduction followed those of

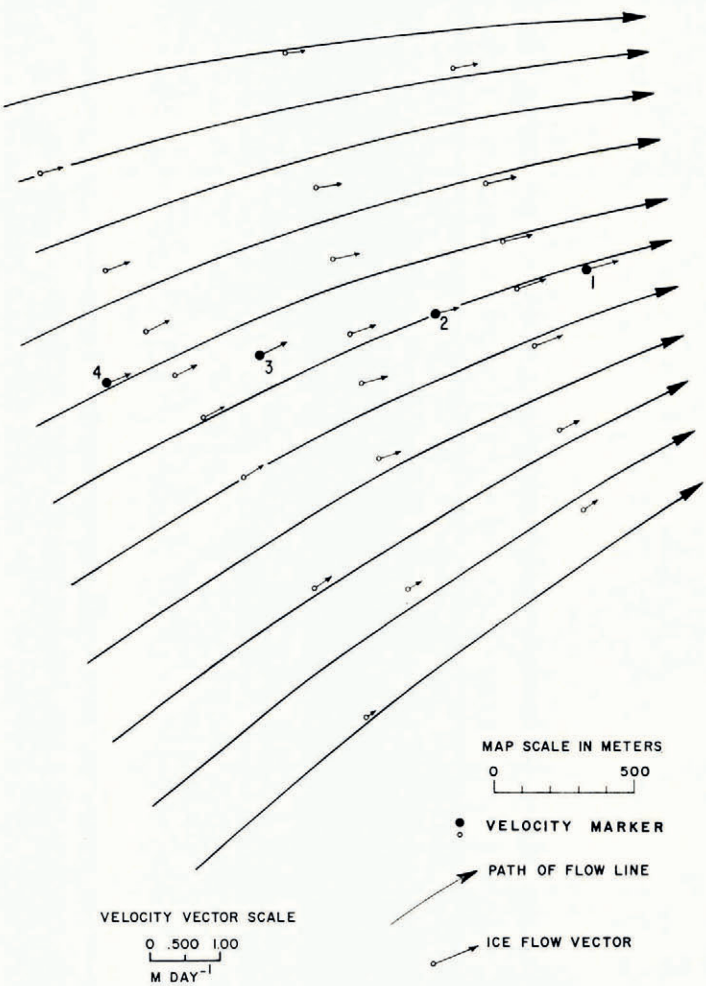

Fig. 4. Configuration of flow-line field based on velocity data, Kaskawulsh Glacier.

Nye (1959[b]). Values of regional strain-rate have been computed from the areal velocity measurements over an area much larger than that of the direct strain-rate measurements. A value of the regional critical extending strain-rate is obtained by overlaying a plot of the crevasse traces on the principal extending strain-rate diagram (Fig. 6a). An estimate of $+3.5 \pm 0.5 \times 10^{-5}\ d^{-1}$ is thus obtained. The associated strain-rate gradient is about

* For purposes of later calculation, the basal slip rate, V_b, was assumed to be about 25% of U_s on the basis of known values for glaciers of similar geometry, and on the basis of the maximum surface flow-rate variations (Brecher, 1966).

$0.005 \times 10^{-5}\ \mathrm{d^{-1}\ m^{-1}}$. Table Ia shows component values of theoretical longitudinal extending strain-rate calculated from the terms in Equation (1). Each of these terms will be discussed briefly.

The accumulation term is positive in sign. Values of mean net annual accumulation have been obtained from R. H. Ragle (personal communication) for 1961 to 1963, and from pit studies in 1964 (Holdsworth, 1965, p. 37).

Table I. Component and Total Theoretical Strain-rates; all Components $\times 10^{-5}\ \mathrm{d^{-1}}$

Point	h m	$\bar{V}_c$ m d^{-1}	tan α	tan β	$\bar{V}_c\, \kappa_\alpha \cot \alpha$	$\bar{V}_c\, \kappa_\beta \cot \beta$	$\frac{a}{h}$	$\frac{\bar{V}}{W}\frac{\mathrm{d}w}{\mathrm{d}x}$	$\dot{\epsilon}_y$ meas.	$\frac{1}{h}\frac{\mathrm{d}h}{\mathrm{d}t}$	$\frac{1}{2}h\bar{V}\frac{\mathrm{d}\kappa}{\mathrm{d}x}$	$\dot{\epsilon}_x$ surface dimens.	$\dot{\epsilon}_x$ bed dimens.	$\dot{\epsilon}_x$ meas.
						(a) *Kaskawulsh Glacier*								
			+	+										
4	660	0.320	0.030	0.093	+6.4	−5.18	+0.7	−5.5	—	−0.5	−0.12	+7.5 ±0.5	+0.91 ±0.5	+2.5 ±0.2
3′A	670	0.315	0.026	0.075	term not valid		0.64	−5.6	—	−0.5	−0.12			2.8
3′	670	0.329	0.016	0.035			0.64	−5.6	−9.1	−0.5	−0.15			4.3
				−										
2′A	660	0.342	0.012	0.033			0.64	−5.4	−11.5	−0.5	+0.17			5.2
				+										
2′	660	0.362	0.009	0.008			0.56	−5.9	−10.2	−0.3	0.39			10.2
1′A	665	0.389	0.008	0.043	term not valid		0.49	−6.5	−12.0	−0.3	0.10			11.0
1′	690	0.405	0.005	0.078			0.46	−7.3	−14.4	−0.3	—			14.1
						(b) *Meserve Glacier*								
				+										
1	220	0.07	0.049	0.077	+0.20	+0.07	0.04	−0.26	—	−0.25	−0.015	+0.74 ±0.10	+0.61 ±0.10	+0.69 ±0.05
2–5	—	—	—	—	term not valid		—	—	—	—	—			—

Kaskawulsh Glacier points 1′ through 4 lie on the "geophysical center-line" of the glacier. This line has been selected on the basis of symmetry of velocity and strain distribution data.

The so-called curvature term, $U_s\, \kappa_\alpha \cot \alpha$, is valid only if $|h\kappa_\alpha| \ll \alpha$ and $h\, \mathrm{d}\alpha/\mathrm{d}x \ll \mathrm{d}h/\mathrm{d}x$. These conditions are seen not to apply except in one short section of the glacier. For convenience curvature and slope are referred to the surface of the glacier but this has never been fully justified. The value of transverse strain-rate that has been used is the value obtained from the strain diamond data, rather than from $\frac{\bar{V}}{W}\frac{\mathrm{d}W}{\mathrm{d}x}$, as suggested by Nye (1959[c], p. 506). To use the surface values means that any variation of $\dot{\epsilon}_y$ with depth is neglected whereas $\frac{\bar{V}}{W}\frac{\mathrm{d}W}{\mathrm{d}x}$ presumably takes account of this. However, since all the measurements in this case are restricted to the surface of the glacier, the surface value of $\dot{\epsilon}_y$ is more appropriate.

The rate of change of ice thickness is positive if the glacier surface at a point is rising with time. Since 1961 there has been a lowering of the surface, hence the term is negative, giving a positive contribution to $\dot{\epsilon}_x$ (see Equation (1)). The contribution of this term is small.

The true bending term, $\frac{1}{2}h\bar{V}\frac{\partial \kappa}{\partial x}$, which takes into account rates of change of curvature, is for the present case generally less than $10^{-5}\ \mathrm{d^{-1}}$, whether the surface or the bed slope is considered. Table I also shows computed and measured values of longitudinal strain-rate.* At station 4 strain-rates of $+0.91 \times 10^{-5}\ \mathrm{d^{-1}}$ to $+7.5 \times 10^{-5}\ \mathrm{d^{-1}}$ are obtained depending on whether surface slopes or bed slopes are used. The measured value of $+2.5 \times 10^{-5}\ \mathrm{d^{-1}}$ lies between these values.

