2024 NORTH AMERICAN ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Iowa State University
Ames, IA, USA
May 14–17, 2024

The 2024 North American Annual Meeting of the Association for Symbolic Logic was held at Iowa State University in Ames, Iowa, May 14–17, 2024. There were six plenary talks, two tutorials, six special sessions, four sessions of contributed talks, and the Karp prize lecture. A welcoming reception was held on Tuesday, May 14.

The program committee consisted of Barbara Csima (Chair), Nick Galatos, Alex Kruckman, Tim McNicholl, and Itay Neeman. The local organizing committee consisted of Samik Basu, Tim McNicholl (Chair), Konstantin Slutsky, Eric Weber, and Kristin Yvonne-Rozier.

There were 139 registered participants at the meeting, of which 59 were graduate students. Generous financial support was provided by the Association for Symbolic Logic, the National Science Foundation and Iowa State University, including the College of Liberal Arts and Sciences, the Department of Computer Science, the Department of Mathematics and the Scott Hanna fund. The plenary addresses at the meeting were:

Monroe Eskew (Kurt Gödel Research Center), *Dense ideals*.

Phokion G. Kolaitis (University of California Santa Cruz and IBM Research), *Characterizing rule-based languages*.

Katherine Kosaian (Iowa State University), *Formalizing mathematics in Isabelle/HOL, ft. algorithms for real quantifier elimination*.

Joel Nagloo (University of Illinois Chicago), *Model theory and automorphic functions*.

Tommaso Moraschini (Universitat de Barcelona), *Spectra of Heyting algebras*.

Henry Towsner (University of Pennsylvania), *Proofs that modify proofs*.

There were two tutorials, consisting of three one-hour sessions each:

Anush Tserunyan (McGill University), *How a logician proves pointwise ergodic theorems*.

Ross Willard (University of Waterloo), *The constraint satisfaction problem dichotomy theorem*.

The tenth Carol Karp prize was awarded to John Steel in 2023 for his work in set theory, especially for his book *A Comparison Process for Mouse Pairs*. The Karp prize lecture, *Descriptive inner model theory*, was given by Grigor Sargsyan (Instytut Matematyczny Polskiej Akademii Nauk).

The program included six special sessions: Algebraic Logic (Guram Bezhanishvili and José Gil-Ferez), Computability Theory (Rachael Alvir and Steffen Lempp), Logic in Computer Science (Larry Moss and Scott Weinstein), Model Theory (Chris Laskowski and Adele Padgett), Set Theory (Clinton Conley and Nam Trang), and Universal Algebra (Charlotte Aten and Keith Kearnes). There were 13 contributed talks delivered at the meeting.

Abstracts of the invited talks and the contributed talks by members of the Association for Symbolic Logic follow.

For the Program Committee
BARBARA CSIMA

Abstract of the Karp Prize Lecture

▶ GRIGOR SARGSYAN, *Descriptive inner model theory*.
Department of Foundations of Mathematics, Instytut Matematyczny Polskiej Akademii Nauk, Poland.
E-mail: gsargsyan@impan.pl.
The talk is a tribute to Steel's *A Comparison Process for Mouse Pairs*.

Abstract of invited tutorials

▶ ANUSH TSERUNYAN, *How a logician proves pointwise ergodic theorems*.
Mathematics & Statistics Department, McGill University, 805 rue Sherbrooke O, Montréal, QC, H3A 0B9, Canada.
E-mail: anush.tserunyan@mcgill.ca.
Descriptive set theory offers methods for studying global properties of sets and functions on a Polish space via local combinatorics on the level of points of the space. I will illustrate this mantra by showing how to reduce pointwise ergodic theorems to finitary tiling problems. We will give an elementary proof of the classical pointwise ergodic theorem for a probability measure preserving (pmp) transformation, and hint at a similar proof of Lindenstraus's ergodic theorem for pmp actions of amenable groups. We will then discuss a more recent "backward" ergodic theorem for one pmp transformation and pointwise ergodic theorems for pmp actions of free semigroups (joint work with Jenna Zomback).

▶ ROSS WILLARD, *The constraint satisfaction problem dichotomy theorem*.
Pure Mathematics Department, University of Waterloo, 200 University Avenue West, Waterloo N2L 3G1, Canada.
E-mail: ross.willard@uwaterloo.ca.
URL Address: www.math.uwaterloo.ca/~rdwillar/.
In the late 1990s, T. Feder and M. Vardi [3] isolated the class of "constraint satisfaction problems with fixed (finite) template," or CSPs, as an interesting and robust class of computational problems, and they conjectured that every problem in this class is either in P or is NP-complete (or both if P = NP). CSPs are parameterized by finite relational structures in finite signatures, and (as it turns out) the computational problems they encode are profitably studied via techniques coming from universal algebra. Interest in the CSP dichotomy conjecture brought (some) computer scientists and universal algebraists together starting in the early 2000s, culminating in 2017 with two independent proofs of the conjecture by A. Bulatov [1,2] and D. Zhuk [4,5].

Given a finite structure M, the computational problem $\mathrm{CSP}(M)$ is just the problem which, given a formula φ which is a conjunction of atomic formulas in the signature of M, asks whether φ^M is non-empty. There is an "obvious reason" for $\mathrm{CSP}(M)$ to be NP-complete: M pp-interprets the two-element structure encoding 3-SAT. Bulatov and Zhuk proved that, if M fails to be NP-complete for this obvious reason, then $\mathrm{CSP}(M)$ is tractable. To do this, they used algebra (old and new) to develop rudimentary structure theories for sets definable by conjunction-of-atomic formulas in the relevant finite structures M.

In this tutorial I will introduce CSPs, explain the universal algebraic perspective, and attempt to give at least an impression of one of the two structure theories (Zhuk's) and how it is used to prove tractability of CSP(M).

[1] A. A. BULATOV, *A dichotomy theorem for nonuniform CSPs*, **58th Annual IEEE Symposium on Foundations of Computer Science–FOCS 2017** (Berkeley, CA), IEEE Computer Society, 2017, pp. 319–330.

[2] ———, *A dichotomy theorem for nonuniform CSPs simplified*, arXiv:2007:09099 (2020), 42 pp.

[3] T. FEDER AND M. Y. VARDI, *The computational structure of monotone monadic SNP and constraint satisfaction: A study through datalog and group theory*, **SIAM Journal on Computing**, vol. 28 (1999), no. 1, pp. 57–104.

[4] D. ZHUK, *A proof of CSP dichotomy conjecture*, **58th Annual IEEE Symposium on Foundations of Computer Science–FOCS 2017** (Berkeley, CA), IEEE Computer Society, 2017, pp. 331–342.

[5] ———, *A proof of the CSP dichotomy conjecture* **Journal of the ACM**, vol. 67 (2020), no. 5, Art. 30, 78 pp.

Abstracts of invited plenary lectures

▶ MONROE ESKEW, *Dense ideals*.
Kurt Gödel Research Center, University of Vienna, 14-16 Kolingasse, 1090 Vienna, Austria.
E-mail: monroe.eskew@univie.ac.at.

We present recent work with Yair Hayut showing that, relative to a huge cardinal, it is consistent that simultaneously for all $n \geq 1$, there is an ω_n-complete I on ω_n such that $\mathcal{P}(\omega_n)/I \cong \mathrm{Col}(\omega_{n-1}, \omega_n)$. This answers some questions of Foreman [2] and has several striking combinatorial consequences. By theorem of Woodin, our model satisfies a certain "transfer principle" that copies structures on ω_m to ω_n for $m < n$, enabling the solution to several problems in infinite combinatorics concerning graph colorings [3] and partition principles [1].

[1] N. BANNISTER, J. BERGFALK, J. T. MOORE, AND S. TODORCEVIC, *A descriptive approach to higher derived limits* **Journal of the European Mathematical Society**, to appear.

[2] M. FOREMAN, *Ideals and generic elementary embeddings*, **Handbook of set theory, Vol. 2** (M Foreman and A. Kanamori, editors), Springer, Dordrecht, 2010, pp. 885–1147.

[3] S. TODORCEVIC, *Combinatorial dichotomies in set theory* **The Bulletin of Symbolic Logic**, vol. 17 (2011), no. 1, pp. 1–72.

▶ PHOKION G. KOLAITIS, *Characterizing rule-based languages*.
Computer Science and Engineering Department, University of California Santa Cruz and IBM Research.
E-mail: kolaitis@ucsc.edu.

There is a mature body of work in logic aiming to characterize logical formalisms in terms of their structural or model-theoretic properties. The origins of this work can be traced to Alfred Tarski's program to characterize metamathematical notions in "purely mathematical terms" and to Per Lindström's abstract characterizations of first-order logic. For the past forty years, rule-based logical languages have been widely used in databases and in related areas of computer science to express integrity constraints and to specify transformations in data management tasks, such as data exchange and ontology-based data access. The aim of this talk is to present an overview of more recent results that characterize various classes of rule-based logical languages in terms of their structural or model-theoretic properties.

► KATHERINE KOSAIAN, *Formalizing mathematics in Isabelle/HOL, ft. algorithms for real quantifier elimination*.

Department of Aerospace Engineering, Iowa State University, Howe Hall, 537 Bissell Rd, IA 50011, United States.

E-mail: kkosaian@iastate.edu.

There are many mathematical algorithms that are used in safety-critical contexts. Correctness of these algorithms, and the mathematical results underlying them, is crucial. In formal methods, a piece of software called a theorem prover can be used to formally verify algorithms. In this approach, code for an algorithm is accompanied by a rigorous proof of correctness that only depends on the logical foundations of the theorem prover. Algorithms that have been verified in this way are highly trustworthy and thus safe for use in safety-critical applications.

The theorem prover Isabelle/HOL is well-suited for formalizing mathematics. This talk will motivate formalized mathematics, exhibit how mathematics is formalized in Isabelle/HOL, and discuss work on formalizing algorithms for real quantifier elimination as a use case.

► TOMMASO MORASCHINI, *Spectra of Heyting algebras*.

Departament de Filosofia, Facultat de Filosofia, Universitat de Barcelona (UB), Carrer Montalegre, 6, 08001 Barcelona, Spain.

E-mail: tommaso.moraschini@ub.edu.

The *prime spectrum* of a Heyting algebra is the poset of its prime filters. In view of Esakia duality [3,4], a poset is isomorphic to the prime spectrum of a Heyting algebra if and only if it can be endowed with a topology that turns it into an Esakia space. Because of this, posets isomorphic to the prime spectrum of some Heyting algebra have been called *Esakia representable*. The problem of describing Esakia representable posets was first raised in [4] and echoes analogous problems for distributive lattices [5] and commutative rings [6].

While the problem of describing Esakia representable posets remains open, we will describe the Esakia representable well-ordered trees. Furthermore, we shall tackle the problem of determining which posets are the top parts of some Esakia space. More precisely, a poset is said to be *image finite* when its principal upsets are finite. A poset X is the *image finite part* of a poset Y when X is the subposet of Y consisting of all the elements whose principal upset is finite. The problem of determining whether every image finite poset is the image finite part of some Esakia space was raised in [1]. Notably, it can be equivalently phrased as follows: is every profinite Heyting algebra the profinite completion of some Heyting algebra? We answer this problem negatively by showing that every profinite member of an equational class K of Heyting algebra is a profinite completion if and only if K omits the Heyting algebras of upsets of the four posets depicted below.

This talk is based on joint work with G. Bezhanishvili, N. Bezhanishvili, D. Fornasiere, and M. Stronkowski. Some of these results have been collected in [2].
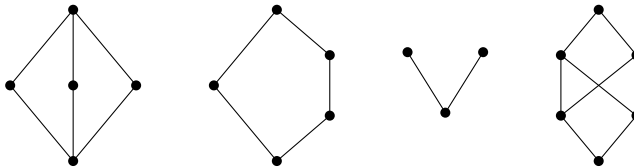
[1] G. BEZHANISHVILI AND P. J. MORANDI, *Profinite Heyting algebras and profinite completions of Heyting algebras* **Georgian Mathematical Journal**, vol. 16 (2009), pp. 29–47.

[2] G. BEZHANISHVILI, N. BEZHANISHVILI, T. MORASCHINI, AND M. STRONKOWSKI, *Profiniteness and representability of spectra of Heyting algebras*. **Advances in Mathematics**, vol. 391 (2021), 107959, 47 pp.

[3] L. ESAKIA, *Topological Kripke models* **Soviet Mathematics Doklady**, vol. 15 (1974), pp. 147–151.

[4] L. ESAKIA, **Heyting Algebras. Duality Theory**, (A Evseev, translator), Springer, 2019.

[5] G. GRÄTZER, **Lattice Theory. First Concepts and Distributive Lattices**, W. H. Freeman and Company, 1971.

[6] I. KAPLANSKY, **Commutative rings**, revised edition, The University of Chicago Press, 1974.

▶ JOEL NAGLOO, *Model theory and automorphic functions.*
University of Illinois Chicago, Department of Mathematics, Statistics, and Computer Science, 851 S. Morgan Street, Chicago, IL 60607-7045, USA.
E-mail: jnagloo@uic.edu.

Roughly speaking, an automorphic function is a function on a space that is invariant under the action of some group and as such, a function on the quotient space. These functions appear centrally in many areas of mathematics, especially in Diophantine geometric problems such as the André-Oort and the Zilber–Pink conjectures.

Over the past two decades, automorphic functions—and more generally covering maps—have been studied from a variety of model theoretic perspectives. In this talk, I will discuss three such interactions:

1. Definability of (restrictions of) automorphic functions in o-minimal expansions of the reals.
2. Strong minimality, in differentially closed fields, of the definable sets/differential equations satisfied by automorphic functions.
3. $L_{\omega_1,\omega}$-categoricity in power of the theory of a fixed automorphic function.

I will highlight the history of those approaches and point out some of the connections between them.

