PROBLEMS FOR SOLUTION

P 125. Let p be a prime > 5. Show that there exist primes q and n, both less than p, such that q is a quadratic residue and n is a quadratic non-residue (mod p).

J. Dixon, University of New South Wales

P 126. On any σ-finite, infinite measure space there exists a strictly positive, bounded function vanishing at infinity, but with infinite integral, and a similar function with finite integral.

J.E. Marsden, Princeton University

P 127. A spread in euclidean 3-space is a collection of skew lines with one line through every point. Give an easily visualized example.

J. Wilker, University of British Columbia

SOLUTIONS

P 110. Find the order, class, number of nodes, and number of cusps of the curve

$$x_1^{2/3} + x_2^{2/3} + x_3^{2/3} = 0$$

in the complex projective plane.

H.S.M. Coxeter, University of Toronto

Solution by G.J. Griffith, University of Saskatchewan.

The curve $x^{2/3} + y^{2/3} + z^{2/3} = 0$ is rational with parametric representation

$$\rho x = (1 - t^2)^3 \ ; \ \rho y = 8t^3 \ ; \ \rho z = i(1 + t^2)^3 \ .$$

Since it is clear that $x = \cos^3\theta$, $y = \sin^3\theta$, $z = i$ satisfies the equation. By letting $t = \tan{}^{\theta}/2$ we immediately obtain the above representation. Therefore the order of the curve is six.

The homogeneous line coordinates of the curve are

$$\sigma u = 2t(1 + t^2) \ ; \ \sigma v = 1 - t^4 \ ; \ \sigma w = 2it(1 - t^2) \ .$$

Therefore the order of the dual, and hence the class of the original curve is four. Since the curve is rational, it has genus zero.

The equation

$$p = \frac{1}{2}(n - 1)(n - 2) - (\delta + \kappa)$$

[1] equation (3), page 100, and the so called first Plücker equation

$$m = n(n - 1) - 2\delta - 3\kappa$$

[1] equation (1), page 99, yield

$$\kappa = 6 \quad \text{and} \quad \delta = 4 \ .$$

Equations (1') and (3) pages 99, 100 of [1] yield

$$\iota = 0 \quad \text{and} \quad \tau = 3 \ .$$

It is readily verified that the cusps are located at the points $(1, i, 0)$, $(1, -i, 0)$, $(1, 0, i)$, $(1, 0, -i)$, $(0, 1, i)$ and $(0, 1, -i)$ all lying on the conic $x^2 + y^2 + z^2 = 0$ and that the nodes are at the points

$$(1, 1, 1), \ (1, 1, -1), \ (1, -1, 1) \ (-1, 1, 1) \ ,$$

these real points being acnodes of the curve, whilst the bitangents are the sides of the triangle of reference having coordinates

$$[1, 0, 0], \ [0, 1, 0], \ [0, 0, 1] \ ,$$

these being the cuspidal tangents.

It should be noted that this curve is projectively equivalent to the hypocycloid having four real cusps.

REFERENCE

1. Algebraic Plane Curves. J.L. Coolidge, Dover Publications, 1959.

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