* The term $mV_b/(mV_b+U_s)$ is evaluated using the values $m = 2$, $V_b = \frac{1}{4}U_s$, see the section on velocity determination above.

The fact that there is very little correspondence between measured and computed strain-rates is probably due to the rapidly changing geometry of the glacier in the restricted length of glacier studied and to the presence of the crevasses. One of the most important terms in Equation (1) is $U_s \kappa \cot \alpha$. Since this term holds only for low rates of change of curvature, it is suggested that if the bending term is used, it must be used on smoothed-out profiles, as is done in the present case.

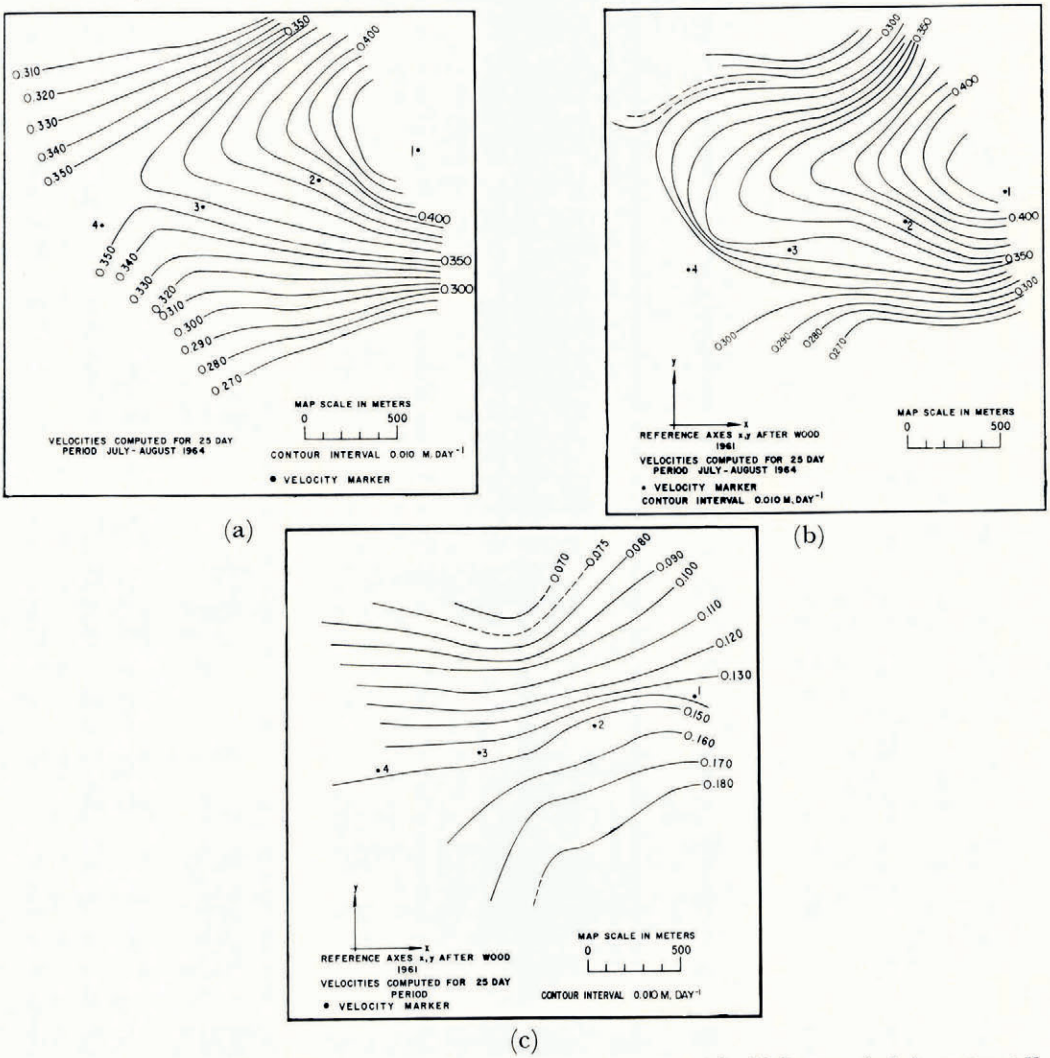

Fig. 5. Velocity contours for Kaskawulsh Glacier. (a) Contours of total velocity V, m d^{-1}. (b) Contours of velocity component V_x, m d^{-1}. (c) Contours of velocity component V_y, m d^{-1}.

Strain-rates measured on laboratory ice which is tested to failure in tension, generally show little agreement with strain measurements made on glaciers. These differences may be attributed to inhomogeneity of the surficial layers of a glacier where firn and firn-ice predominate.

Thermal shock may be significant and the rate of development of thermal stresses important in determining the local value of σ_c and hence of $\dot{\epsilon}_c$.

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Figure 8 shows that the direction of the principal extending strain-rate ($\dot{\epsilon}_1$) is in most cases nearly perpendicular to the crevasse traces.

Crevasse depths. Nye and others (1954) and Nye (1955) have discussed, generally, the problem of the depth of crevasses. Schuster and Rigsby (1954) state that observed crevasse depths generally vary from 50 ft to 100 ft (15 m to 30 m) but may extend to 150 ft (45 m) or more. According to Seligman (1955) most crevasses in the European alpine glaciers do not exceed 30 m in depth, but Loewe (1955, p. 511) cites a crevasse in the Bernese Oberland as being nearly 40 m deep (the position was probably in cold ice).