▶ HENRY TOWSNER, *Proofs that modify proofs.*
Department of Mathematics, University of Pennsylvania.
E-mail: htowsner@upenn.edu.
URL Address: http://www.math.upenn.edu/~htowsner.

One of the main projects of proof theory has been *ordinal analysis*, which measures the consistency strength of a theory by assigning the theory a computable ordinal which captures the combinatorial complexity of proofs in the theory. Gentzen famously showed that $\epsilon_0 = \omega^{\omega^{\cdot^{\cdot^{\cdot}}}}$ is the proof-theoretic ordinal of Peano arithmetic, and nearly a century of work in proof theory has been devoted to extending this analysis to stronger theories.

A recurring idea in proof theory is the interplay between infinitary mathematical objects and finite descriptions of these objects. This has played an important role in ordinal analysis, which often introduces infinitary proof systems—for instance, systems with the ω rule, which deduces $\forall x\, \phi(x)$ from proofs of $\phi(n)$ for each n—and nonetheless uses these infinitary proof systems to give finite, constructive proofs by replacing the ω rule with a *computable* ω rule which deduces $\forall x\, \phi(x)$ from a function which computably assigns, to each n, a proof of $\phi(n)$.

In this talk, I will discuss how this approach can be extended to functions on proofs. We will describe an infinitary proof system whose (no longer well-founded) proofs can be interpreted as algorithms describing a function from proofs to proofs. By iterating this process, obtaining

proofs which represent algorithms transforming algorithms into other algorithms, we are able to give an ordinal analysis of full second order arithmetic.

Abstracts of invited talks in the Special Session on Algebraic Logic

▶ MARCO ABBADINI*, VINCENZO MARRA, AND LUCA SPADA, *A duality for metrically complete lattice-ordered groups*.

School of Computer Science, University of Birmingham, University Rd W, B15 2TT Birmingham, UK.
E-mail: marco.abbadini.uni@gmail.com.
URL Address: https://marcoabbadini-uni.github.io.
Dipartimento di Matematica "Federigo Enriques", Università degli Studi di Milano, via Cesare Saldini 50, 20133 Milano, Italy.
E-mail: vincenzo.marra@unimi.it.
Dipartimento di Matematica, Università degli Studi di Salerno, Piazza Renato Caccioppoli, 2, 84084 Fisciano (SA), Italy.
E-mail: lspada@unisa.it.

Just like Boolean algebras are the algebraic semantics of classical propositional logic, Abelian lattice-ordered groups provide an algebraic tool for certain nonclassical logics. For example (due to their equivalence with MV-algebras), they are fundamental for the algebraic investigation of Łukasiewicz many-valued logic.

In a landmark paper in 1936, Stone obtained a representation theorem for Boolean algebras. In modern terms, his result amounts to a dual equivalence between Boolean algebras and Stone spaces. We provide an analogous duality for a class of Abelian lattice-ordered groups (with a designated element called unit). Our work takes inspiration from Yosida's duality (1941) between compact Hausdorff spaces and metrically complete unital vector lattices. We extend Yosida's theorem to metrically complete unital lattice-ordered groups, thus dropping the structure of real vector spaces. This calls for a generalised notion of compact Hausdorff space whose points carry an arithmetic character. The arithmetic character of a point is a metrically complete additive subgroup of $\mathbb{R}$ containing 1—namely, either $\frac{1}{n}\mathbb{Z}$ for an integer $n = 1, 2, ...$, or $\mathbb{R}$. The original Yosida duality is obtained by considering the full subcategory of spaces every point of which is assigned $\mathbb{R}$.

This presentation is based on [1].

[1] M. ABBADINI, V. MARRA, AND L. SPADA, *Stone-Gelfand duality for metrically complete lattice-ordered groups*, arXiv/2210.15341.

▶ PAPIYA BHATTACHARJEE, *Max(dL) and fusible frames*.

Department of Mathematical Sciences, Florida Atlantic University, 777 Glades Road, Boca Raton, FL 33431, USA.
E-mail: pbhattacharjee@fau.edu.

The space of maximal d-ideals of $C(X)$ is well known and the concept of d-elements has been generalized for algebraic frames with the FIP by J. Martìnez and E. Zenk. In a recent article, maximal d-subgroups of a lattice-ordered group has been introduced and studied by the speaker along with W. McGovern. Finally, the space of maximal d-elements and some properties of this topological space with respect to the hull-kernel topology has been studied for algebraic frames with FIP (by the speaker). It turns out that even though certain behaviors of Max_d-objects hold for $C(X)$ and any W-objects, these behaviors do not follow through in frames. The speaker will discuss different properties of $Max(dL)$ objects and introduce the concept of "fusibility" for frames. It will be revealed that fusible frames play an important role in the study of $Max(dL)$.

[1] P. Bhattacharjee, *Maximal d-elements of an algebraic frame* **Order**, vol. 36 (2019), pp. 377–390.

[2] P. Bhattacharjee, and W. Wm. McGovern, *Maximal d-subgroups and ultrafilters* **Rendiconti del Circolo matematico di Palermo**, vol. 67 (2018), no. 3, pp. 421–440.

[3] M. Henriksen, J. Vermeer, and R. G. Woods., *Quasi F-covers of Tychonoff spaces* **Transactions of the American Mathematical Society**, vol. 303 (1987), no. 2, pp. 779–803.

[4] C. B. Huijsmans, and B. de Pagter, *Maximal d-ideals in a Riesz space* **Canadian Journal of Mathematics**, vol. 35 (1983), no. 6, pp. 1010–1029.

[5] J. Martìnezand E. Zenk, *When an algebraic frame is regular* **Algebra Universalis**, vol. 50 (2003), pp. 231–257.

▶ XAVIER CAICEDO, *The model completion of projectable abelian ℓ-groups*.
Departamento de Matemáticas, Universidad de los Andes.
E-mail: xcaicedo@uniandes.edu.co.

Projectable abelian lattice ordered groups are Boolean products of linearly ordered abelian groups, and they have a model completion which consists on the divisible members of the class, without weak units, and having atomless Boolean skeleton of each closed interval. This allows us to identify the model completion of the variety of abelian lattice ordered groups supporting an additional operation, $(AL)_{\rhd}$, introduced recently by Metcalfe and Reggio (*Model completions for universal classes of algebras: necessary and sufficient conditions*, JSL 2022).

▶ NIKOLAOS GALATOS AND ISIS A. GALLARDO*, *Decidability and generation of the variety of distributive ℓ-pregroups*.
Department of Mathematics, University of Denver, Denver, CO, USA.
E-mail: isis.gallardo@du.edu.

Lattice-ordered pregroups (ℓ-pregroups) represent a natural generalization of lattice-ordered groups (ℓ-groups). It is well-established that every ℓ-group can be embedded into a symmetric one, as demonstrated by Cayley–Holland's embedding theorem. Analogously, a Cayley–Holland's embedding theorem exists for distributive ℓ-pregroups, asserting that any distributive ℓ-pregroup can be embedded into a functional one.

In this work, we enhance this result by establishing that any distributive ℓ-pregroup can be embedded into a functional one over a chain that is locally isomorphic to $\mathbb{Z}$. Utilizing this, we demonstrate that the variety of distributive ℓ-pregroups is generated by the (single) functional algebra over the integers. We will later use this to prove the decidability of the variety.

▶ WESLEY H. HOLLIDAY, *Modal logic, fundamentally*.
Department of Philosophy, University of California, Berkeley, CA, USA.
E-mail: wesholliday@berkeley.edu.

Non-classical generalizations of classical modal logic have been developed in the contexts of constructive mathematics and natural language semantics. In this talk, we will discuss a general approach to the semantics of non-classical modal logics via algebraic representation theorems. We begin with complete lattices L equipped with an antitone operation $\neg$ sending 1 to 0, a completely multiplicative operation $\Box$, and a completely additive operation $\Diamond$. Such lattice expansions can be represented by means of a set X together with binary relations $\lhd$, R, and Q, satisfying some first-order conditions, used to represent $(L, \neg)$, $\Box$, and $\Diamond$, respectively. Indeed, any lattice L equipped with such a $\neg$, a multiplicative $\Box$, and an additive $\Diamond$ embeds into the lattice of propositions of a frame $(X, \lhd, R, Q)$. Building on our recent study of *fundamental logic* [1], we then focus on the case where $\neg$ is dually self-adjoint ($a \leq \neg b$

implies $b \leq \neg a$) and $\Diamond \neg a \leq \neg \Box a$. In this case, our representations can be constrained so that $R = Q$, i.e., we need only add a single relation to $(X, \lhd)$ to represent both $\Box$ and $\Diamond$. Using these results, we can prove that a system of fundamental modal logic is sound and complete with respect to an elementary class of bi-relational structures $(X, \lhd, R)$.

[1] W. H. Holliday, *A fundamental non-classical logic*, **Logics**, vol. 1 (2023), pp. 36–79.

▶ JAMES MADDEN, *Preservation of subfitness of semilattices under certain constructions*.
Department of Mathematics, Louisiana State University, Baton Rouge LA, 70808, USA.
E-mail: mmmad@lsu.edu.

Let $(S, \vee, 0)$ be a join-semilattice with 0. S said to be *subfit* if S has a largest element 1 and for all $x, x' \in S$, if $\forall y \in S$ $(x \vee y = 1 \iff x' \vee y = 1)$, then $x = x'$. Subfitness was first introduced in 1938 (under a different name) by Wallman [3] as a condition on the semilattice of open sets of a topological space, and it was studied in a purely algebraic setting by Piece in 1954 [2].

For $a \in S$, let $(a) := \{ x \in S \mid x \leq a \}$. Recall that S is said to be *distributive* if, whenever $z \leq x \vee y$ in S, there are $x', y' \in S$ such that $x' \leq x$, $y' \leq y$ and $z = x' \vee x'$. At the 2022 BLAST Conference, Madden posed the following question: *Suppose S is a distributive semilattice, $a, b \in S$ and $a \vee b = 1$. If the subsemilattices (a) and (b) are subfit, does it follow that S is subfit?*

A simple finite example shows that the distributive hypothesis is necessary. When the problem was announced, the answer was known to be "Yes" for finite semilattices. In late 2022, Tressl proved that if S is a distributive lattice, then the answer is, "Yes," and in late 2023, Bezhanishvili produced an example showing that in general, the answer is "No."

Subfitness is an important separation property in the theory of locales; see [1]. The question above is the simplest of a family of local-global questions concerning preservation of separation properties in (pointfree) topology. Tressl's result implies that if a locale L is the union of two open sublocales, each subfit, then L is subfit. Contrast this with regularity: if X is the union of two open subspaces, each regular, then X need not be regular, as the line with two origins shows. Subfitness is also an important idea in algebra. A commutative ring with 1 is said to be *Jacobson* if every radical ideal in it is an intersection of maximal ideals. A ring is Jacobson if and only if its frame of radical ideals (i.e., the frame of opens of in its Za7riski spectrum) is conjunctive. Our results are relevant to questions about the preservation of the Jacobson property under ring-theoretic constructions.

This is joint work with Guram Bezhanishvili, Andrew Moshier, Marcus Tressl, and Joanne Walters-Wayland.

[1] J. Picado and A. Pultr, **Frames and Locales**, Springer, 2012.
[2] R. S. Pierce, *Homomorphisms of semi-groups* **Annals of Mathematics**, vol. 59 (1954), pp. 287–291.
[3] H. Wallman, *Lattices and topological spaces* **Annals of Mathematics**, vol. 39 (1938), pp. 112–126.

▶ CHASE MEADORS, *Local finiteness in varieties of MS4-algebras*.
Department of Mathematics, University of Colorado.
E-mail: chase.meadors@colorado.edu.

It is a classic result of Segerberg and Maksimova that a variety of S4-algebras is locally finite iff it is of finite depth. Since the logic MS4 (monadic S4) axiomatizes the one-variable fragment of QS4 (predicate S4), it is natural to try to generalize the Segerberg–Maksimova theorem to this setting. We obtain several results in this direction. We provide a semantic criterion for local finiteness in MS4 that is in general hard to verify, but provide an application

showing a direct generalization of the Segerberg–Maksimova theorem holds for a family of varieties containing $S4_u$ ($S4$ with a universal modality). Our main negative result is a translation of varieties of $S5_2$-algebras into semisimple varieties of MS4-algebras of depth-2 which preserves and reflects local finiteness (the fusion $S5_2 = S5 \oplus S5$ is the bimodal logic of two $S5$ modalities). In particular, this shows that the problem of characterizing locally finite varieties of MS4-algebras is at least as hard as that of characterizing locally finite varieties of $S5_2$-algebras, a well-known open problem in modal logic. Furthermore, we demonstrate with an example that our translation does not naturally extend to depth-3 or higher, suggesting that the problem is strictly harder. Finally, we end by discussing some conjectures and ongoing work concerning another natural extension of MS4 where an analog of Segerberg–Maksimova may hold. This is joint work with Guram Bezhanishvili.

▶ ADAM PŘENOSIL, *Unital lattice subreducts of integral residuated lattices*.
Departament de Filosofia, Universitat de Barcelona, Carrer de Montalegre 6, Spain.
E-mail: adam.prenosil@ub.edu.
URL Address: https://sites.google.com/site/adamprenosil.

Integral residuated lattices form a large variety of algebras whose subclasses provide algebraic semantics for various non-classical logics. A particular subvariety of interest is the variety of integral cancellative residuated lattices, which was first systematically studied in [1]. The authors of that paper in particular show that each lattice is a subreduct of a simple integral cancellative residuated lattice. This in particular means that integral cancellative residuated lattices do not satisfy any non-trivial quasi-equation in the signature of lattices. Whether the same holds for *commutative* integral cancellative residuated lattices has, however, long remained an open problem.

We provide an affirmative answer to this problem. In addition, we describe the subreducts of commutative integral cancellative residuated lattices in the signature of *unital* lattices (lattices equipped with a constant designating the top element) and show that they coincide with the unital lattice subreducts of integral residuated lattices, and therefore also of integral commutative and integral cancellative residuated lattices. This class is the quasivariety generated by unital lattices with a join irreducible top element.

