reviewer feels that the clarity might have been enhanced by the inclusion of more illustrations. Italics, we feel, are used to excess, sometimes for definitions and sometimes for emphasis, and the reader is not always certain for which purpose they are employed. We would have welcomed also introductory paragraphs outlining the purpose and chief results of each chapter, for, as is common in many works on algebra, the reader often finds it difficult to decide which results are of prime importance and which results are secondary. The printing is excellent.

D. E. RUTHERFORD

PICCARD, SOPHIE, *Sur les bases des groupes d'ordre fini* (Neuchatel, Secrétariat de l'Université, 1957), pp. xxiv + 242.

Mlle. Professor Piccard is known not only as an enthusiast but also as the expert on the bases of finite permutation groups. It follows that those who are interested in this topic will give her book a warm welcome. To describe the subject matter to the uninitiated it is perhaps best to illustrate it with an example. The symmetric group $\mathscr{S}_4$ of all permutations on four letters which we may designate simply as 1, 2, 3, 4, is generated by the three transpositions (12), (23), (34). These three independent generators form a system of order 3. Quite apart from equivalent systems of generators obtained from the above merely by renumbering the letters, other systems of independent generators are possible and those of the smallest possible order are called bases. In $\mathscr{S}_4$ the bases are of order 2 and indeed this group can be generated by any one of the following pairs of permutations (123), (34); (1234), (12); (1234), (123); (1234), (132); (1234), (1324). Thus $\mathscr{S}_4$ has five different types of base. For any given base there are a set of characteristic relations which define the group. For instance if $S = (1234)$ and $T = (1324)$ the relations which define $\mathscr{S}_4$ are $S^4 = TS^2T^{-3}S^2 = TSTS^3T^{-1}S^3 = 1$. The book under review is a study of such bases and characteristic relations, especially those of the symmetric and alternating groups but also of many other permutation groups with special types of base. The value of the subject is that it throws some light on the difficult problem of classifying all finite groups of a given order according to their structure.

Parts of this volume are necessarily in the nature of a catalogue of bases of those groups which have been fully investigated. For instance there are 2308320 bases of the alternating group $\mathscr{A}_7$. As might be expected Hölder's theorem that $\mathscr{S}_6$ is the only symmetric group possessing outer automorphisms can be established by considering characteristic relations.

The book is carefully composed and excellently printed. A very useful feature of the work is a preliminary synopsis defining all those group concepts with which the reader is expected to be familiar, and including some results established in Mlle. Piccard's two previous works on the symmetric group. These features make the book self-contained. It is a pity that there is no index in addition to the Table of Contents.

D. E. RUTHERFORD

KHINCHIN, A. Y., *Three Pearls of Number Theory* (Graylock Press, Rochester, N.Y., 1956), \$2.00, 64 pp., 16s.

This little book contains an account of three very beautiful results in Number Theory. They are typical of the subject in three respects: the deceptive simplicity of statement; the complication of their proofs (which even Khinchin's lucid exposition cannot entirely hide); and the fact that each one was first proved by a young man on the threshold of his career, after defeating the efforts of learned mathematical scholars.

The book is thus not merely a lifeless exposition of abstractions, but a human document, full of challenge and stimulation. It is excellent reading for the discouraged research student, who will take heart from the story of how the mathematical world was made to sit up and take notice by the young van der Waerden, Mann, and Linnik.

It should be recorded that, since Chapter II was first written in 1945, it has been partly superseded by the work of van der Corput, Martin Kneser, and others.

A. M. MACBEATH

APOSTOL, T. M., *Mathematical Analysis* (Addison-Wesley, Reading, Mass., 1957), 553 pp., 76s.

This book aims to fulfil the long-felt want of a textbook which will deal rigorously with the part of the subject now known as "advanced calculus". The author proves very carefully, with a proper statement of conditions, theorems like Green's theorem, which are unsatisfactorily dealt with in most textbooks.

Inevitably, the book suffers slightly from the complication which so rigorous treatment must at first involve. Some of this is a result of the intrinsic difficulty of the material, but there are places at which simplification would be possible. For instance, the statement of the Mean Value Theorem of the Differential Calculus is needlessly complicated by allowing the function to have, at the endpoints, two jump discontinuities which cancel one another out.

The following features are particularly welcome:

(i) use of vector notation and the treatment of functions mapping one Euclidean space into another,

(ii) the use of set theory and simple topological terms and ideas,

(iii) a satisfactory treatment of Gauss's, Stokes's and Green's theorems.

The book also includes chapters on Riemann-Stieltjes integration, on Fourier analysis, and on Cauchy's theorem and calculus of residues. It seems a pity that the Stieltjes integral $\int f(x)dg(x)$ is treated without first dealing with the special case $\int f(x)dx$. One also regrets the absence of a definition of real numbers, either using Dedekind section or Cauchy sequences. And one would have liked to see a treatment from first principles of the exponential and trigonometric functions.

There would appear to be a misprint in the statement of the inverse function theorem on p. 144. Surely condition (ii) ought to be not $X = f^{-1}(Y)$, but $X \subset f^{-1}(Y)$. Incidentally the proof given of the inverse function theorem is unusual and rather interesting.

By and large, the book succeeds in its aim. Teachers of analysis should not be without it, though for students it is at times a little severe. A. M. MACBEATH

SPRINGER, G., *Introduction to Riemann Surfaces* (Addison-Wesley, Reading, Mass., 1958), pp. viii + 305, 76s.

This is a modern presentation of the classical theory of Riemann surfaces. The author assumes that his reader has a knowledge of elementary complex variable theory and a little algebra and real variable theory, but gives a sufficient introduction to topology and Hilbert space for his purpose. The material is clearly and carefully explained and frequently illustrated by figures. The book is not intended to be an account of modern work in the subject but would be a useful introductory text for advanced undergraduate reading. E. M. WRIGHT

HAYMAN, W. K., *Multivalent Functions* (Cambridge University Press, 1958), 168 pp., 27s. 6d.

The object of this tract is to study the growth of univalent and multivalent functions $f(z)$ which are regular in the unit circle, and, in particular, to obtain bounds for the absolute values and coefficients of such functions.

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