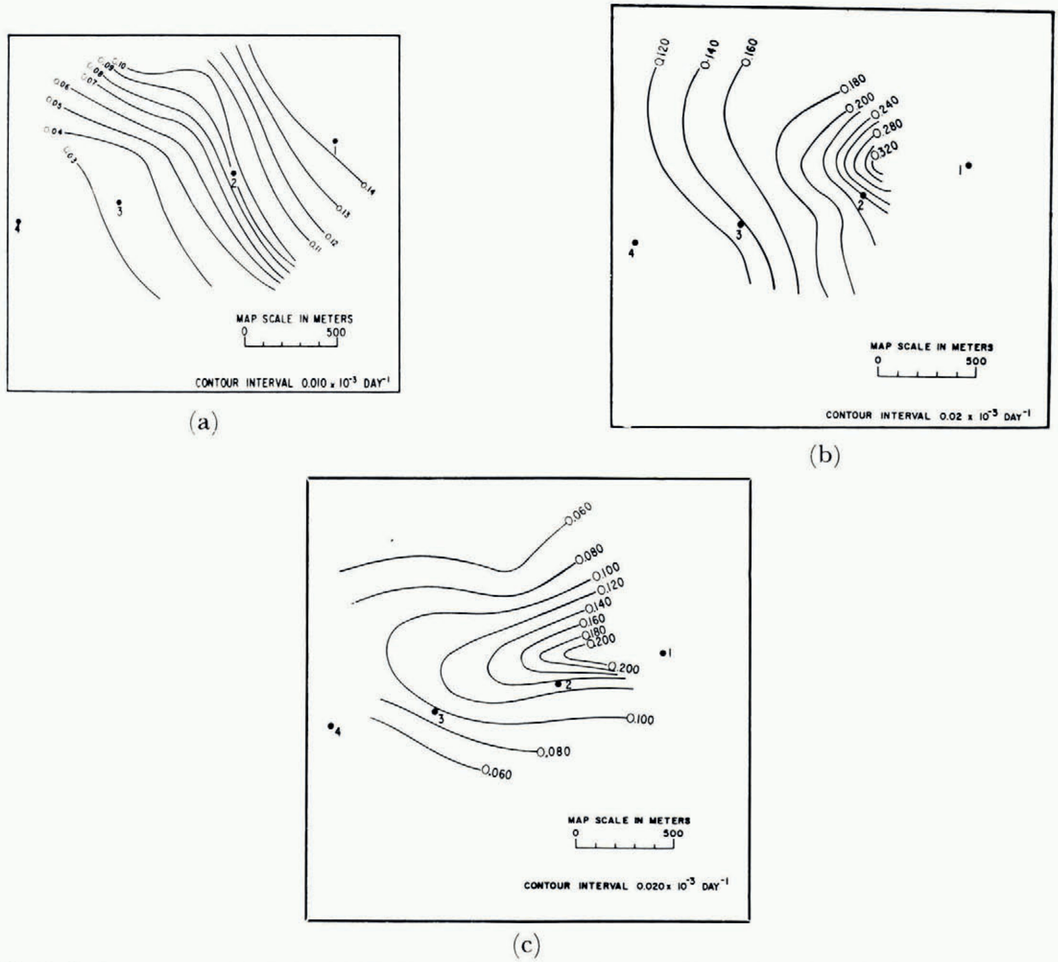

Fig. 6. Strain-rate contours for Kaskawulsh Glacier. (a) Contours of principal extending strain-rate ($\dot{\epsilon}_1$). (b) Contours of principal contracting (least extending) strain-rate ($\dot{\epsilon}_2$). (c) Contours of maximum shearing strain-rate ($|\dot{\epsilon}_1-\dot{\epsilon}_2|$)

Estimates of crevasse depths from Alaskan glaciers range from 20 m to 40 m. Some of these values have been obtained from seismicdata (e.g. Goldthwait, 1936, p. 503; Miller in Nye, 1955). Crevasses in Greenland and Antarctic glaciers have been reported as being up to 150–200 ft (45–60 m) in depth. In few of these reports has the type of crevasse been specified.

Five of the primary transverse crevasses on Kaskawulsh Glacier show a range of measured depths between 24 ± 0.5 m and 28 ± 1 m.

Meyerhof (in Nye and others, 1954, p. 340) obtains an expression which gives fracture depths of from 23 m to 46 m, but since soil mechanics theory is used, the validity of the method is suspect.

Before proceeding it must be pointed out that previous investigators (Meier and others, 1957; Holdsworth, 1965) incorrectly used the relationship attributed to Nye (1955)

$$d = \frac{1}{\bar{\rho}g}\left(\frac{\dot{\epsilon}}{B}\right)^{1/n}$$

where B is the constant in the relation between tensile strain-rate and tensile stress on the glacier surface. A corrected relationship is found in Nye (1957, p. 128) which is obtained by putting $\sigma_x = 0$ in the stress equation (p. 118) and solving for d (Equation (2)). Because the surface slope α is small, $(3 \sin^2 \alpha + 1)^{-1/2} \approx 1$ and Equation (2) reduces to

$$d = \frac{2}{\bar{\rho}g}\left(\frac{\dot{\epsilon}_x}{B}\right)^{1/n}. \tag{7}$$

Thus table IV in Meier and others (1957, p. 40) and table 2 in Holdsworth (1965, p. 40) must be recalculated.

Table II. Crevasse Depths, Kaskawulsh Glacier

$\rho = 0.70$ Mg m^{-3}; $n = 3.17$

Crevasse No.	Temperature θ °C		B bar^{-n} year^{-1}	$\dot{\epsilon}_K$ across crevasse year^{-1}	$\dot{\epsilon}_x$ regional year^{-1}	d calculated $\dot{\epsilon}_K$ m	d calculated $\dot{\epsilon}_x$ m	d measured m
6	(a)	0	0.17	1.6	0.020	57.1	14.6	23.5–24
	(b)	−1.5	0.023				27.3	
7	(a)	0	0.17	1.5	0.022	—	15.1	26
	(b)	−1.5	0.023	1.5	0.022		28.5	
8		0		1.7	0.025	—	15.6	25.5–26
9		0		2.1	0.032	—	16.8	28–28.5
10		0		1.3	0.037	—	—	27
14		0		0.8	0.046	—	—	26.5
18		0		—	0.053	—	19.8	25

Table III. Recalculation of d from Data of Meier and others (1957, p. 40)

$$d \approx \frac{2}{\bar{\rho}g}\left(\frac{\dot{\epsilon}}{B}\right)^{1/n}$$

Crevasse No.	$\bar{\rho}$ Mg m^{-3}	n	B bar^{-n} year^{-1}	θ °C	$\dot{\epsilon}$ year^{-1}	d calculated m	d measured m
2	0.65	3.3	0.022	−1.5	0.0056	18.8	22
			0.011	−3.0	0.0056	25.0	
3	0.65	3.3	0.022	−1.5	0.0097	22.0	26
				−3.0		29.5	

The corrected results for Holdsworth's data are shown in Table II and for Meier and others, data in Table III. In both cases there is now a close correspondence between the observed and the computed values of d (especially in Table III), obtained by taking values of B appropriate to the mean temperature (−1.5°C) within the first few meters of firn. The use of a value of B corresponding to $\theta = -12$°C (which is actually not in agreement with the temperature data) is therefore considered to be incorrect (see also Meier and others, 1957, p. 52).

Using the expression obtained by Nye (1957), Equation (7), an estimate of the theoretical crevasse depth corresponding to a known strain-rate is obtained. Data from Glen (1955, p. 519) have been used to provide appropriate values of B and n. Regional as well as "across

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crevasse" strain-rates have been used to compute d in Table II. Only values of *regional* strain-rate produce a close correspondence between computed and observed depths. In a previous paper, Holdsworth (1965) used $\dot{\epsilon}_K = \dot{W}_c/W_c$ to obtain $\dot{\epsilon}_K$ across a crevasse, where W_c is the crevasse width. Regional values were obtained from the velocity distribution and from $\dot{\epsilon} = \dot{W}_c/s$.

In this latter case it was assumed that strain on inter-crevasse blocks is negligible. That this is not so has been demonstrated by placing strain diamonds between crevasses on the Meserve Glacier, discussed in the second part of this paper. Further evidence that the assumption is incorrect is furnished by a plot of crevasse spacings (Fig. 7) through the "ice fall" on the Kaskawulsh Glacier, where it is seen that crevasse spacings decrease down-glacier to about half of their maximum value up-glacier, indicating secondary rupture of the primary crevasse system. This is also supported by direct measurements of $\dot{\epsilon}$ on inter-crevasse blocks on the Meserve Glacier.

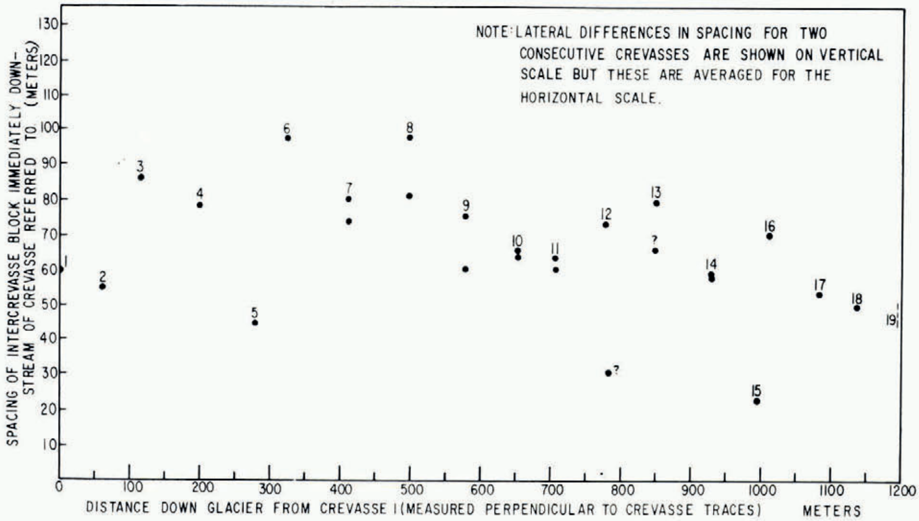

Fig. 7. Plot of crevasse spacing with distance down-glacier, Kaskawulsh Glacier.