[1] P BAHLS, J. COLE, N. GALATOS, P. JIPSEN, AND C. TSINAKIS, *Cancellative residuated lattices* **Algebra Universalis**, vol. 50 (2003), no. 1, pp. 83–106.

[2] F. MONTAGNA AND C. TSINAKIS, *Ordered groups with a conucleus* **Journal of Pure and Applied Algebra**, vol. 214 (2010), no. 1, pp. 71–88.

▶ MELISSA SUGIMOTO, *Integrality: A local perspective on residuated structures*.
Chapman University, One University Drive, Orange, CA 92866, USA.
E-mail: msugimoto@chapman.edu.

Residuated lattices are the algebraic semantics of substructural logics. Integrality ($x \leq 1$) is an important property for residuated lattices since it is equivalent to the proof-theoretical rule called weakening. In this talk, we investigate a class of structures that decompose into integral components and hence are referred to as *locally integral*.

In particular, we focus on *involutive partially ordered semigroups* (*ipo-semigroups*), which are structures of the form $\mathbf{A} = (A, \leq, \cdot, \sim, -)$ such that $(A, \leq)$ is a partially ordered set and $(A, \cdot)$ is a semigroup with two order-reversing operations $\sim$ and $-$ satisfying *involution* $\sim\!-x = x = -\!\sim x$ and *rotation* $x \cdot y \leq z \iff y \cdot \sim z \leq \sim x \iff -z \cdot x \leq -y$. In the case that the semigroup has an identity, we call it an *ipo-monoid*.

We show that every locally integral ipo-semigroup $\mathbf{A}$ decomposes in a unique way into a family of integral ipo-monoids. We also solve the reverse problem, that is, we provide necessary and sufficient conditions so that the glueing of a system of integral ipo-monoids becomes an ipo-semigroup. This is joint work with José Gil-Férez and Peter Jipsen.

▶ SARA UGOLINI, *Equational anti-unification*.
Artificial Intelligence Research Institute (IIIA), Spanish National Research Council (CSIC), Campus de la UAB, E-08193 Bellaterra, Barcelona, Spain.
E-mail: sara@iiia.csic.es.

The problem of finding a common generalization or *anti-unifier* of a set of terms has first been formalized around 1970 by Plotkin [2], Popplestone [3], and Reynolds [4]. Let us fix some algebraic language $\mathcal{L}$; we call an *anti-unification problem* a finite set $T = \{t_1, \ldots, t_k\}$ of terms of $\mathcal{L}$, and a solution, or *generalization*, of T is another term t of $\mathcal{L}$ for which there exist substitutions $\sigma_1, \ldots, \sigma_k$ such that $\sigma_i(t) = t_i$ for all $i = 1, \ldots, k$. In this contribution we are interested in anti-unification *up to some equational theory* $\mathcal{E}$; i.e., given an anti-unification problem $T = \{t_1, \ldots, t_k\}$, a solution is given by a term t for which there exist $\sigma_1, \ldots, \sigma_k$ such that for all $i = 1, \ldots, k$

$$\mathcal{E} \models \sigma_i(t) = t_i.$$

Given the fact that anti-unification problems always have a solution (mapping a fresh variable to each term), it becomes relevant to know whether there is a *least general solution*: a solution that can be obtained by all other possible solutions by further substitution. We can then define a notion of anti-unification type, which intuitively gives the cardinality of the set of least general solutions.

We show that both equational anti-unification problems and their type can be studied algebraically, in analogy with Ghilardi's approach to equational unification [1], with the use of projective algebras in the variety determined by the equational theory under consideration. Moreover, we show that in varieties where the finitely generated subalgebras of free algebras are projective (e.g., Boolean algebras and Heyting algebras), the study of the anti-unification type can be reduced to the study of the congruence lattice of the 1-generated free algebra.

This contribution is based on joint work with Tommaso Flaminio.

[1] S. GHILARDI, *Unification through projectivity* **Journal of Logic and Computation**, vol. 7 (1997), no. 6, pp. 733–752.

[2] G. D. PLOTKIN, *A Note on Inductive Generalization* **Machine Intelligence**, vol. 5 (1970), pp. 153–163.

[3] R. POPPLESTONE, *An experiment in automatic induction* **Machine Intelligence**, vol. 5 (1970), pp. 203–215.

[4] J. C. REYNOLDS, *Transformational systems and the algebraic structure of atomic formulas* **Machine Intelligence**, vol. 5 (1970), pp. 135–151.

▶ AMANDA VIDAL, *Modal fuzzy logics: general and standard semantics*.
Artificial Intelligence Research institute (IIIA), Spanish National Research Council (CSIC).
E-mail: amanda@iiia.csic.es.

Fuzzy logics arise as the 1-assertional logics of classes of bounded integral residuated lattices with a continuous monoidal operation. There are three main subvarieties which allow to generate any other algebra in the class though the so-called ordinal sum construction: the varieties of Gödel ($\mathbb{G}$), MV ($\mathbb{MV}$) and Product ($\mathbb{P}$) algebras, respective algebraic semantics of Gödel (G), Łukasiewicz ($\L$), and Product (Π) logics. It is immediate that the previous logics are complete with respect to the corresponding linearly ordered algebras of their respective classes, but turns out that each one of them is complete with respect to the 1-assertional logic arising from a particular single algebra of its corresponding class, named the standard algebra, whose universe is $[0, 1]$.

The F.O. versions of the previous logics behave, however, differently. The logics arising from F.O. models evaluated over all algebras in the corresponding variety do coincide with those arising from F.O. models evaluated over the corresponding chains, which is usually

referred to as the *general semantics*. The *standard semantics* are those arising from F.O. models evaluated over the corresponding standard algebras. In the Gödel case, the general and the standard semantic coincide. However, this is not the case for the Łukasiewicz nor Product logics: for the Łukasiewicz case, it follows as a corollary from the fact that the general logic is recursively enumerable (R.E.), but the set of theorems of the standard logic is not [4], and an analogous proof can be done for the product case.

Modal fuzzy logics can be understood as the restriction of the previous F.O. logics to the fragment resulting from the usual translation of modal operators (and formulas in variables $\mathcal{V}$) to the formulas in the predicates language $\{R/2\} \cup \{P/1 \colon P \in \mathcal{V}\}$ as it is done in the classical case. This approach yields the so-called valued Kripke models, which are Kripke models where the accessibility relation and the variables at each world are evaluated over an algebra from the ones above. It is interesting to point out that, over the same class of models, two modal logics (i.e., consequence relations) are defined: the local and the global one. In the latter one, the truth of the premises is taken to be global (in the whole model), while in the former it is only world-wise. Nevertheless, their sets of theorems coincide.

Analogously to the F.O. case, we can refer to the *general* or *standard modal fuzzy logics* whenever the evaluation is considered over all algebras (or equivalently, chains) of the corresponding variety, or only over the standard one. Furthermore, the particular cases when the accessibility relation in the Kripke model is taken as a classical binary relation (crisp) are also of special interest, since the underlying Kripke frames are the classical ones.

In this talk we will compare the previous general and standard modal fuzzy logics. While in the Gödel case, the F.O. behavior immediately implies that, in all cases, the general and standard modal Gödel logics coincide, we will see how for the other two logics, the results are more varied. For what concerns the global modal logics, they behave as the F.O. ones, namely, the general and standard logics differ. For the crisp-accessibility cases, a reasoning similar to the one from F.O. can be done: on the one hand, we know that global modal standard Łukasiewicz and product logics with crisp accessibility relation are not R.E. [5]; on the other, since the general F.O. Łukasiewicz and Product logics are axiomatizable, the corresponding general modal logics are R.E. The previous implies that the standard and general logics do not coincide. This approach cannot be used to tackle the question for global logics with valued accessibility, since their computational classification is not known. Nevertheless, two non-trivial examples can be built to prove that also these logics differ, exploiting peculiarities of a model over the Chang algebra for the Łukasiewicz logic, and of models over the analogous product algebra (arising from the same group) for the Product logic.

For what concerns local modal logics, however, we will see that the general and standard logics coincide, both for the crisp accessibility and for the valued one. This implies that the theorems of these logics (which are the same as the ones from the global logics) coincide too. In the Łukasiewicz cases, this equality can be proven relying in the F.O. completeness with respect to witnessed models (those in which, for each quantified formula, there is an assignment in the model where the formula without the quantifier takes the same value as the quantified one) [1]. For the Product logic, we can prove the claim for the models with valued accessibility relation by relying in the details of a proof of decidability of the Description Logic over the standard product algebra [3]. This approach, however, does not help to answer the case with crisp accessibility, which can nevertheless be proven by a different approach using the completeness of F.O. product logic with respect to models evaluated over a particular single algebra (the one arising via Cignoli–Torrens functor from the lexicographic sum $\mathbb{R}^{Q}$) [2]). Using models valued over this algebra we can identify certain conditions about infinite models, that can nevertheless be expressed with finitely many propositional formulas, and that capture all relevant information about the modal operations. In this fashion, the modal general product logic can be faithfully encoded within the the propositional one, and so, it

is possible to rely in the standard completeness of the latter to prove that the general and standard modal logics also coincide.

As a corollary of this latter proof, the decidability of (local) crisp-accessibility modal product logic is also established.

[1] P. HÁJEK, *Metamathematics of fuzzy logic*, Kluwer Academic Publishers, Dordrecht, 1998.

[2] M. C. LASKOWSKI AND S. MALEKPOUR, *Provability in predicate product logic* **Archive for Mathematical Logic**, vol. 46 (2007), no. 5, pp. 365–378.

[3] F. E. M. CERAMI, *On decidability of concept satisfiability in description logic with product semantics* **Fuzzy Sets and Systems**, vol. 445 (2021), pp. 1–21.

[4] B. SCARPELLINI, *Die Nichtaxiomatisierbarkeit des unendlichwertigen Prädikatenkalküls von Łukasiewicz* **The Journal of Symbolic Logic**, vol. 27 (1962), pp. 159–170.

[5] A. VIDAL, *Undecidability and non-axiomatizability of modal many-valued logics* **The Journal of Symbolic Logic**, vol. 87 (2022), no. 4, pp. 1576–1605.

Abstracts of invited talks in the Special Session on Logic in Computer Science

▶ SIDDHARTH BHASKAR, *Transfinite semantics for programming languages*.
Computer Science Department, James Madison University, 701 Carrier Dr., USA.
URL Address: https://w3.cs.jmu.edu/bhask2sk/.
E-mail: bhask2sk@jmu.edu.

The Turing–Church analysis of effective computation is claimed as an origin story by both recursion theory and the theory of programming languages, yet the two fields diverged sharply from there. While the latter considers a much richer family of models of computation than does the former, none of these model *transfinite* computation. On the other hand, the most common models of transfinite computation are machines, such as infinite-time Turing or register machines, rather than languages. In this talk I shall attempt to bridge the gap by describing a transfinite operational and denotational semantics for a simple language of while-programs, and prove them equivalent.

▶ ELSA L. GUNTER, *A biased overview of influences of symbolic logic on computer science*.
Department of Computer Science, University of Illinois, Urbana–Champaign, 201 N Goodwin Ave, Urbana, IL 61801-2302, USA.
E-mail: egunter@illinois.edu.

Symbolic logic (and abstract algebra) has been overtly influencing computer science for more than 50 years. Obvious early influences were in the development of formal systems for the specification and verification of program properties. Early systems included Axiomatic Logic (aka Floyd–Hoare Logic), the Logic of Computable Functions, and Linear Temporal Logic. Use of these systems gave rise to interactive theorem provers and model checkers for there implementation. From this grew general purpose theorem provers based on logical systems for the formal foundation of mathematics, ranging from classical simple type theory (HOL, Isabelle) to intuitionistic dependent type theory (NuPRL, Coq, Agda, LF, and more). These have developed into extremely powerful, versatile tools for the rigorous development and checking of a wide range of mathematics as well as the specification and verification of properties of complex computer systems.

A somewhat (then) surprising side-effect of the creation of interactive theorem provers was the growth of new areas of programming language theory. This includes a shift from the imperative model of programming languages to the declarative model and

the introduction of static type systems incorporating ideas from universal algebra via algebraic types, polymorphism, and type inference. These type systems fundamentally are logics, with a natural deduction presentation. Evolving from these type systems has been a chain of other more specialized logics for guaranteeing an array of specialized properties.

But type systems are specification languages, and type checking is verification, of a form that can be fully automated. Symbolic logic has found its way deeper into computer systems in many ways. Just one that shows some of the richness brought to systems development though the incorporation of abstractions suggested by formal logics and abstract algebra is the area of compiler optimizations. A large class of optimizations may be understood as conditional graph rewrites, with conditions given by temporal logic formulae and implemented by model finding, with a composition system making it a Kleene algebra. And all this is buried out of sight in some modern compilers.

▶ GIORGI JAPARIDZE, *Strong alternatives to weak theories*.
Computing Sciences Department, Villanova University, 800 Lancaster Avenue, Villanova, PA 19085 USA.
E-mail: giorgi.japaridze@villanova.edu.

I shall briefly survey number theories based on Computability Logic, the game-semantically conceived alternative to classical logic and a conservative extension of the latter. Such theories, dubbed "clarithmetics", allow us to naturally and systematically capture various computational complexity classes, and do this in a stronger sense than weak arithmetics (e.g., bounded arithmetic) do. Specifically, due to being extensions rather than restrictions of Peano Arithmetic, clarithmetics achieve not only extensional but also intensional completeness with respect to their target complexity classes. The underlying concept of computability in clarithmetics is also more general than the traditional one, in that it is about interactive problems rather than merely about functions. In this world of interactive computability, some unusual phenomena occur. E.g., space complexity is not necessarily upper-bounded by time complexity; not all computable problems have computable time complexities; interactive P can be provably separated from interactive PSPACE; and more.