According to Equation (7) the depth of an individual crevasse will vary as $\dot{\epsilon}_x$ varies. Therefore once a crevasse has formed, its depth will fluctuate with time.

From Equation (7) it can be shown that

$$\frac{\mathrm{d}d}{d} = \frac{1}{n} \cdot \frac{\mathrm{d}\dot{\epsilon}_x}{\dot{\epsilon}_x}. \tag{8}$$

Thus for a 10% change in $\dot{\epsilon}$, and assuming $n \approx 3$, the percentage change in depth given by Equation (8) is 3.3%.

The empirical constant B in the flow law is extremely temperature sensitive, so that an accurate value of englacial temperature must be determined. Recalculating the data of Meier and others (1957) shows that if the minimum recorded temperature is used to determine B, a value of $d = 30$ m is obtained using Equation (7). The actual temperatures ranged from $-0.5°$ to $-6.5°$C, with a mean of about $-3.0°$C, corresponding to a value of B which would yield a value of $d = 25$ m (crevasse 2). It is not fully clear why a value of $-12°$C was used, although this produces a good agreement between the measured depths and those computed from the incorrect equation,

$$d = \frac{1}{\bar{\rho} g}\left(\frac{\dot{\epsilon}}{B}\right)^{1/n}.$$

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In the case of temperate ice, B is fixed and any anomalies are produced by the value of $\dot{\epsilon}$ used. For Kaskawulsh Glacier $\bar{U}_s \approx 130$ m year^{-1} and s varies between 55 m to 96 m for the first six crevasses (or an average of $s \approx 69.5$ m). That is, two crevasses are formed each year, on the average.

There are two possibilities:* Either (i) the crevasses are formed at different times, for example, one in the winter and one in the summer, or (ii) the crevasses form in pairs during the winter when surface temperatures are lower and a lower value of $\dot{\epsilon}_x$ is required to break the ice, although it must be noted that the value of σ_c is roughly the same for the temperature range considered (see Appendix II).

Assumption (i) is based on a consideration of Equation (1) from which one may conclude that the position of fracture is dependent only on the physical geometry of the glacier boundaries at that point and on the rate of flow both at the surface and at the base. Consequently it is reasonable to conclude that, given the values of $\bar{U}_s$ and s for Kaskawulsh Glacier, the crevasses form at the rate of two per year, approximately six months apart. In saying this we are not neglecting temperature changes as might first appear to be the case. Appendix I shows that a 10% change in V_b, which is in effect temperature controlled, produces only a 6.6% change in $\dot{\epsilon}$, or a 0.23×10^{-5} d^{-1} departure from $\dot{\epsilon}_c = 3.5\times10^{-5}$ d^{-1} which is within the limits of error in the measurement of $\dot{\epsilon}_c$.

Assumption (ii) is, on the other hand, worthy of consideration. First we have to establish what is the criterion of failure: strain-rate or stress. If the general flow law is used for the temperate and polar glaciers with appropriate values of B, the critical stresses σ_c are not greatly different in the two cases (Appendix II). Hence, applying this reasoning to the Kaskawulsh Glacier alone, for summer and winter conditions, if we lower the temperature at a point where $\dot{\epsilon} < \dot{\epsilon}_c$ in the summer, σ will correspondingly rise and may reach the tensile strength of the material even though $\dot{\epsilon} < \dot{\epsilon}_c$ (summer) has remained substantially the same. This reasoning prompts us to abandon the strain-rate as a criterion for failure. However, since the latter parameter is so easily measured directly, its use is understandable.

The dilemma is therefore not satisfactorily resolved. In concluding, some remarks on the strain-rate gradient, thermal stresses and stress concentrations will be added.

(a) Strain-rate gradient: From the point of view of the response of material to the rate of stress application, this parameter should be considered because it is known that the ultimate strength of certain materials is dependent on the rate of application of the deforming stress.

(b) Thermal stresses: Although the effect of temperature has been considered in conjunction with the flow-law calculations (see Tables II, III, IV and Appendix II), direct stresses due to thermal shock have not been considered independently.

If, for ice, the horizontal components of stress are given by

$$\sigma_x = \sigma_y = -\frac{E/\alpha}{(1-\mu)} \,.\, \Delta T \tag{9}$$

where E is Young's modulus (bars), α is the coefficient of linear expansion (deg^{-1}), μ is Poisson's ratio and ΔT is the temperature change (deg). Then the thermal stresses which will be tensile for a decrease of temperature may be estimated.

Dorsey (1940, p. 446 and p. 472) gives for ice $E \approx 935$ kg mm^{-2} $\approx 9.16\times10^4$ bar, $\mu \approx 0.35$, $\alpha \approx 50\times10^{-6}$ deg^{-1}.

Using these values, $\sigma_x = \sigma_y \approx 7$ bars per degree, which is significant compared with the stress produced by the flow of ice induced by boundary conditions of a purely geometrical nature. The existence of these stresses has been recognized by the author on areas of bare ice on Meserve Glacier. After a significant air temperature drop of 5 to 10 deg, the passage of a

* These could be tested by installing a pattern of seismographs on the surface of the glacier and allowing them to record for a year. The position and time of an energy pulse would thus be known.

person wearing crampons over the surface caused the latter to fracture violently. This phenomenon was both audible and visible.

However, primary transverse crevasses occur almost exclusively in areas of firn and consequently the values of the parameters used in Equation (9) will be different. In the absence of information about μ and α for snow, only a rough estimate of σ_x can be made using $E \approx 50$ kg mm^{-2} (≈ 5 kbar) (Dorsey, 1940, p. 445). Then $\sigma_x \approx 0.377$ bar, which is still comparable with the stresses generated by the dynamic factors. To be considered also is the rate of temperature change on which any adjustment of the snow pack tending to relax the stresses will depend.

Since thermal shock is a near-surface phenomenon and since we have postulated that crevasses form initially at the surface, it may be seen that thermal stresses may be significant in certain cases.

(c) Stress concentrations: In a previous paper (Holdsworth, 1965) it was found necessary to appeal to local concentrations of stress in order to produce a computed value of d which would agree closely with the measured values. With the use of the correct depth formula, Equation (7), this concept need not be pursued, except to add that *once* a fracture has formed, it may be propagated by stress concentrations developed at the bottom and ends of the crack. In metals, before rupture in tension, "Griffith cracks" are considered to exist and the local concentrated stress is given by $\sigma = 2\sigma_r (l/R)^{1/2}$ where $2l$ is the crack length, R is the radius of curvature of the ends and σ_r is the regional stress acting perpendicular to the crack length. Thus as R tends to small values, the local stress σ can attain very high values. Using this information we may suppose that the crevasse may propagate laterally into regions where $\sigma_x \ll \sigma_c$, although the direction of crack propagation is governed by the relative values of σ_x, σ_y, and τ_{yx} (see Appendix III). With depth, the crack terminates when the tensile stress is less than the tensile strength of the ice at that depth.

Field measurements (Meier and others, 1957; Nye, 1959[b]; Paterson, unpublished; Wu and Christensen, 1964; Holdsworth, 1965) indicate that large variations of the direction and magnitude of $\dot{\epsilon}_1$ do occur on the surface of a glacier, tending to support the concept of stress concentrations.

Crevasse spacing. Measurements of crevasse spacing are plotted against position on the glacier (Fig. 8). Where a significant lateral variation in spacing between two adjacent crevasses occurred, the individual values are plotted together; the mean was taken. Otherwise, in some cases a 20% variation was recorded. Figure 8 shows that crevasse spacings vary from about 30 m to 100 m with a mean of about 75 m. A decrease in spacing appears to occur with increasing age of the crevasses and this can probably be explained by secondary fracture on intercrevasse blocks, meaning that strain on such blocks is not negligible as had been assumed previously (Meier and others, 1957; Holdsworth, 1965).

Another approach, in order to explain variations in s, would be to appeal to surface and depth changes in the structural properties of firn and ice. Laboratory tests on the ultimate tensile strength of ice (see for example [SIPRE], 1951) commonly show variations over 300%. Such a variation of σ_c would be more than sufficient to account for the observed variations in s (e.g. 70 ± 35 m). However, the concept of secondary fracture explains satisfactorily the apparent gradient of spacing and is supported by the observation that crevasse 18 was only 20 cm wide but was flanked by crevasses at least 5 m wide, suggesting a difference of age.

Using the formula of Nielsen (1958, p. 47) and putting $h = 650$ m, $Q = 2$ bars (estimated, from laboratory tests, etc.) and $\rho = 0.70$ Mg m^{-3}, a value of $s \approx 79$ m is obtained. This agreement with the observed spacing is considered to be fortuitous, however, as indicated in the previous discussion of the formula and because the value of s in Nielsen's treatment is dependent on h, which is unreasonable.

As a matter of interest, if Equation (4) is used for the Kaskawulsh Glacier, with $\dot{\epsilon}_c = 0.0128$ year^{-1} (3.5×10^{-5} d^{-1}), the value obtained, $s < 25$ m, is a spacing which has not been

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recorded, so Nielsen's formula does not give correct results when using observed values of strain-rate.

(II) *Meserve Glacier*

Velocity determination. Surface velocity measurements in the *névé* region are shown in Figures 2 and 3. Central horizontal components of ice flow rates indicate extending flow, being 0.47 cm d^{-1} at B and 0.88 cm d^{-1} at A for the period 3 December 1966 to 27 January 1967. It is not yet established whether the flow rate shows significant variations from season

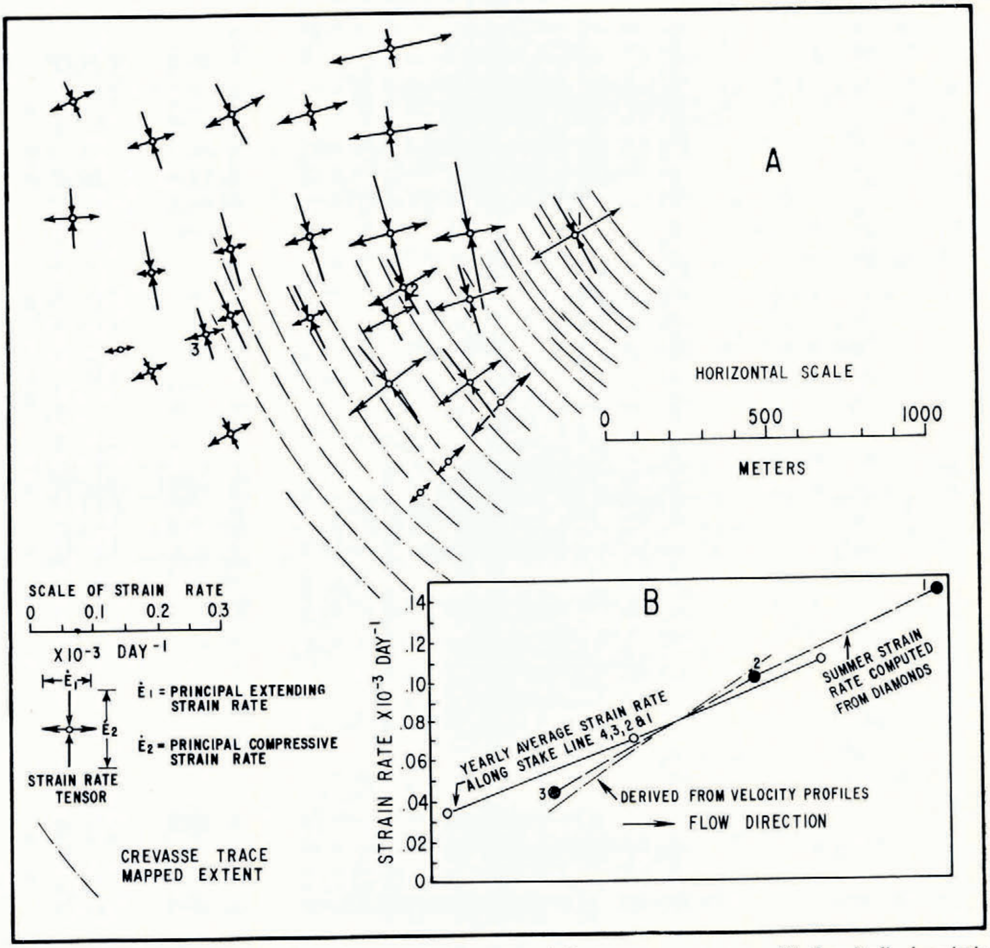

Fig. 8. (a) Configuration of principal strain-rate field in region of the transverse crevasses. (b) Longitudinal variation of principal extending strain-rate.

to season. Because there is no basal slip, any variations in U_s can only be due to changes in accumulation rate and in temperature, neither of which are likely to produce variations of U_s beyond the standard error of measurement.

Strain-rate determination. Figure 9 shows the pattern of strain diamonds, arranged so that the principal strain-rates spanning consecutive crevasses and those confined to intercrevasse "blocks" could all be measured. The results show that the strain on intercrevasse "blocks" is not negligible but is almost the same as the value obtained from the diamond spanning the crevasses.

Principal strain-rates for the three compound diamonds show a consistency, although for the second net the strain-rate spanning crevasses is a little less than the inter-crevasse value, a condition which is not readily understood. On the other hand, regional values of strain-rate tensors show marked variation of $\dot{\epsilon}_1$ and $\dot{\epsilon}_2$ as is usually the case (Meier and others, 1957; Wu and Christensen, 1964; Holdsworth, 1965; and others). A value of the critical strain-rate has therefore been obtained from the diamond network ($0.60 \times 10^{-5} \pm 0.05 \times 10^{-5}$ d^{-1}) and this is

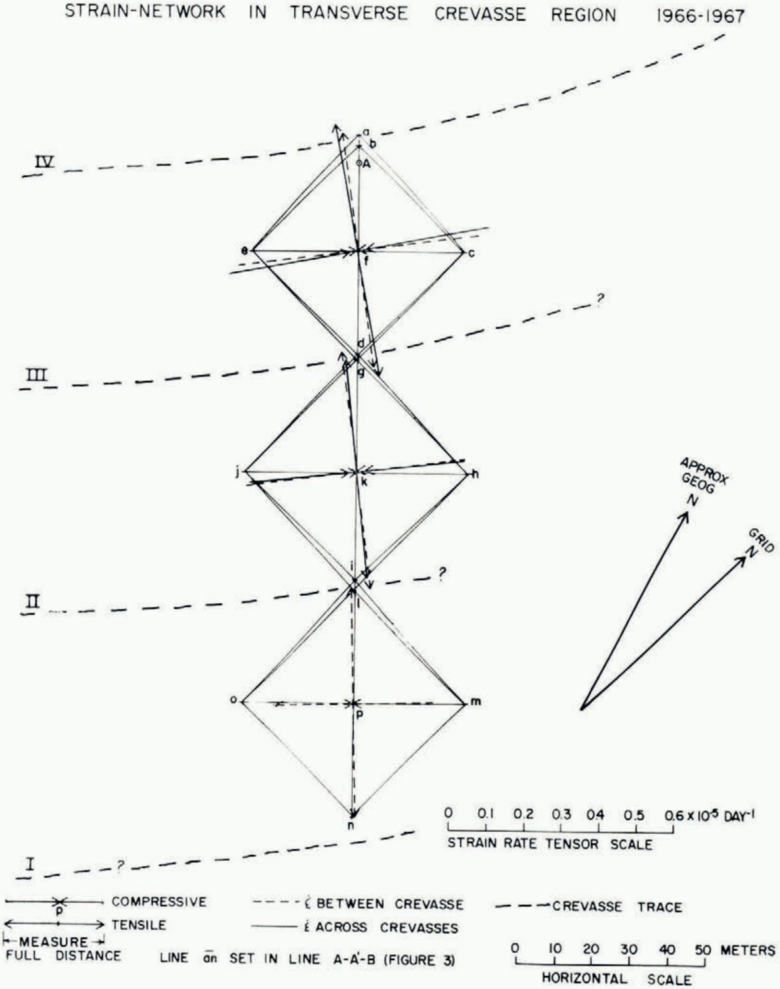

Fig. 9. Strain network in transverse crevasse region, Meserve Glacier, 1966–67.