▶ GYÖRGY TURÁN, *Explanations in AI and connections to logic*.
University of Illinois at Chicago, HUN-REN-SZTE Research Group on AI.
E-mail: gyt@uic.edu.

Recent developments in AI are based on machine learning using complex learned models, such as deep neural networks. These models are black boxes in the sense that there is no understanding of why an input is decided to belong to a certain class. In several applications, including societal and scientific ones, it is necessary to have an explanation of the decisions.

What is an explanation? This is a difficult question, thoroughly studied in the philosophy of science. Finding a formal definition seems elusive. Yet, for the important goal of building trustworthy AI it is necessary to have an understanding of the notion(s) of explanation and of methods to find and evaluate explanations.

Logic seems to be a natural candidate for approaching this question. This talk will give an introduction to the machine learning background needed for explainability, survey various logic-based approaches, present some results and formulate open problems.

A historical note: at the first Logic in Computer Science (LICS) conference in 1986 Anil Nerode gave an invited talk titled "A Logician Looks at Expert Systems: Areas for Mathematical Research." A lot has changed since then, but the issues discussed in that talk remain relevant.

▶ WEI-LIN WU, *A study of the expressive power of homomorphism counts*.

Department of Computer Science and Engineering, University of California, Santa Cruz, 1156 High St., Santa Cruz, CA 95064, USA.
E-mail: wwu53@ucsc.edu.

A theorem by Lovász [1] asserts that two graphs G and H are isomorphic if and only if $\hom(F, G) = \hom(F, H)$ for all graphs F, where $\hom(A, B)$ denotes the number of homomorphisms from A to B. This characterization of graph isomorphism in terms of graph homomorphism counts motivated a wealth of research that seeks to characterize different *relaxations of isomorphism* — equivalence relations that are coarser than isomorphism — in terms of the numbers of homomorphisms "from" certain graphs F. Symmetrically, a theorem by Chaudhuri and Vardi [2] says that two graphs G and H are isomorphic if and only if $\hom(G, F) = \hom(H, F)$ for all graphs F. This dual characterization prompts to characterize relaxations of isomorphism in terms of the numbers of homomorphisms "to" certain graphs F. We show that no relaxation of isomorphism sandwiched between C^1-equivalence and C^{k+1}-equivalence is characterized this way, however, where C^k is first-order logic with counting limited to $k \geq 1$ variables. A different view of these characterizations is that they give rise to *query algorithms* for testing membership in a class that answer "yes" or "no" based on the evaluation of homomorphism-count queries [3]. As a variant, *homomorphism-existence* queries have values 0 or 1. We prove that for classes closed under homomorphic-equivalence, an algorithm of queries that are homomorphism counts from certain structures exists precisely when there is an algorithm of queries that are homomorphism existence from certain structures, which is surprising since the former is more capable in general. This is joint work with Atserias, ten Cate, Dalmau, and Kolaitis.

[1] L Lovász, *Operations with structures* **Acta Mathematica Hungarica**, vol. 18 (1967), no. 3-4, pp. 321–328.

[2] S. Chaudhuri and M. Y. Vardi, *Optimization of Real Conjunctive Queries*, **Proceedings of the Twelfth ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems** (New York, NY, USA), Association for Computing Machinery, 1993, pp. 59–70.

[3] Y. Chen, J. Flum, M. Liu, and Z. Xun, **47th International Symposium on Mathematical Foundations of Computer Science (MFCS 2022)**, Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2022.

Abstracts of invited talks in the Special Session on Computability Theory

▶ DAMIR DZHAFAROV, *The Ginsburg-Sands theorem and computability theory*.

Department of Mathematics, University of Connecticut.
E-mail: damir@math.uconn.edu.

In their 1979 paper "Minimal Infinite Topological Spaces", Ginsburg and Sands proved that every infinite topological space has an infinite subspace homeomorphic to exactly one of the following five topologies on ω: indiscrete, discrete, initial segment, final segment, and cofinite. The proof, while nonconstructive, features an interesting application of Ramsey's theorem for pairs (RT^2_2). We analyze this principle in computability theory and reverse mathematics, using Dorais's formalization of CSC spaces. Among our results are that the Ginsburg–Sands theorem for CSC spaces is equivalent to ACA_0, while for Hausdorff spaces it is provable in RCA_0. Furthermore, if we enrich a CSC space by adding the closure operator on points, then the Ginsburg–Sands theorem turns out to be equivalent to the Chain-antichain principle (CAC). The most surprising case is that of the Ginsburg–Sands theorem restricted to T_1 spaces. Here, we show that the principle lies strictly between ACA_0 and RT^2_2, yielding perhaps the first natural theorem of ordinary mathematics (i.e., conceived outside of logic) to occupy this interval. I will discuss the proofs of both the implications and separations, which

feature several novel combinatorial elements, and survey a new class of purely combinatorial principles below ACA_0 and not implied by RT_2^2 revealed by our investigation. This is joint work with Heidi Benham, Andrew DeLapo, Reed Solomon, and Java Darleen Villano.

▶ DAVID GONZALEZ, *Scott sentence complexities of linear orderings*.

Department of Mathematics, University of California, Berkeley, Evans Hall, University Dr, Berkeley, CA 94720.
E-mail: david_gonzalez@berkeley.edu.

The concept of Scott sentence complexity was introduced by Alvir, Greenberg, Harrison-Trainor, and Turetsky and gives a way of assigning countable structures to elements of the Borel hierarchy. By calculating the Scott sentence complexities occurring in a class of structures we obtain a detailed picture of the descriptive complexity of its isomorphism relation. We study possible Scott sentence complexities of linear orderings using two approaches. First, we investigate the effect of the Friedman–Stanley embedding on Scott sentence complexity and show that it only preserves Π_α complexities. We then take a more direct approach and exhibit linear orderings of all Scott sentence complexities except Σ_3 and $\Sigma_{\lambda+1}$ for λ a limit ordinal. We show that the former can not be the Scott sentence complexity of a linear ordering. In the process we develop new techniques which appear to be helpful to calculate the Scott sentence complexities of structures in general.

This talk is based on joint work with Dino Rossegger.

▶ MATTHEW HARRISON-TRAINOR, *A return to degree spectra of relations on a cone*.

Department of Mathematics, Statistics and Computer Science, University of Illinois Chicago.
E-mail: mht@uic.edu.

Given an $\mathcal{L}$-structure $\mathcal{A}$, consider an additional relation R on $\mathcal{A}$ not in $\mathcal{L}$. Given an isomorphic copy $\mathcal{B} \cong_f \mathcal{A}$, there is a copy $R^{\mathcal{B}}$ of R on $\mathcal{B}$. The degree spectrum of R is the collection of all Turing degrees of $R^{\mathcal{B}}$ as $\mathcal{B}$ varies over all computable $\mathcal{B} \cong \mathcal{A}$. As is usual in computability theory, the degree spectra are quite wild and unclassifiable. Montalbán introduced degree spectra on a cone with the hope that, by reducing to structural properties of relations, some structure might be found. In my thesis, I showed that this was not the case, and the area lay dormant for some time. One of the primary open questions was to determine what happens in one of the simplest non-trivial cases, that of relations R on $(\omega, <)$. I will talk about recent joint work with Jad Damaj where we solved this question in an exciting way.

▶ URI ANDREWS, MENG-CHE "TURBO" HO*, AND LUCA SAN MAURO, *Word problem of groups as ceers*.

Department of Mathematics, University of Wisconsin-Madison, Madison, WI 53706-1388, USA.
Department of Mathematics, California State University, Northridge, 18111 Nordhoff Street, Northridge, CA 91330-8313, USA.
E-mail: turbo.ho@csun.edu.
Institute for Discrete Mathematics and Geometry, Vienna University of Technology, 1040 Vienna, Austria.

Since Dehn proposed the word problem in 1911 [1], it has been an important topic of study in both group theory and computability theory. Most naturally occurring groups are c.e., and their word problem is classically defined to be the c.e. set of words equal to the identity of the group and analyzed using Turing reductions. By the Higman embedding theorem, any c.e. degree is realized as a word problem of a finitely presented group [2].

In this talk, we consider the word problem of a group in the framework of computably enumerable equivalence relations (ceer), which has seen a lot of growth recently [3,4,5]. We

compare ceers using computable reductions. We see that the Higman embedding theorem preserves only the Turing degrees of word problems but not the ceer degrees, thus failing to prove that every ceer degree is realized as a word problem. Indeed, not every ceer degree is realized as the word problem of some group. We will investigate some natural questions about the ceer degrees which contain a word problem and discuss some recent results.

[1] *Über unendliche diskontinuierliche Gruppen* **Mathematische Annalen**, (1911), no. 1, 116–144.

[2] *An embedding theorem for finitely generated groups*, **Proceedings of the London Mathematical Society. Third Series**, vol. 17 (1967), pp. 419–430.

[3] S. Gao, and P. Gerdes, *Computably enumerable equivalence relations* **Studia Logica**, vol. 67 (2001), no. 1, pp. 27–59.

[4] U. Andrews, S. Lempp, J. S. Miller, K. M. Ng, L S Mauro, and A. Sorbi, *Universal computably enumerable equivalence relations* **The Journal of Symbolic Logic**, vol. 79 (2014), no. 1, pp. 60–88.

[5] U. Andrews, S. Badaev, and A. Sorbi, *A survey on universal computably enumerable equivalence relations*, **Computability and complexity** Springer, Cham, 2017, pp. 418–451.

▶ JOSIAH JACOBSEN-GROCOTT, *Strong minimal pairs in the enumeration degrees*.
Department of Mathematics, University of Wisconsin-Madison, 480 Lincoln Drive, Madison, WI 53706.
E-mail: jacobsengroc@wisc.edu.

We prove that there are strong minimal pairs in the enumeration degrees and that the degrees of the left and right sides of strong minimal pairs include Σ^0_2 degrees, although it is unknown if there is a strong minimal pair in the Σ^0_2 enumeration degrees. We define a stronger type of minimal pair we call a strong super minimal pair, and show that there are none of these in the enumeration degrees, answering a question of Lempp, Slaman, and Soskova [1]. We leave open the question of the existence of a super minimal pair in the enumeration degrees.

[1] S. Lempp, T. A. Slaman, and M. I. Soskova, *Fragments of the theory of the enumeration degrees* **Advances in Mathematics**, vol. 383 (2021), 107686, 39 pp.

▶ BJØRN KJOS-HANSSEN, *Formal marginalia in computability theory*.
Department of Mathematics, University of Hawai'i at Mānoa, Honolulu, HI 96822.
E-mail: bjoernkh@hawaii.edu.

Formal marginalia are formal proofs of small parts of a mathematical theorem or publication. Research in computability theory makes extensive use of Church's thesis, making full formalization laborious. I present some examples of formal marginalia for sources such as [1,3,2].

[1] B. Kjos-Hanssen, *The probability distribution as a computational resource for randomness testing* **Journal of Logic and Analysis**, vol. 2 (2010), no. 10, pp. 1–13.

[2] ———, *A tractable case of the Turing automorphism problem: bi-uniformly E_0-invariant Cantor homeomorphisms*, **Higher recursion theory and set theory**, Lecture Notes Series, Institute of Mathematical Sciences, National University of Singapore 44 (2024), World Scientific Publishing, Hackensack, NJ.

[3] R. I. Soare, **Turing computability: Theory and Applications**, Theory and Applications of Computability, Springer, 2016.

▶ RUSSELL MILLER, *Computability for absolute Galois groups of computable fields*.
Math. Dept., Queens College – CUNY, 65-30 Kissena Blvd., Queens, NY 11355, USA.
E-mail: russell.miller@qc.cuny.edu.

Fix a computable presentation $\overline{F}$ of the algebraic closure of a computable field F. With such a presentation, the automorphisms of $\overline{F}$ are naturally given as paths through a strongly computable finite-branching tree. If F has a splitting algorithm, then this tree has no terminal nodes. The operations of composition and inversion on these automorphisms (i.e., on these paths) are both type-2 computable. Thus we have an effective way of presenting the absolute Galois group $\mathrm{Gal}(F)$ of F.

Each automorphism has a Turing degree, and for each Turing ideal I, the automorphisms with degrees in I form a subgroup G_I of $\mathrm{Gal}(F)$. When I is a Scott ideal, it is known that G_I is elementary for existential formulas within $\mathrm{Gal}(F)$. We consider whether this might hold for all Turing ideals I. This requires examining the usual method of building a computable infinite subtree of $2^{<\omega}$ with no infinite paths and asking whether this method can be applied in the context of $\mathrm{Gal}(F)$. The specific answers may differ for different computable fields, including the field $\mathbb{Q}$ of rational numbers.

This is joint work with the number theorist Debanjana Kundu.

▶ KENG MENG NG, *Separating notions in effective topology and analysis*.
School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore.
E-mail: kmng@ntu.edu.sg.

We discuss some results in effective topological spaces and present some attempts to classify spaces using computability notions. We discuss notions such as universality, metrisability, and presentability from the effective point of view. Specifically, we discuss a series of recent results separating the degree of presentability of Polish spaces.

▶ JAN REIMANN, *Complexity notions on monoids and pointwise dimension*.
Department of Mathematics, Penn State University, University Park, PA 16802, USA.
E-mail: jan.reimann@psu.edu.

We study axiomatic complexity notions on monoids. We are particularly interested in conditions under which the complexity induces an admissible distance function in the sense of Bennett et al. Embedding complexity monoids into metric spaces induces a pointwise dimension notion as well as a function mapping a real a to the set of a reals of pointwise dimension a. We compute the Hausdorff dimension of this set for a class of well-behaved complexity notions and use this to give new proofs of some results in metric Diophantine approximation, such as the Jarnik–Besicovitch theorem.