seen to be 25% less than the regional value to which less confidence is given. A local value of the gradient of strain-rate is given as 0.0003×10^{-5} d^{-1} m^{-1} which is an order of magnitude lower than the value obtained for Kaskawulsh Glacier.

Table I(b) shows component values of theoretical longitudinal strain-rate calculated from the terms in Equation (1). The accumulation term is positive (based on a net gain of 3.0 cm of ice year^{-1} in this area of the basin). The conditions for the curvature term to be valid occur at point 1 (Fig. 3). The value of bed curvature has been used and thus $\dot{\epsilon}_y$ has been calculated from $\frac{\bar{V}}{w}\frac{dw}{dx}$ since there is no longer any restriction on the surface strain-rates but rather the average over the depth is considered. The rate of change of ice thickness has only been recorded at A with sufficient accuracy for calculations to be made. Allowing for surface slope, a drop in h of $dh = 3$ cm in $54.92\ d^{-1}$ gives a value of $-0.248\times10^{-5}\ d^{-1}$ for $\frac{1}{h}\frac{dh}{dt}$.

Figure 10 shows cross-sections of the Meserve Glacier in the crevasse region obtained by radio-sounding equipment. Table I(b) shows computed and measured values of longitudinal

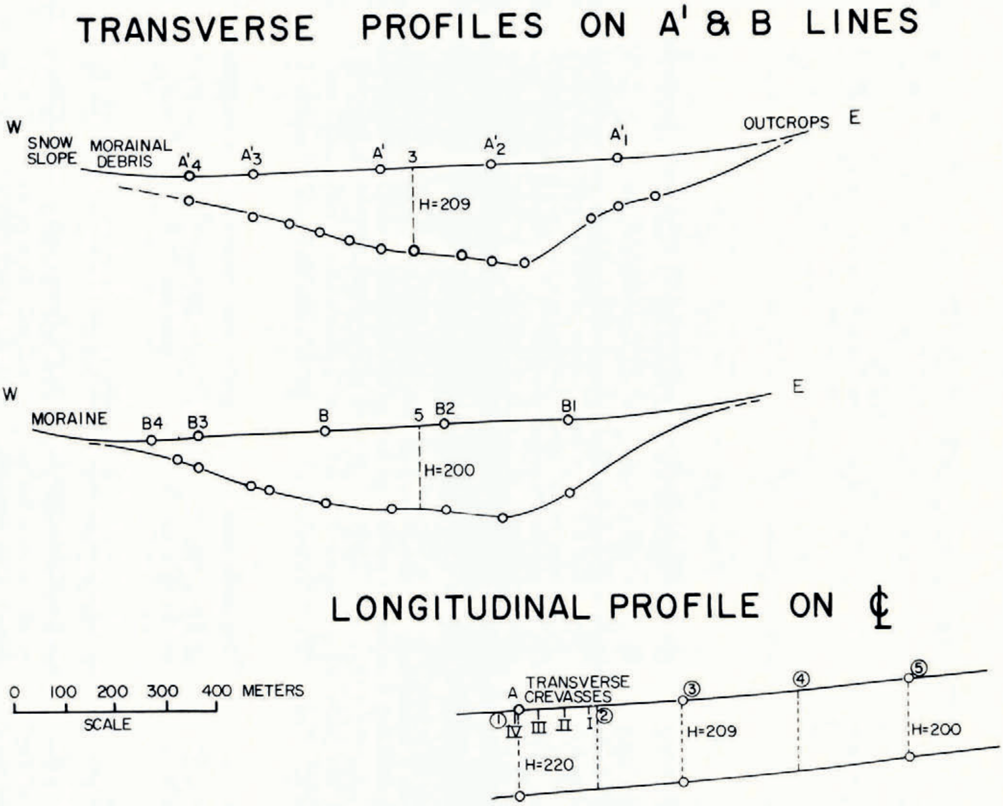

Fig. 10. Depth profiles for Meserve Glacier.

strain-rates. At point 1 there is a close agreement, but as in the case of Kaskawulsh Glacier, the boundary requirements for the bending term to be valid are rarely met. Consequently calculations for points 2–5 (Fig. 3) have been omitted.

Crevasse depths. Only one crevasse (IV) (Fig. 11) could be plumbed for depth with sufficient accuracy; a value of 23.9 ± 0.5 m was obtained. This is very similar to true transverse crevasse depth measurements on most other glaciers and is at the lower end of the range of values for Kaskawulsh Glacier. Table IV shows computed crevasse depths. Several assumptions have been made: (1) the fracture initiates at or near the surface, (2) the flow law holds for material of density less than 0.9 Mg m^{-3}, and (3) the horizontal strain-rate is constant with depth.

The first assumption implies that a temperature appropriate to the first few meters of firn should be used in order to compute B and n in the flow law. Glen's (1955) data extend as far as -13°C, which is about the mean summer temperature of the first meter.

That the density of the material in this zone is about half that of ice may be a serious objection. Nevertheless, the *ultimate* depth of the fracture *must* depend on the density of the material below; hence a mean density (0.7 Mg m^{-3}) over the depth d has been used. By this

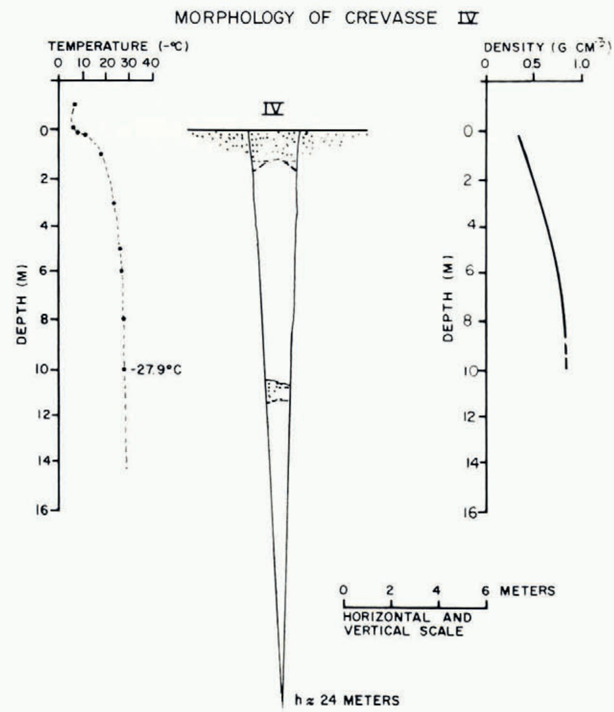

Fig. 11. Morphology of crevasse IV, Meserve Glacier.

TABLE IV. CREVASSE DEPTHS, MESERVE GLACIER

Crevasse No.	θ °C	$\bar{\rho}$ Mg m^{-3}	B bar^{-n} year^{-1}	n	$\dot{\epsilon}_K$ *across crevasse*	$\dot{\epsilon}_x$ *regional*	d (m) *Calculated using* $\dot{\epsilon}_K$	d (m) *Calculated using* $\dot{\epsilon}_x$	d (m) *Measured*
IV	-13	0.70	0.0017	3.15	704×10^{-5} year^{-1}	254×10^{-5} year^{-1}	47	32.3	23.9 ± 0.5
III		0.70	0.0017	3.15	—	232×10^{-5} year^{-1}	—	31.7	—
II		0.70	0.0017	3.15	—	216×10^{-5} year^{-1}	—	30.6	—
I	-13	0.70	0.0017	3.15	—	219×10^{-5} year^{-1}	—	31	—
	-20	0.70	0.00019	3.15	—	219×10^{-5} year^{-1}	—	64.5	—

Surface slope $<5°$.

reasoning, the fracture, once formed at the surface, propagates to a depth d by local increase in σ at the lower end of the crack as it penetrates colder ice.* Glen's (1955) flow-law constants are only determined as far as -13°C and extrapolation of his graphs is of dubious validity. If this is done, then $B_{-28} \approx 1.35\times10^{-5}$ bar^{-n} year^{-1} and $d \approx 154$ m. Similarly $B_{-20} \approx 19.05\times10^{-5}$ bar^{-n} year^{-1}, $d \approx 64.5$ m.

* Since $\dot{\epsilon} = B\sigma^n = B_0\, e^{-kT}\, \sigma^n$ a decrease in temperature is accompanied by an increase in σ, if $\dot{\epsilon}$ remains constant; n is also assumed constant, over the depth concerned.

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Hence it is apparent that it is the stress distribution rather than the strain which must be analyzed in an attempt to understand the limiting depth of a transverse fracture. In using Equation (2) it is evident that one must be consistent in taking a temperature known to exist in the region where $\dot{\epsilon}$ was measured.

In order to further convince oneself that the use of the strain-rate across a crevasse is inappropriate, the calculation of d is made using

(i) $\dot{\epsilon}_K$ across a crevasse, and

(ii) $\dot{\epsilon}_x$ regional.

It can be stated that (i) gives unreasonably large values of d in this case. For crevasse IV, $\dot{\epsilon}_K \approx 1.932 \times 10^{-5}\ \text{d}^{-1} \approx 704 \times 10^{-5}\ \text{year}^{-1}$ and hence $d \approx 47$ m for B appropriate to a temperature of $-13°$C, or $d \approx 230$ m for B appropriate to a temperature of $-28°$C, bearing in mind that the latter value of B is a gross extrapolation.

Crevasse spacing. Three measurements showed values of $s = 57$, 60 and 66 m, and for the case of crevasse IV, $s = 2.5\ d$, which is close to the empirical relation obtained for Kaskawulsh Glacier.

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APPENDIX I

Equation (1) provides a means of analysing the factors which produce variations in the longitudinal strain-rate in a glacier. If we consider only the accumulation and the curvature terms, then for a glacier which is sliding

$$\dot{\epsilon}_x = \frac{mV_b}{mV_b+U_s}\left(\frac{a}{h}+U_s\,\kappa\cot\alpha\right) \tag{10}$$

or

$$\dot{\epsilon}_x = \frac{mV_b}{mV_b+U_s}\left(\frac{a}{h}+(V_b+U)\,\kappa\cot\alpha\right). \tag{11}$$

Therefore

$$\mathrm{d}\dot{\epsilon}_x = \frac{mU}{[(m+1)\,V_b+U]^2}\left(\frac{a}{h}+U_s\,\kappa\cot\alpha\right)\mathrm{d}V_b+\frac{mV_b\,\kappa\cot\alpha}{[(m+1)\,V_b+U]}\cdot\mathrm{d}V_b \tag{12}$$

assuming that any changes in a/h are so small as to be negligible. Dividing Equation (12) by Equation (10) we get

$$\frac{\mathrm{d}\dot{\epsilon}_x}{\dot{\epsilon}_x} = \frac{U}{(m+1)\,V_b+U}\cdot\frac{\mathrm{d}V_b}{V_b}+\frac{V_b\,\kappa\cot\alpha}{(a/h+U_s\,\kappa\cot\alpha)}\cdot\frac{\mathrm{d}V_b}{V_b} \tag{13}$$

assuming that the accumulation rate is constant and that variations in $\dot{\epsilon}_x$ are due to changes in V_b only.

For the purposes of discussing the simple variation of $\dot{\epsilon}_x$ with V_b we now take the case where the term $\kappa \to 0$, that is the base is approximately planar.

Thus Equation (13) becomes

$$\frac{\mathrm{d}\dot{\epsilon}_x}{\dot{\epsilon}_x} = \frac{U}{(m+1)\,V_b+U}\cdot\frac{\mathrm{d}V_b}{V_b}. \tag{14}$$

Taking values of $U_s = 0.35\ \mathrm{m\,d^{-1}}$, $V_b = 0.09\ \mathrm{m\,d^{-1}}$, $\dfrac{\mathrm{d}\dot{\epsilon}_x}{\dot{\epsilon}_x} = 0.49\,\dfrac{\mathrm{d}V_b}{V_b}$,

we see that for a 10% change in the basal sliding rate, there will be a corresponding change in the longitudinal strain-rate of about 5%.

If the second term in Equation (13) is retained, then substituting the values appropriate to point 4 (Table I, p. 115), we obtain

$$\frac{\mathrm{d}\dot{\epsilon}_x}{\dot{\epsilon}_x} = 0.74\cdot\frac{\mathrm{d}V_b}{V_b}. \tag{15}$$

Hence a 10% change in V_b produces a 7.4% change in strain-rate. Such changes in basal sliding (Weertman, 1957, 1962, 1964) are not unfounded and could be exceeded in extreme cases such as surging or "catastophic advances".

If changes in a are considered to be significant (changes in h being neglected), then we may write, assuming V_b and a are functions of a common variable:

$$\frac{d\dot{\epsilon}_x}{\dot{\epsilon}_x} = \frac{U}{(m+1)\,V_b+U}\,\frac{dV_b}{V_b} + \frac{V_b\,\kappa\cot\alpha}{[a/h+U_s\,\kappa\cot\alpha]}\,\frac{dV_b}{V_b} + \frac{a}{h}\left[\frac{a}{h}+U_s\,\kappa\cot\alpha\right]^{-1}\frac{da}{a}. \tag{16}$$

Using the values for point 4 in Table I, Equation (16) reduces to

$$\frac{d\dot{\epsilon}_x}{\dot{\epsilon}_x} = 0.74\frac{dV_b}{V_b} + 0.1\,\frac{da}{a}.$$

For a 10% change in both V_b and a in the same sense, there is thus a corresponding 8.4% change in $\dot{\epsilon}_x$.

Such changes in strain-rate at a point on the glacier could be used to explain the irregularities in the spacings of crevasses. In the case of the cold glacier $V_b = 0$; therefore, changes of $\dot{\epsilon}_x$ can only come about by changes in a which have been shown to produce only small changes in $\dot{\epsilon}_x$. One would expect, therefore, crevasse spacings on cold glaciers to be more uniform, and in fact this has been shown to be the case.