▶ LUCA SAN MAURO, *On the learning power of equivalence relations*.
Department of Humanistic Research and Innovation, University of Bari, Piazza Umberto I, Bari, Italy.
E-mail: luca.sanmauro@uniba.it.

Borel reducibility has proven to be a decisive tool in understanding whether familiar classes of countable structures may be nicely classified. However, isomorphism problems with countably many isomorphism types have all the same Borel complexity; thus, a finer notion is required to investigate them properly. We present a framework [1,2] that borrows ideas from computational learning theory and is suitable for this task. In a nutshell, a learner is given increasingly larger pieces of an arbitrary copy of a structure and, at each stage, is required to output a conjecture about the observed isomorphism type. The learning is successful if the conjectures eventually stabilize into a correct guess.

It turns out that, in this framework, a countable family of countable structures is learnable if and only if the related isomorphism problem continuously reduces to E_0 (the relation of eventual agreements on reals). Then, by replacing E_0 with other Borel equivalence relations

E, we unlock a natural hierarchy for ranking isomorphism relations that escape the Borel analysis.

We explore this hierarchy and the learning power of Borel equivalence relations, studying both well-established benchmark relations and relations associated to classic learning criteria. We also revisit the Friedman–Stanley tower through the lenses of learning.

The talk is based on a number of different projects pursued together with N. Bazhenov, V. Cipriani, E. Fokina, S. Jain, A. Marcone, and F. Stephan.

[1] N. Bazhenov, V. Cipriani, and L. San Mauro, *Learning algebraic structures with the help of Borel equivalence relations* **Theoretical Computer Science** vol. 951 (2023), 113762.

[2] N. Bazhenov, E. Fokina, and L. San Mauro, *Learning families of algebraic structures from informant* **Information and Computation** vol. 275 (2020), 104590.

▶ ISABELLA SCOTT, *Constructions surrounding existentially closed groups*.
Department of Mathematics, University of Chicago, 5734 S. University Ave, USA.
E-mail: iscott@uchicago.edu.

Existentially closed groups were introduced in 1951 in analog with algebraically closed fields. Since then, they have been further studied by Neumann, Macintyre, and Ziegler, who elucidated deep connections with model theory and computability theory. We review some of the literature on existentially closed groups and present new results that further refine these connections.

▶ DON STULL, *Dimensions of pinned distance sets*.
Department of Mathematics, University of Chicago, Chicago, IL 60637, USA.
E-mail: dmstull@uchicago.edu.

Recent work has shown that techniques from computability theory and algorithmic randomness can be used to understand questions in classical geometric measure theory. One of the central problems in geometric measure theory is Falconer's distance set problem. Give a set E in the plane, and a point $x \in \mathbb{R}^2$, the pinned distance set of E with respect to x is the set of distances between x and the points in E. In this talk, we will discuss ongoing progress on this problem, and present improved lower bounds for both the Hausdorff and packing dimensions of pinned distance sets. We also discuss the computability-theoretic methods used to achieve these bounds.

This is joint work with Jacob Fiedler.

Abstracts of invited talks in the Special Session on Model Theory

▶ BENJAMIN CASTLE, *Advances on Zilber's trichotomy and its applications*.
Department of Mathematics, University of Maryland, 4176 Campus Drive, College Park, MD, USA.
E-mail: bcastle@umd.edu.

The last two years have seen significant progress toward several open problems around Zilber's trichotomy for strongly minimal sets. Most notably, this included a complete proof of the trichotomy for strongly minimal structures interpreted in algebraically closed fields. Meanwhile, significant progress has been made on the analogous statements for algebraically closed valued fields (ACVF) and o-minimal expansions of fields. In this talk, I will give an overview of the various relevant conjectures and their status reports, highlighting new work in ACVF. Time permitting, I will also discuss some applications of the trichotomy over algebraically closed fields.

▶ GABRIEL CONANT, *Stable arithmetic regularity for functions*.

Department of Mathematics, The Ohio State University, 231 W 18th Ave, Columbus, OH 43201, USA.
E-mail: conant.38@osu.edu.

I will present a structure theorem for stable functions on amenable groups, which mirrors the arithmetic regularity lemma for stable subsets of finite groups (proved first in [6] for $\mathbb{F}_p^n$, and generalized to all finite groups in [4]; see also [7,2]). Here a function $f : G \to [-1, 1]$ is called stable if the binary function $f(x \cdot y)$ is stable in the sense of continuous logic. Roughly speaking, our main result says that if G is amenable, then any stable function on G is approximately constant on all translates of a unitary Bohr set in G of bounded complexity. The proof uses ingredients from topological dynamics and continuous model theory, along with an ultraproduct construction with metric structures. We also establish a fair amount of local stable group theory in continuous logic. The key difference compared to the discrete case is that the space of generic types is no longer finite (or even profinite), which explains the necessity for Bohr sets in the final structure theorem. Finally, I will explain how our main result can be used to obtain applications with no ambient assumption of stability. For example, as a quick corollary we obtain the uniform version of Bogolyubov's Lemma for general amenable groups (proved originally for finite abelian groups by Ruzsa [5], then for finite groups in [1], and finally for amenable groups in [3]). Further potential applications will be discussed as time permits.

This is joint work with Anand Pillay.

[1] G. CONANT, *On finite sets of small tripling or small alternation in arbitrary groups* **Combinatorics, Probability and Computing**, vol. 29 (2020), no. 6, pp. 807–829.

[2] ———, *Quantitative structure of stable sets in arbitrary finite groups* **Proceedings of the American Mathematical Society**, vol. 149 (2021), no. 9, pp. 4015–4028.

[3] G. CONANT, E. HRUSHOVSKI, AND A. PILLAY, *Compactifications of pseudofinite and pseudo-amenable groups*, arXiv:2308.08440, 2023.

[4] G. CONANT, A. PILLAY, AND C. TERRY, *A group version of stable regularity* **Mathematical Proceedings of the Cambridge Philosophical Society**, vol. 168 (2020), no. 2, pp. 405–413.

[5] I. Z. RUZSA, *Generalized arithmetical progressions and sumsets* **Acta Mathematica Hungarica**, vol. 65 (1994), no. 4, pp. 379–388.

[6] C. TERRY AND J. WOLF, *Stable arithmetic regularity in the finite field model* **Bulletin of the London Mathematical Society**, vol. 51 (2019), no. 1, pp. 70–88.

[7] ———, *Quantitative structure of stable sets in finite abelian groups* **Transactions of the American Mathematical Society**, vol. 373 (2020), no. 6, pp. 3885–3903.

▶ SEBASTIAN ETEROVIĆ, *Geometric properties of transcendental holomorphic functions*.

School of Mathematics, University of Leeds, Woodhouse Lane, Leeds, UK.
E-mail: s.eterovic@leeds.ac.uk.
URL Address: https://sites.google.com/view/sebastianeterovic/home.

The complex exponential function has been studied extensively for centuries, but there are still important algebraic aspects of it that are not well-understood. For example, it is unknown whether the transcendental numbers e and π are algebraically independent over $\mathbb{Q}$.

Zilber's seminal work on pseudo-exponentiation [1,2] proposes a possible explicit axiomatization of the algebraic properties of complex exponentiation which is $\aleph_1$-categorical. The proposed axioms expose natural connections between important problems in number theory and model-theoretic conditions. This impulsed a systematic model-theoretic study of many other important holomorphic maps, especially those arising in number theory such as the exponential maps of semi-abelian varieties, the modular j-function, automorphic functions, and the Γ-function.

In this talk I will introduce a few of the main problems in the model-theoretic study of these functions of interest, in particular we will discuss the categoricity and the existential closedness problems, and present some of the main results obtained recently.

[1] B. Zilber, *Exponential sums equations and the Schanuel conjecture*, **Journal of the London Mathematical Society. Second Series**, vol. 65 (2002), no. 1, pp. 27–44.

[2] ———, *Pseudo-exponentiation on algebraically closed fields of characteristic zero*, **Annals of Pure and Applied Logic**, vol. 132 (2005), no. 1, pp. 67–95.

▶ BRADD HART, *An update on the model theory of operator algebras*.
Department of Mathematics and Statistics, McMaster University, Hamilton ON, Canada.
E-mail: hartb@mcmaster.ca.

Over the past 15 years, the model theory of operator algebras has grown from what might have seemed like a one-off application of continuous model theory into a programme with many interesting branches. In this talk, I would like to review where we've been, what the status is now and highlight some of the current open problems.

▶ LÉO JIMENEZ, *Bounding non-orthogonality using algebraic groups*.
Department of Mathematics, The Ohio State University, 231 W. 18th Ave., Columbus, OH 43210, United States.
E-mail: jimenez.301@osu.edu.

It is a well-known stability theory result that if p and q are stationary non-orthogonal types over the same set of parameters, then there are n, m such that $p^{(n)}$ and $q^{(m)}$ are not weakly orthogonal. Is there a bound on the smallest such n and m?

In this talk, I will present such a bound for differentially closed fields of characteristic zero and give a differential-algebraic interpretation of the result. The proof uses geometric stability machinery to reduce the problem to a question about algebraic group actions. This is part of a joint work with James Freitag and Rahim Moosa.

▶ ELLIOT KAPLAN, *Generic derivations on o-minimal structures*.
Department of Mathematics and Statistics, McMaster University, 1280 Main Street West, Hamilton, Ontario L8S 4L8, Canada.
E-mail: kaplae2@mcmaster.ca.

Let T be a model complete o-minimal theory that extends the theory of real closed ordered fields (RCF). We introduce T-derivations: derivations on models of T which cooperate with T-definable functions. The theory of models of T expanded by a T-derivation has a model completion, in which the derivation acts "generically." If T = RCF, then this model completion is the theory of closed ordered differential fields (CODF) as introduced by Singer. We can recover many of the known facts about CODF (open core, distality) in our setting. We can also describe thorn-rank for models of T with a generic T-derivation. This is joint work with Antongiulio Fornasiero.

▶ MARYANTHE MALLIARIS, *On stability*.
Department of Mathematics, University of Chicago, 5734 S. University Avenue, Chicago, IL 60637, USA.
E-mail: mem@math.uchicago.edu.
URL Address: https://math.uchicago.edu/ mem/.

The talk will take a new look at model-theoretic stability from some very recent points of view.

▶ SCOTT MUTCHNIK, *Identifying genericity*.
Department of Mathematics, Statistics and Computer Science, University of Illinois Chicago, 851 S Morgan St, Chicago, IL 60607 USA.
E-mail: mutchnik@uic.edu.

Independent indiscernible sequences, or Morley sequences, can tell us about genericity in a first-order theory. This leads us to ask: when can we define whether an indiscernible sequence is independent? If an indiscernible sequence is dependent, how close can it be to being independent - or how far? We discuss some applications to forking in supersimple homogeneous structures, Freitag and Moosa's degree of nonminimality, and the simple Kim-forking conjecture. This is on joint work with James Freitag.

▶ MARGARET E. M. THOMAS, *Definable topological spaces in o-minimal structures*.
Department of Mathematics, Purdue University, 150 N. University St., IN 47907-2067, USA.
E-mail: memthomas@purdue.edu.

O-minimal structures have been analyzed extensively as a framework for tame topology, where the main focus has been on understanding the underlying Euclidean order topology. Also key, however, to the development of o-minimality has been the study of the topological nature of various definable objects, such as groups, manifold spaces, orders, function spaces, and metric spaces, where in many cases the topological spaces involved go beyond that of the underlying euclidean topology. We present some progress towards a more general understanding of the nature of topological spaces definable in this setting, in particular focussing on one-dimensional definable topological spaces. This is based on a long-term joint project with Pablo Andújar Guerrero and Erik Walsberg, and intersects with work carried out independently by Peterzil and Rosel. We consider various classification results given in terms of decomposition and embedding theorems and, in parallel, the identification of suitable definable analogues of classical properties such as separability, compactness, and metrizability. This leads to a variety of applications, including definable versions of conjectures from classical topology due to Gruenhage and Fremlin (on the nature of regular Hausdorff and perfectly normal compact Hausdorff spaces), as well as universality results for certain classes of spaces.

Abstracts of invited talks in the Special Session on Set Theory

▶ TOM BENHAMOU, *Cofinal types of ultrafilters on measurable and non-measurable cardinals*.
Department of Mathematics, Rutger University.
E-mail: tom.benhamou@rutgers.edu.
URL Address: https://sites.math.rutgers.edu/ tb822/.

The Tukey order finds its origins in the concept of Moore–Smith convergence in topology, and is especially important when restricted to ultrafilters with reverse inclusion. The Tukey order of ultrafilters over ω was studied intensively by Blass, Dobrinen, Isbell, Raghavan, Shelah, Todorcevic, and many others, but still contains fundamental unresolved problems. In the first part of this talk, I will present a recent development in the theory of the Tukey order restricted to ultrafilters on measurable cardinals, and explain how different the situation is when compared to ultrafilters on ω. Moreover, we will see an important application to the Galvin property of ultrafilters. In the second part of the talk we will demonstrate how ideas and intuition from ultrafilters over measurable cardinals lead to new results on the Tukey order restricted to ultrafilters over ω. This is joint with Natasha Dobrinen.

▶ SUMUN IYER, *Extremely amenable groups of homeomorphisms*.
Department of Mathematics, Cornell University, 212 Garden Avenue, Ithaca, NY 14853 USA.
E-mail: ssi22@cornell.edu.

A topological group is *extremely amenable* if every continuous action of it on a compact, Hausdorff space has a fixed point. We discuss a construction due to Uspenskij which gives a condition equivalent to extreme amenability for the setting of homeomorphism groups of compact, metrizable spaces. We then show a Ramsey-type statement for subsets of simplices and discuss its connection with Uspenskij's construction and consequences for extreme amenability. This is joint work with Lukas Michel and Alex Scott.

▶   JOHN KRUEGER, *Forcing over a free Suslin tree*.
Department of Mathematics, University of North Texas, 1155 Union Circle #311430 Denton, TX 76203 USA.
E-mail: jkrueger@unt.edu.

This talk is based on joint work with Šárka Stejskalová. A Suslin tree is *free* if all of its derived trees are Suslin. We introduce a general framework for forcing over a free Suslin tree and present several applications. Let T be a free Suslin tree. (1) We show how to specialize derived trees of T of dimension $n + 1$ while preserving that T is n-free. (2) We force a sequence of almost disjoint subtrees of T of any length while preserving that T is Aronszajn. (3) We add any number of automorphisms to T while preserving that T is Suslin and not adding cofinal branches to ω_1-trees appearing in intermediate extensions. All of the above forcings are totally proper and ω_2-c.c. under CH. As a consequence, we are able to produce a model in which there exists a non-saturated Aronszajn tree and in which there does not exist a Kurepa tree, answering an open problem of Moore. And we are able to produce a model in which there exists an almost Kurepa Suslin tree but no Kurepa tree, answering open problems of Bilaniuk and Jin-Shelah.

▶   FARMER SCHLUTZENBERG, *Correctness of inner models and optimal wellorders*.
Institute of Discrete Mathematics and Geometry, TU Vienna, Wiedner Hauptstraße 8–10/104, Austria.
E-mail: farmer.schlutzenberg@tuwien.ac.at.
URL Address: https://sites.google.com/site/schlutzenberg/home-1.

John Steel and Mitch Rudominer conjectured, assuming $AD + V = L(\mathbb{R})$, that every mouse M has an "optimal" well-order of its reals, definable right at the point that determinacy fails in $L(\mathbb{R})^M$. We will discuss recent progress on this conjecture and its current status. This is joint work with Steel.

[1] F. SCHLUTZENBERG AND J. STEEL, Σ_1 *gaps as derived models and correctness of mice*, arXiv:2307.08856.
[2] M. RUDOMINER AND J. STEEL, *Inner models with wellorders in* $L(\mathbb{R})$, unpublished notes.

▶   FORTE SHINKO, *Hyperfiniteness of generic actions on Cantor space*.
Department of Mathematics, University of California Berkeley, 970 Evans Hall 33840, Berkeley, CA 94720 USA.
E-mail: forteshinko@berkeley.edu.

A countable discrete group is exact if it has a free action on Cantor space which is measure-hyperfinite, that is, for every Borel probability measure on Cantor space, there is a conull set on which the orbit equivalence relation is hyperfinite. For an exact group, it is known that the generic action on Cantor space is measure-hyperfinite, and it is open as to whether the generic action is hyperfinite; an exact group for which the generic action is not hyperfinite would resolve a long-standing open conjecture about whether measure-hyperfiniteness and hyperfiniteness are equivalent. We show that for any countable discrete group with finite asymptotic dimension, its generic action on Cantor space is hyperfinite. This is joint work with Sumun Iyer.

▶ IIAN SMYTHE, *A descriptive approach to manifold classification*.

Department of Mathematics and Statistics, University of Winnipeg, 515 Portage Avenue, Winnipeg, Manitoba, R3B 2E9, Canada.
E-mail: i.smythe@uwinnipeg.ca.

We propose a unified descriptive set-theoretic framework for studying the complexity of classification problems arising in geometric topology. We establish several precise complexity results, such as for the classification of surfaces up to homeomorphism, and for classes of hyperbolic manifolds up to isometry. The latter is intimately connected with the conjugation actions of certain Lie groups on their spaces of discrete subgroups. This work is joint with Jeffrey Bergfalk.

▶ TREVOR M. WILSON, *Characterizing large cardinals in terms of Löwenheim–Skolem and weak compactness properties of strong logics*.

Department of Mathematics, Miami University, 123 Bachelor Hall, 301 S. Patterson Ave., Oxford, OH 45056, USA.
E-mail: twilson@miamioh.edu.

We discuss characterizations of large cardinals in terms of properties of strong logics, such as Löwenheim–Skolem and weak compactness properties of certain fragments of infinitary second-order logic. For example, a cardinal κ is strong if and only if the Löwenheim–Skolem property holds for a certain fragment of infinitary second-order logic, namely for the sentences of $\mathcal{L}^2_{\infty,\infty}$ in negation normal form whose conjunctions and existential quantifiers have length less than κ.

We also discuss recent work with Jonathan Osinski obtaining similar results for weak compactness properties. Namely, a cardinal κ is Shelah if and only if a kind of weak compactness property holds for the dual of the aforementioned logic fragment, namely for theories consisting of sentences in $\mathcal{L}^2_{\infty,\infty}$ in negation normal form whose disjunctions and universal quantifiers have length less than κ. This property says that if such a theory T has size κ and every subtheory of T of size less than κ has a model of size less than κ, then T has a model. This connection to logic was somewhat unexpected because Shelah cardinals are sometimes thought to be a technical notion with few connections outside the large cardinal hierarchy.

Finally, we will mention some virtual analogs of these results, meaning results pertaining to large cardinal properties that are defined in terms of elementary embeddings existing in generic extensions of the universe.

Abstracts of invited talks in the Special Session on Universal Algebra

▶ CHARLOTTE ATEN, *Varieties of algebras and algebraic varieties*.

Department of Mathematics, University of Denver, 2390 S. York St., Denver 80208, USA.
E-mail: charlotte.aten@du.edu.
URL Address: https://aten.cool.

To each variety $\mathcal{V}$ of algebras, we can assign an $\mathbb{N}$-indexed family of algebraic sets $\{\mathcal{A}_n\}_{n\in\mathbb{N}}$ (or "algebraic varieties", although they are not in general irreducible) whose points are linear models of the axioms for $\mathcal{V}$. One might imagine that these algebraic sets carry information about the structure of $\mathcal{V}$. The case where $\mathcal{V}$ is the variety of semigroups has been considered, for instance, in [1]. In that work, the authors used tools from nonstandard analysis to determine the open components of $\mathcal{A}_2$. These techniques did not seem to generalize well, and there were few subsequent results in this direction.

Part of my thesis resulted from an attempt to develop more general methods for understanding such families of algebraic sets [2]. Through some motivating examples from universal algebra, I found that any first-order property of finite structures could be seen

to hold (or not) for a given structure by counting certain small substructures. Despite the generality of this result, its application is much more effective in the context of universally quantified equations. This parallels work such as [3], where primitive positive formulas are particularly compatible with techniques from universal algebra.

[1] J. M. A. Bermúdez, J. Fresán, and J. S. Hernández, *On the variety of two dimensional real associative algebras* **International Journal of Contemporary Mathematical Sciences**, vol. 2 (2007), no. 26, pp. 1293–1305.

[2] C. Aten, ***Finite Generation of Families of Structures Equipped with Compatible Group Actions***, ProQuest Dissertations and Theses, ProQuest, 2022.

[3] S. Bova, H. Chen, and M. Valeriote, *Generic expression hardness results for primitive positive formula comparison* **Information and Computation** vol. 222 (2013), pp. 108–120.

▶   ANDREI A. BULATOV, *Operator CSP and satisfiability gap*.
School of Computing Science, Simon Fraser University, 8888 University Drive, Burnaby BC, V5A 1S6, Canada.
E-mail: abulatov@cs.sfu.ca.
   The Constraint Satisfaction Problem (CSP) asks to find a satisfying assignment to a collection of variables subject to specified constraints. Graph Coloring and Graph Homomorphism problems are standard examples of CSPs. In the CSP and other homomorphism problems, we are looking for an assignment of discrete values such as colors of vertices of a graph. However, one may follow the lead of quantum mechanics and convert bits to qubits, that is, look for satisfying assignments by matrices or linear operators. More precisely, constraints in a CSP are interpolated by polynomials (real or complex) that allows for using linear operators as values of variables. Sometimes a CSP that is not satisfiable 'classically' may have a satisfying operator assignment. In this talk, we study what kind of CSPs demonstrate such a satisfiability gap.

▶   JOHN HARDING, *Monadic ortholattices*.
Department of Mathematical Sciences, New Mexico State University.
E-mail: jharding@nmsu.edu.
   Halmos [1] introduced monadic algebras as an algebraic version of the one-variable fragment of predicate calculus, and extended this to polyadic algebras as an algebraic version of predicate calculus. At about the same time, Henkin, Monk, and Tarski [3,4] introduced cylindric algebras for the same purpose. A monadic algebra is a Boolean algebra with an additional operation called a quantifier that is a closure operator with the property that the complement of a closed element is closed. Polyadic and cylindric algebras are Boolean algebras with families of related quantifiers. Since then, quantifiers have been studied on various types of algebraic structures, most notably Heyting algebras [6], and also on orthomodular lattices [5].
   We consider monadic and cylindric ortholattices and orthomodular lattices. A primary source of examples comes from operator algebras. Each von Neumann algebra provides a monadic orthomodular lattice, as does each von Neumann subfactor. Further, commuting squares of subfactors provide examples of cylindric orthomodular lattices. We describe some of the basic theory of monadic ortholattices. We also present an axiomatization of set cylindric orthomodular lattices, which are obtained in an obvious way from a tensor power of a Hilbert space. Finally, we consider MacNeille and canonical completions of monadic ortholattices, and a closely related duality theory.

[1] P. Halmos, ***Algebraic Logic***, Chelsea Publishing Company, New York, 1962.

[2] J. Harding, *Quantum monadic algebras* **Journal of Physics A**, vol. 55 (2022), no. 39, 394001, 27 pp.

[3] L. Henkin, J.D. Monk, and A. Tarski, *Cylindric algebras Part I*, North-Holland, Amsterdam, 1985.

[4] ———, *Cylindric algebras. Part II*, North-Holland, Amsterdam, 1985.

[5] M. Janowitz, *Quantifiers and orthomodular lattices* **Pacific Journal of Mathematics**, vol. 13 (1963), pp. 1241–1249.

[6] A. Monteiro and O. Varsavsky, *Algebras de Heyting moná dicas*, **Actas de las X Jornadas de la Unión Matemática Argentina**, Instituto de Matemáticas, Universidad Nacional del Sur, Bahía Blanca, 1957, pp. 52–62.

▶ MICHAEL KINYON, *Loops with squares in two nuclei*.
Department of Mathematics, University of Denver, Denver CO 80208.
E-mail: michael.kinyon@du.edu.

The varieties of loops (quasigroups with identity elements) of Bol–Moufang type are defined by identities with the following properties: (i) they involve three distinct variables, each occurring on both sides of the equal sign; (ii) the variables occur in the same order on both sides; (iii) exactly one of the variables appears twice on both sides. For example, the identity $(xy)(zx) = x((yz)x)$ is of Bol–Moufang type and defines the variety of Moufang loops. (This is half of the rationale for the name "Bol–Moufang type").

Various interesting varieties of loops (interesting, at least, to quasigroup theorists) — such as Moufang loops, Bol loops, C loops and others — can be defined by identities of Bol–Moufang type. There are others which, up until now, have not been garnered much interest because there does not seem to be much that can be said about them. For instance, the variety of loops with *left nuclear squares* is defined by $(xx)(yz) = ((xx)y)z$; essentially nothing interesting can be proven about these or the similarly defined varieties of loops with middle nuclear or right nuclear squares.

It turns out however, that one can say interesting things about the pairwise intersection of these varieties (from whence comes the title of this talk). This is a bit surprising because on the face of it, the condition that squares associate with all other elements in certain positions does not seem like a very strong property.

In particular, it turns out that in such loops, the intersection of the corresponding nuclei is a normal subloop. In addition, we consider the subvarieties of loops satisfying the automorphic inverse property (AIP) $(xy)\backslash 1 = (x\backslash 1)(y\backslash 1)$. These turn to have endomorphic squaring $(xy)^2 = x^2 y^2$ and all squares lie in the center. In the finite case, these satisfy a direct product into an abelian group of odd order and a loop consisting of elements of order a power of 2. This is analogous to what happens in other Bol–Moufang type varieties in the presence of the additional assumption of the AIP.

This talk will not assume any background in loop theory beyond perhaps having seen the definition of loop. This is joint work with J.D. Phillips (Northern Michigan University).

▶ PETER MAYR* AND NIK RUŠKUC, *Filtered Boolean powers of simple algebras*.
Department of Mathematics, University of Colorado Boulder, USA.
E-mail: peter.mayr@colorado.edu.
School of Mathematics and Statistics, University of St Andrews, Scotland, UK.
E-mail: nik.ruskuc@st-andrews.ac.uk.

Let **A** be a finite simple non-abelian Mal'cev algebra, for example, a finite simple non-abelian (quasi)group or a matrix ring over a finite field. Then the class K of finite direct powers of **A** has the joint embedding property (JEP) and the amalgamation property (AP) but in general not the hereditary property (HP). There exists a (generalized) Fraïssé limit **D** of K, which we can describe as a filtered Boolean power (see [2]) of **A** and the countable atomless Boolean algebra.

The automorphism group **G** of the countable atomless Boolean algebra satisfies the following topological and combinatorial properties among others.

1. **G** has the small index property (SIP): every subgroup of **G** of index less than $2^{\aleph_0}$ is open in the topology of pointwise convergence (Truss [3]);
2. **G** has uncountable cofinality: **G** is not the union of a countable chain of proper subgroups (Droste and Göbel [1]);
3. **G** has the Bergman property: for each generating set E of **G** with $1 \in E = E^{-1}$ there exists $k \in \mathbb{N}$ such that $E^k = G$ (Droste and Göbel [1]).

We extend these results to show that the automorphism group of the filtered Boolean power **D** has the SIP, uncountable cofinality, and the Bergman property as well.

[1] M. DROSTE AND R. GÖBEL, *Uncountable cofinalities of permutation groups*. **Journal of the London Mathematical Society**, vol. 71 (2005), no. 2, pp. 335–344

[2] D.M. EVANS, *Examples of $\aleph_0$-categorical structures*, **Automorphisms of first-order structures** (R. Kaye and D. Macpherson, editors), Oxford University Press, New York, 1994, pp. 33–72.