APPENDIX II

If we consider the flow law

$$\dot{\epsilon} = B_0\,e^{-kT}\,\sigma^n = B\sigma^n$$

for the case where $\dot{\epsilon} = \dot{\epsilon}_c$ corresponding to $\sigma = \sigma_c$,

then
$$\sigma_c = \left(\frac{\dot{\epsilon}_c}{B}\right)^{1/n}$$

and assuming n to be roughly constant ≈ 3, then for the temperate case (Table II, p. 118)

$$\sigma_c = 0.43 \text{ bar},$$

and for the cold glacier case, taking $T \approx 260°$K (Table IV, p. 125)

$$\sigma_c = 1.09 \text{ bar}.$$

If in the first case a value of B appropriate to $-1.5°$C is taken

$$\sigma_c = 0.82 \text{ bar}.$$

So if we consider that the first crevasse on the temperate glacier is formed in the winter, the mean temperature of the upper few meters will certainly be $-1.5°$C or less, and allowing for a small percentage change in $\dot{\epsilon}_c$ (see Appendix I), it is seen that the values of σ_c for the "temperate" and cold glacier cases are not greatly different and are approximately 1 bar, a value hitherto assumed (e.g. Nye, 1951).

APPENDIX III

Consider a model glacier in which flow is confined to a linear channel (Fig. 12) of width $2a$. If we consider only the stresses acting in the plane of the glacier, σ_x, σ_y and τ_{yx}, then if we express σ_y and τ_{yx} in terms of σ_x (Hill, 1950), it should be possible to plot crevasse traces on the model according to different values of $\sigma_y = \alpha|\sigma_x|$ and $\tau^{\star}_{yx} = \beta|\sigma_x|$, that is, for varying lateral constraints to flow. $\tau^{\star}_{yx}$ is the boundary value of $\tau^{\star}_{yx}$, and α and β are pure numbers.

We may proceed with the result (e.g. Jaeger, 1962, p. 7)

$$\tan 2\phi = \frac{2\tau_{yx}}{\sigma_x-\sigma_y} \tag{17}$$

where ϕ is the inclination of the principal stress axis to the center-line of the glacier (x-axis).

Now
$$\tau^{\star}_{yx} \leqslant \left|\frac{\sigma_x}{\sqrt{2}}\right|$$

(Hill, 1950) depending on the side constraint. At the boundary, let

$$\tau^{\star}_{yx} = \beta\sigma_x$$

where $\beta \leqslant 1/\sqrt{2}$. For a point distance y from the center-line

$$\tau_{yx} = \beta\sigma_x y/a$$

where the assumption is made that the variation of τ_{yx} across the surface is linear, in order to simplify the problem.

Let
$$\sigma_y = \alpha\sigma_x$$

be uniform across the width for a particular value of α.

Substituting these values in Equation (17) we get

$$\tan 2\phi = \frac{2\beta}{1-\alpha}\cdot\frac{y}{a},$$

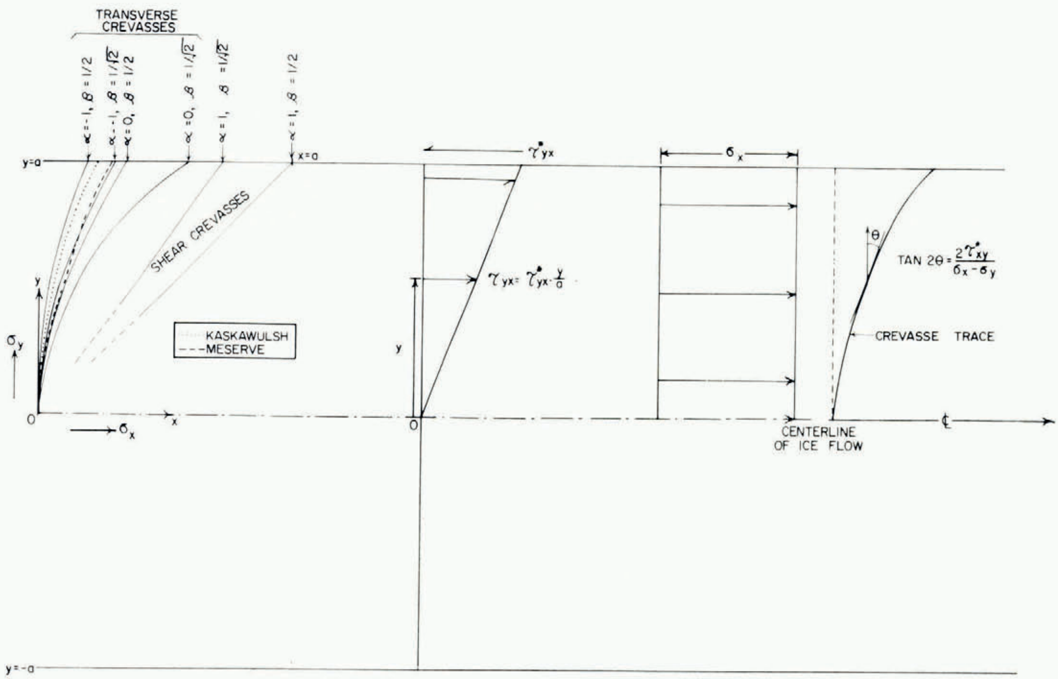

Fig. 12. Plot of crevasse traces for various values of α and β, where $\alpha = \sigma_y/\sigma_x$; $\beta = \tau^{\star}_{yx}/\sigma_x$ (see Appendix III).

$$2\sec^2 2\phi \, \mathrm{d}\phi = \frac{2\beta}{1-\alpha}\frac{\mathrm{d}y}{a},$$

therefore

$$\frac{\mathrm{d}\phi}{\mathrm{d}y} = \frac{\beta}{a(1-\alpha)}\cos^2 2\phi$$

which has a maximum when $\phi = 0$, i.e. on the center-line.

Now

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \cot\phi = \frac{1+\cos 2\phi}{\tan 2\phi \cos 2\phi};$$

on substituting for ϕ,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{[a^2(1-\alpha)^2+4\beta^2y^2]^{\frac{1}{2}}+a(1-\alpha)}{2\beta y}$$

therefore

$$\int_0^y \frac{2\beta y \, \mathrm{d}y}{[a^2(1-\alpha)^2+4\beta^2y^2]^{\frac{1}{2}}+a(1-\alpha)} = \int_0^x \mathrm{d}x.$$

For the left-hand integral put

$$u^2 = a^2(1-\alpha)^2+4^2\beta y^2$$

therefore

$$2u\,\mathrm{d}u = 8\beta^2 y\,\mathrm{d}y$$

then

$$\int_{a(1-\alpha)}^{[a^2(1-\alpha)^2+4\beta^2y^2]^{1/2}} \frac{1}{2\beta}\cdot\frac{u\,\mathrm{d}u}{u+a(1-\alpha)} = \frac{1}{2\beta}\Big[u-a(1-\alpha)\ln\{a(1-\alpha)+u\}\Big]_{a(1-\alpha)}^{[a^2(1-\alpha)^2+4\beta^2y^2]^{1/2}}$$

or

$$x = \frac{1}{2\beta}\left\{[a^2(1-\alpha)^2+4\beta^2y^2]^{\frac{1}{2}}-a(1-\alpha)-a(1-\alpha)\ln\left[\frac{a(1-\alpha)+[a^2(1-\alpha)^2+4\beta^2y^2]^{\frac{1}{2}}}{2a(1-\alpha)}\right]\right\}. \qquad (18)$$

We can now proceed to plot values of (x, y) given by Equation (18) for various values of α and β, except that there is a restriction that $\beta > 0$.

The crevasse traces are shown in Figure 12 for various values of α and β. The traces of crevasses on the Kaskawulsh and Meserve Glaciers are plotted also. For Meserve Glacier we know that $\sigma_y \approx -\sigma_x$ (Fig. 9). Therefore $\alpha \approx -1$, and taking $\beta = 1/\sqrt{2}$, the resulting trace falls very close to the trace measured in the field; therefore, we are led to believe that in a cold glacier the maximum boundary shearing stress is developed.

For the Kaskawulsh Glacier $\alpha = -1$ as a near approximation, β just exceeds $\frac{1}{2}$, so that a lower degree of lateral constraint at the margins of the glacier occurs here.