[3] J.K. TRUSS, *Infinite permutation groups II. Subgroups of small index.* **Journal of Algebra**, vol. 120 (1989), no. 2, pp. 494–515.

▶ ANDREW MOORHEAD* AND REINHARD PÖSCHEL, *When are bounded arity polynomials enough?*
Institute Für Algebra, TU Dresden, Zellescher Weg 12-14, 01069 Dresden, Germany.
E-mail: andrew_paul.moorhead@tu-dresden.de.
E-mail: reinhard.poeschel@tu-dresden.de.

Equivalence relations appear everywhere in mathematics, in particular, if they are compatible with some mathematical structure (and therefore allow factor constructions). They have a remarkable property: An n-ary operation f preserves an equivalence relation ρ if and only if each unary polynomial function which can be obtained from f by substituting constants at all but one argument preserves ρ (such unary functions are called *basic translations* of f). This preservation criterion can be generalized to a condition that ranges over those n-ary polynomial functions that are obtained from f by substituting constants at all but n many arguments. In fact, there are many examples of such relations for each positive n which play an important role in higher commutator theory [2].

In [1], Jakubíková-Studenovská, Pöschel, and Radeleczki provide an essentially complete characterization of all finitary relations ρ with the property that f preserves ρ if and only if all basic translations of f preserve ρ. In this talk we will explain how to generalize this characterization to relations with the analogous 'higher dimensional' compatibility criterion with respect to n-ary polynomial functions for some positive n.

[1] D. JAKUBÍKOVÁ-STUDENOVSKÁ, R. PÖSCHEL, AND SÁNDOR RADELECZKI, *Generalized Quasiorders and the Galois Connection End-gQuord*, arXiv:2307.01868.

[2] A. MOORHEAD, *Supernilpotent Taylor algebras are nilpotent* **Transactions of the American Mathematical Society**, vol. 374 (2021), no. 2, pp. 1229–1276.

▶ GAVIN NOP AND JONATHAN SMITH*, *Quasilattices and complex concept analysis.*
Department of Mathematics, Iowa State University, Ames, Iowa 50011, USA.
E-mail: gavin.nop@gmail.com.
E-mail: jdhsmith@iastate.edu.
URL Address: https://jdhsmith.math.iastate.edu/.

Consider the following question: Why do we have tailbones, but no tail? Universal algebra provides a framework that underlies the answer to questions of this type.

Quasilattices are algebraic structures that comprise a semilattice-ordered system of lattices [4,5]. In a forthcoming paper [3], certain quasilattices (that are characterized abstractly by a local completeness property) provide an extension of Wille's *concept analysis* from data science [1] and Hardegree's *logic of natural kinds* [2] to the study of complex systems that function on a number of distinct levels. In an important special case, a chain semilattice serves to represent a time series governing the evolution of a single system.

Natural set representations of locally complete quasilattices comprise *opposed* set inclusions describing order relations within a complete lattice, and *parallel* set inclusions tracking homomorphisms that connect distinct lattice fibers. In the time series model, the sets that appear within the set representation accumulate successive layers at each time step, establishing a logical basis for historical phenomena such as the presence of tailbones in the absence of a tail.

[1] B. Ganter and R. Wille, *Formale Begriffsanalyse*, Springer, Berlin, 1996.

[2] G.M. Hardegree, *An approach to the logic of natural kinds* **Pacific Philosophical Quarterly**, vol. 63 (1982), pp. 122–132.

[3] G.N. Nop and J.D.H. Smith, *Quasilattices and complex concept analysis* **Quaestiones Mathematicae**, vol. 47 (2024), pp. 173–200.

[4] R. Padmanabhan, *Regular identities in lattices* **Transactions of the American Mathematical Society**, vol. 158 (1971), pp. 179–188.

[5] J. Płonka, *On distributive quasilattices* **Fundamenta Mathematicae**, vol. 60 (1967), pp. 191–200.

▶ ANDREJA TEPAVČEVIĆ* AND BRANIMIR ŠEŠELJA, *Axiomatic approach to lattice characterization of groups*.

Mathematical Institute of the Serbian Academy of Sciences and Arts, Kneza Mihaila 36, Belgrade, Serbia.
E-mail: andreja@dmi.uns.ac.rs.
Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad, Trg Dositeja Obradovica 4, Novi Sad, Serbia.

We introduce a new class of algebraic lattices by a system of axioms. A subclass of these lattices can be modeled by weak-congruence lattices of groups. These lattices consist of all congruences on all subgroups of a group, equivalently, of all normal subgroups of all subgroups. It is possible to formulate main group properties in a lattice framework, e.g., normality among subgroups, homomorphism and quotient subgroups, particular chains, and systems of subgroups. In this way, we prove that many structural properties of groups turn out to be lattice-theoretic. This axiom system is not complete in the sense that not every lattice fulfilling the axioms represents a weak-congruence lattice of some group. Still, these axioms enable the characterization of many classes of groups. We give necessary and sufficient conditions that should be satisfied by the weak-congruence lattice of a group under which this group is Hamiltonian, abelian, solvable, nilpotent, and many others.

By our axioms, a special role in these lattices has a particular codistributive element which should correspond to a diagonal relation of the weak congruence lattice of a group (or any algebra in a more general setting). Therefore, we compare the axioms motivated by group properties with lattice properties of so-called suitable elements in algebraic lattices which should represent weak-congruence lattices of arbitrary algebras.

[1] G. Czédli, M. Erné, Ě. Sešelja, and A. Tepavčević, *Characteristic triangles of closure operators with applications in general algebra* **Algebra Universalis**, vol. 62 (2009), pp. 399–418.

[2] *Lattice characterization of some classes of groups by series of subgroups* **International Journal of Algebra and Computation**, vol. 33 (2023), no. 2, pp. 211–235.

[3] *Lattice characterization of finite nilpotent groups* **Algebra Universalis**, vol. 82 (2021), no. 3, article 40.

[4] ——— *Lattice characterization of finite nilpotent groups Lattices with normal elements* **Algebra Universalis**, vol. 83 (2022), no. 1, article 2.

▶ PATRICK WYNNE, *The subpower membership problem for nilpotent Mal'cev algebras*.
Department of Mathematics, University of Colorado Boulder, Campus Box 395 Boulder, CO 80309, USA.
E-mail: patrick.wynne@colorado.edu.

The Subpower Membership Problem for an algebra $\mathbb{A}$, SMP($\mathbb{A}$), is the following computational problem: given tuples $a_1, \ldots, a_k, b \in \mathbb{A}^n$, is b in the subalgebra of $\mathbb{A}^n$ generated by $a_1, \ldots, a_k$? For certain finite algebras, SMP($\mathbb{A}$) has a polynomial time algorithm (such as Gaussian elimination for vector spaces) while for other finite algebras, SMP($\mathbb{A}$) is EXPTIME-complete. We investigate the structure of 2-nilpotent Mal'cev algebras by decomposing the term clone using an associated clonoid between abelian Mal'cev algebras. A clonoid from an algebra $\mathbb{A}$ to an algebra $\mathbb{B}$ is a set of finitary functions from $\mathbb{A}$ to $\mathbb{B}$ that is closed with respect to composition with term operations of $\mathbb{A}$ on the domain side and closed with respect to composition with term operations of $\mathbb{B}$ on the codomain side. We use this decomposition via clonoids to show that a large class of finite 2-nilpotent Mal'cev algebras have Subpower Membership Problem solvable in polynomial time.

Abstracts of contributed talks

▶ MEI ROSE CONNOR, *Developments in eight–valued lattice logic inspired by the seven–valued logic of the Saptabhangi*.
Department of Mathematics, University of Wisconsin–Madison, Van Vleck Hall, 480 Lincoln Dr, Madison, WI 53706, USA.
E-mail: meirconnor@gmail.com.
URL Address: meiroseconnor.com.

The work of this paper centers around an eight–valued lattice logic system developed to build a formalization of the seven–valued Jaina system of sevenfold predication, called *Saptabhangi* in the original Sanskrit. The work addresses the issues surrounding linguistic and philosophical objections to reducing the number of truth values to fewer than seven, and presents a solution that preserves the seven truth values while adding an eighth, null, value, that allows the system to become a distributed lattice. In other words, the eighth truth value, ∗, turns the system into a Boolean algebra, which has well–known associations between its join and meet and conjunction and disjunction, respectively. What remains to be developed is the syntax of a proof in such a system, as well as the semantics of a model in such a system.

▶ ANDREW DELAPO, *Index sets of CSC spaces*.
Department of Mathematics, University of Connecticut, 341 Mansfield Rd, Storrs, CT 06269, USA.
E-mail: andrew.delapo@uconn.edu.

The study of countable second-countable (CSC) topological spaces within computability theory was initiated by Dorais in 2011 in the context of reverse mathematics. In this talk, we define natural number indices corresponding to computable CSC spaces, and these form a Π^0_2-complete set which we denote by CSC. We then examine the complexity of several index sets within CSC, including the sets of indices of CSC spaces which are homeomorphic to the discrete or cofinite topologies on the natural numbers. More generally, we also consider

index sets of CSC spaces having certain topological properties, such as those which are T_1, Hausdorff, discrete, or have a certain specified subspace. This project is a work-in-progress, and there are several questions remaining to be answered which we also discuss.

▶ NICK GALATOS*, VITOR GREATI, REVANTHA RAMANAYAKE, AND GAVIN ST. JOHN, *Knotted substructural logics: complexity and finite models*.
University of Denver, C.M. Knudson Hall, 2390 S. York St., Denver, CO 80208, USA.
E-mail: ngalatos@du.edu.

Substructural logics are generalizations of classical logic that include, intuitionistic logic (constructive mathematics), relevance logic (philosophy), many-valued logic (engineering), and linear logic (functional programming), among others. Their algebraic semantics, *residuated lattices*, include diverse structures such as lattice-ordered groups, lattices of ideals of rings, and algebras of binary relations. Subctructural logics take their name from the fact that, when formulated in Getzen-style sequent calculi, they may lack one or more of the three structural rules of exchange, contraction and weakening (which are special cases of *knotted* inference rules).

We study various extensions of knotted substructural logics, which we formulate in hypersequent calculi. We prove that, they have the finite embeddability property (hence also the finite model property) by making use of existing and novel results in the theory of well-quasi orders, as well as utilizing the semantical method of residuated frames [2]. We also provide analytic calculi (that enjoy cut elimination) and employ a proof-theoretic analysis to establish complexity upper bounds (both for theorem-hood and deducibility). Finally, we prove that our complexity bounds are actually tight by using suitable variants of counter machines [3]. Our results extend to the level $\mathcal{P}_3$ of the substructural hierarchy [1].

[1] A. CIABATTONI, N. GALATOS, AND K. TERUI, *Algebraic proof theory for substructural logics: hypersequents* **Annals of Pure and Applied Logic**, vol. 168 (2017), no. 3, pp. 693–737.
[2] N. GALATOS AND P. JIPSEN, *Residuated frames with applications to decidability* **Transactions of the American Mathematical Society**, vol. 365 (2013), no. 3, pp. 1219–1249.
[3] N. GALATOS AND G. ST. JOHN, *Most simple extensions of FLe are undecidabe*. **The Journal of Symbolic Logic**, vol. 87 (2022), no. 3, pp. 1156–1200.

▶ KENNETH GILL, *Probabilistic automatic complexity*.
Independent Scholar.
E-mail: gillmathpsu@posteo.net.
URL Address: https://nowheredense.github.io.

We define a new complexity measure for finite strings using probabilistic finite-state automata (PFAs), inspired by similar existing notions using DFAs and NFAs. The PFA complexity of x, $A_P(x)$, is the least number of states of a PFA for which x is the most likely string of its length to be accepted. A parameterized variant $A_{P,\delta}(x)$ stipulates a real-valued lower bound δ on the gap in probabilities between x and other strings. We classify all binary strings with $A_P = 2$ and see that the NFA complexity of such strings can be arbitrarily high. The classification also serves to vindicate A_P as capturing some intuitive structural properties of strings. Unlike the Kolmogorov complexity, the DFA and NFA complexities are both computable, and A_P also turns out to be computable. Indeed, $A_{P,\delta}$ is computable for all δ, and there is a uniform algorithm computing $A_{P,\delta}(x)$ for all x and all but finitely many δ (depending on x).

▶ DEXUAN HU* AND SŁAWOMIR SOLECKI, *Polish modules over subrings of* $\mathbb{Q}$.
Department of Mathematics, Cornell University, Malott Hall, 310 Garden Ave, Ithaca, NY 14853, USA.
E-mail: dh623@cornell.edu.

Department of Mathematics, Cornell University, Malott Hall, 310 Garden Ave, Ithaca, NY 14853, USA.
E-mail: ssolecki@cornell.edu.

We give a method of producing a Polish module over an arbitrary subring of $\mathbb{Q}$ from an ideal of subsets of $\mathbb{N}$ and a sequence in $\mathbb{N}$. The method allows us to construct two Polish $\mathbb{Q}$-vector spaces, U and V, such that

— both U and V embed into $\mathbb{R}$ but
— U does not embed into V and V does not embed into U,

where by an embedding we understand a continuous $\mathbb{Q}$-linear injection. This construction answers a question of Frisch and Shinko [1]. In fact, our method produces a large number of incomparable with respect to embeddings Polish $\mathbb{Q}$-vector spaces.

[1] J. Frisch, F. Shinko, *A dichotomy for Polish modules*, ***Israel Journal of Mathematics***, vol. 254(2023), pp. 97–112.

▶ HANUL JEON, *Separating the Wholeness axioms*.
Department of Mathematics, Cornell University, Ithaca, NY, USA.
E-mail: hj344@cornell.edu.

Corazza [1] introduced the Wholeness axiom, which asserts the existence of an elementary embedding $j : V \to V$. Unlike the Reinhardt embedding, the Wholeness axiom is consistent with the axiom of choice. Hamkins [2] formulated finer versions WA_n of the Wholeness axioms and proved that WA_0 does not prove WA_1, but it was open whether WA_{n+1} proves the consistency of WA_n. In this talk, I will briefly sketch the proof for $\mathsf{Con}(\mathsf{ZFC} + \mathsf{WA}_0)$ from $\mathsf{ZFC} + \mathsf{WA}_1$.

[1] P. Corazza, *The Wholeness axiom and Laver sequences* ***Annals of Pure and Applied Logic***, vol. 105 (2000), pp.157–260.
[2] J. D Hamkins, *The Wholeness Axioms and $V = HOD$* ***Archive for Mathematical Logic***, vol. 40 (2001), pp. 1–8.

▶ ANG LI, *Introenumerability, autoreducibility, and randomness*.
Department of Mathematics, University of Wisconsin–Madison, 480 Lincoln Drive, USA.
E-mail: ali233@wisc.edu.

It is known that the class of total enumeration degrees has measure zero. In this talk, we define Ψ-autoreducible sets given an autoreduction procedure Ψ. Then, we show that a measurable class of Ψ-autoreducible sets has measure zero for a fixed Ψ. Using this, we show that classes of cototal, uniformly introenumerable, introenumerable, and hyper-cototal enumeration degrees all have measure zero. Then, we discuss what level of randomness these enumeration degrees could and could not have because sufficiently random sets must avoid such enumeration degrees.

▶ DICLE MUTLU, *Residue field domination in Henselian valued fields*.
Department of Mathematics & Statistics, McMaster University, Hamilton, Ontario, Canada.
E-mail: mutlud@mcmaster.ca.

In algebraically closed valued fields (ACVF), Haskell, Hrushovski, and Macpherson demonstrated that certain types are controlled by the stable part of the structure, known as stable domination. Later, it was generalized to residue field domination in various henselian valued fields of equicharacteristic zero by multiple authors including Ealy, Haskell, Maříková, Simon, and Vicaria. In this talk, we will present our results around residue field domination in henselian valued fields. Specifically, we will show the equivalence between stable domination

and residue field domination over models. Additionally, we will talk about the ongoing work on residue field dominated abelian groups inspired by the work of Hrushovski and Rideau-Kikuchi on stably dominated groups.

▶ DIEGO A. ROJAS, *Effective tightness of measures and Prokhorov's Theorem*.
Department of Mathematics, University of Dallas, 1845 E Northgate Dr, Irving, TX 75062, USA.
E-mail: darojas@udallas.edu.

Prokhorov's Theorem in probability theory states that, given a Polish space X, a family Γ of Borel probability measures on X is tight (i.e., every measure in Γ is concentrated on a compact subset of X) if and only if every sequence in Γ has a weakly convergent subsequence. To this end, we propose an effective notion of tightness for families of computable measures on a computable metric space. We will explore the connections between effective tightness and effective weak convergence of measures in the style of McNicholl and Rojas [1]. In particular, we derive an effective analog to Prokhorov's Theorem for computable sequences of measures under our effective tightness notion.

[1] T. H. McNicholl and D. A. Rojas, *Effective notions of weak convergence of measures on the real line*, **Information and Computation**, vol. 290 (2023), 104997

▶ WIM RUITENBURG, *Predicate logic for transitive Kripke models*.
Department of Mathematical and Statistical Sciences, Marquette University, P.O. Box 1881, Milwaukee, WI 53201, USA.
E-mail: wim.ruitenburg@marquette.edu.

This is joint work with Mohammad Ardeshir.

We consider a predicate logic for transitive Kripke models in a way similar to how intuitionistic logic is the predicate logic for reflexive transitive Kripke models. This logic and model theory can also be seen as a direct generalization of classical logic and model theory. No philosophical concerns are required.

▶ LUKE SERAFIN, *The infinite hat game and other graph 2-coloring problems*.
Department of Mathematics, Cornell University, 310 Malott Hall Ithaca, NY 14853, United States.
E-mail: lss255@cornell.edu.

Answering a question of Geschke, Lubarsky, and Rahn concerning an infinite-player version of a well-known recreational puzzle known as the "hat game," we demonstrate in the context of ZF + DC with one inaccessible cardinal that for a broad class of Borel bipartite graphs, the existence of a 2-colouring does not imply the existence of either a nonprincipal ultrafilter on ω or a Vitali set. Note here that "bipartite" means having a 2-colouring in every model of ZFC; the axiom of choice will likely be necessary to construct this colouring. The class of graphs for which our result applies is the Borel bipartite graphs of countable colouring number which, for some $n < \omega$, contain no embedded copy of K_{n,ω_1}. The proof relies on the geometric set theory of Larson and Zapletal, and proceeds by adding a 2-colouring of a given graph to the symmetric Solovay model W by means of a natural compactly balanced and Bernstein-balanced forcing.

▶ ILYA SHAPIROVSKY, *On the finite model property of subframe pretransitive logics*.
Department of Mathematical Sciences, New Mexico State University, USA.
E-mail: ilshapir@nmsu.edu.

A binary relation R on a set X is said to be *m-transitive*, if $R^{m+1} \subseteq \bigcup_{i \leq m} R^i$, where R^0 is the diagonal on X, $R^{i+1} = R \circ R^i$, and $\circ$ is the composition. A class of structures (X, R) is *pretransitive*, if these structures are *m*-transitive for some fixed m.

While, for $m > 1$, the finite model property (FMP) of the logic of all m-transitive relations is a long-standing open problem, some positive results are known for pretransitive classes which are closed under taking substructures; logics of such classes are said to be *pretransitive subframe*. Examples of such logics are given by the conditions $R^{m+1} \subseteq R$; their FMP in known since 1970s [1]. The FMP of the logic wK4 of the class of 1-transitive ($R^2 \subseteq R \cup R^0$) relations is shown in [2]. In [4], it is shown that all subframe transitive logics have the FMP; this result is generalized for subframe extensions of wK4 in [3].

I will present a recent result obtained jointly with A. Kudinov in [5]: for each $m > 1$, the logic given by the condition $R^{m+1} \subseteq R \cup R^0$, as well as its canonical subframe extensions, have the FMP.

[1] D. Gabbay, *A general filtration method for modal logics* **Journal of Philosophical Logic**, vol. 1 (1972), no. 1, pp. 29–34.

[2] G. Bezhanishvili, L. Esakia, and D. Gabelaia, *Spectral and T_0-spaces in d-semantics*, **Logic, language, and computation** (Nick Bezhanishvili, Sebastian Löbner, Kerstin Schwabe, and Luca Spada, editors), Springer, Berlin, 2011, pp. 16–29

[3] G. Bezhanishvili, S. Ghilardi, and M. Jibladze, *An Algebraic Approach to Subframe Logics. Modal Case* **Notre Dame Journal of Formal Logic**, vol. 52 (2011), no. 2, pp. 187–202.

[4] K. Fine, *Logics containing K4, Part II* **The Journal of Symbolic Logic**, vol. 50 (1985), no. 3, pp. 619–651.

[5] A. Kudinov and I. Shapirovsky, *Filtrations for wK4 and its relatives*, **Journal name spelled out, no abbreviations**, https://arxiv.org/pdf/2401.00457.pdf.

▶ JAVA DARLEEN VILLANO, *Computable categoricity relative to a c.e. degree*.
Department of Mathematics, University of Connecticut, 341 Mansfield Rd, Storrs, CT 06269, USA.
E-mail: javavill@uconn.edu.

A computable graph $\mathcal{G}$ is computably categorical relative to a degree **d** if and only if for all **d**-computable copies $\mathcal{B}$ of $\mathcal{G}$, there is a **d**-computable isomorphism $f : \mathcal{G} \to \mathcal{B}$. In this paper, we prove that for every computable partially ordered set P and computable partition $P = P_0 \sqcup P_1$, there exists a computable computably categorical graph $\mathcal{G}$ and an embedding h of P into the c.e. degrees where $\mathcal{G}$ is computably categorical relative to all degrees in $h(P_0)$ and not computably categorical relative to any degree in $h(P_1)$.

Abstracts of talks presented by title

▶ BEIBUT KULPESHOV, *Non-trivial expansions of 1-transitive weakly o-minimal orderings*.
Institute of Mathematics and Mathematical Modeling, Kazakh-British Technical University, Almaty, Kazakhstan.
E-mail: b.kulpeshov@kbtu.kz.

Let L be a first-order countable language. We consider L-structures and assume that L contains a binary relation symbol $<$, which is interpreted as the linear ordering in these structures. A linearly ordered structure $M := \langle M, <, ... \rangle$ is said to be 1-*transitive* if $tp(a/\emptyset) = tp(b/\emptyset)$ for any $a, b \in M$.

A subset A of a linearly ordered structure M is called *convex* if for any $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal structure* [1] is a linearly ordered structure $M = \langle M, =, <, ... \rangle$ such that any definable (with parameters) subset of the structure M is a union of finitely many convex sets in M.

We consider definable functions from M in its Dedekind completion $\overline{M}$, more precisely into definable sorts of $\overline{M}$, representing infima or suprema of definable sets. Let $A, D \subseteq M$, D be infinite, $Z \subseteq \overline{M}$ be an A–definable sort and $f : D \to Z$ be an A-definable function.

We say that f is *locally increasing* (*locally decreasing, locally constant*) on D if for any $a \in D$ there exists an infinite interval $J \subseteq D$ containing $\{a\}$ such that f is strictly increasing (strictly decreasing, constant) on J.

THEOREM. *Let T be an 1-transitive weakly o-minimal linear ordering, $M \models T$. Suppose that $R(x, y)$ is a new binary relation such that each of the sets $R(a, M)$ and $R(M, a)$ is convex and infinite for any $a \in M$. Then a weakly o-minimal expansion of T by $R(x, y)$ preserves 1-transitivity if and only if $g(x) := \inf R(x, M)$, $f(x) := \sup R(x, M)$, $g'(y) := \inf R(M, y)$ and $f'(y) := \sup R(M, y)$ are locally monotonic or locally constant on M and there exists an $\emptyset$-definable equivalence relation $E(x, y)$ partitioning M into infinitely many convex classes such that that all functions g, f, g' and f' are strictly increasing on M/E.*

[1] H.D. MACPHERSON, D. MARKER, AND C. STEINHORN, *Weakly o-minimal structures and real closed fields*, **Transactions of The American Mathematical Society**, Vol. 352, No. 12 (2000), pp. 5435–5483.

▶ JOACHIM MUELLER-THEYS, *Universal semantics*.
Independent; Heidelberg, Germany.
E-mail: mueller-theys@gmx.de.

I. Let $L \neq \emptyset$ with $\phi, \psi, \chi, ... \in L$, $\Phi, \Psi, ... \subseteq L$ ("language"). Further, $\Im^* \neq \emptyset$ with $I, J, ... \in \Im^*$ ("set of all interpretations"). We define *abstract satisfaction relations* $\models$ as free subsets of $\Im^* \times L$. $I \models \Phi$ if $I \models \phi$ for all $\phi \in \Phi$. Accordingly, $\mathcal{S} := (L, \Im^*, \models)$ is called an *abstract semantics*. The conception even covers *singular $\mathcal{S}$*, videlicet $|\Im^*| = 1$.

For instance, $\mathcal{S}$ is *alternative* if L is equipped with the operation $\vee : L \times L \to L$ and $I \models \phi \vee \psi$ iff $I \models \phi$ or $I \models \psi$.

II. The *semantic consequence relation* $\models = \models' \subseteq \wp(L) \times L$ (induced by $\models \subseteq \Im^* \times L$) may now be defined in standard manner: $\Phi \models_{(\mathcal{S})} \phi$:iff for all $I \in \Im^*$: $I \models \Phi$ implies $I \models \phi$. Consequently, *premises always are consequences*: $\phi \in \Phi$ *implies* $\Phi \models \phi$. We call ϕ a *zero consequence* if $\emptyset \models \phi$; symbolically, $\models_0 \phi$.

III. *Validity (Allgemeingültigkeit)* $\models = \models' \subseteq L$ may be defined as usual by the definiens: $I \models \phi$ for all $I \in \Im^*$. By means of the sub-lemma $I \models \emptyset$, $\models_0 \phi$ *implies* $\models \phi$. We have introduced *general consequence* (for which we use the symbol $\not\models$) as *consequence of all premise sets*: $\not\models \phi$:iff for all $\Phi \subseteq L$: $\Phi \models \phi$. Obviously, $\not\models \phi$ *implies* $\models_0 \phi$. Moreover, $\models \phi$ *implies* $\not\models \phi$. Thus, despite the intensional differences, *validity, general consequence, and zero consequence generally coincide*, viz. $\models \phi$, $\not\models \phi$, $\models_0 \phi$ *are equivalent*.

IV. $seq(\Phi) := \{\phi : \Phi \models \phi\}$ *never produces further consequences*, viz. $seq(\Phi) \models \phi$ if and only if $\Phi \models \phi$. This follows from $I \models seq(\Phi)$ iff $I \models \Phi$. So $seq(\Phi)$ *is closed*. $seq : \wp(L) \to \wp(L)$ is a *consequence operator*, which is monotonic additionally.

Notes. Joint work with Wilfried Buchholz. Prompted by the talk of Zalán Gyenis in 2023 at the "Logical Universalis Webinar", founded by Jean-Yves Béziau, we introduced abstract semantics as a tool for the "Refutation of Alternativeism", which has shown that, roughly, no logic exists such that "$\Phi \vdash \phi$ or $\Phi \vdash \psi$" is too a *necessary* condition for $\Phi \vdash \phi \vee \psi$, as claimed in intuitionistic meta-logic (**XIX SLALM 2022, ASL@JMM 2023, elaborate paper XX SLALM 2024**). Further, important aspects have occurred.

Thanks. Jan Wolenski, R. Iemhoff; A. & H. Haltenhoff, and Peana Pesen".