V. PHOTOSPHERIC FLUX TUBES

MAGNETOHYDRODYNAMICS OF SUNSPOTS

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ABSTRACT. Recent numerical investigations of fully compressible nonlinear magnetoconvection have clarified the nature of convection in sunspot umbrae. In a shallow layer with a strong vertical magnetic field oscillations give way to travelling waves as the Rayleigh number is increased but in a deep stratified layer oscillatory behaviour only appears after secondary bifurcations. This behaviour leads to a model that explains the formation of umbral dots. Penumbral structure is more difficult to explain owing to the apparent presence of adjacent horizontal and inclined fields in dark and bright filaments. The inner penumbra lies above a transition zone where volume currents are needed to maintain an overall static equilibrium; instabilities in this region may be responsible for filamentary structure in the penumbra as well as for fine structure at the umbral-penumbral boundary.

1. Introduction

Sunspots have been observed for centuries and the ideal circular spot with a filamentary penumbra is a fairly good approximation to reality. Yet this apparently simple structure remains surprisingly difficult to explain. Our inability to provide a proper description of a sunspot is an obvious embarrassment to theoreticians. A few days ago the "Navhind Times" referred to "sunspots, *mysterious* areas on the solar surface that are cooler and darker than the surrounding area". In this review I hope to explain why they continue to be so mysterious.

One sign of our ignorance is the lack of any consensus on the overall structure of a flux-tube beneath a sunspot. Early discussions contrasted a monolithic plug extending deep into the convection zone with a jellyfish that split into innumerable tentacles below the photosphere. Now we know that convection must lead to an irregular structure which still remains sufficiently coherent to support global oscillations. But disagreement persists between those who favour an inhomogeneous plug and those who favour a tight cluster of independent flux tubes. Moreover, surface observations do not allow us to decide between these models.

In what follows I shall give a selective discussion of some major problems; others, such as waves, are treated elsewhere in these Proceedings. First I shall describe recent progress in modelling convection in the umbra. Then I shall outline the

E. R. Priest and V. Krishan (eds.), Basic Plasma Processes on the Sun, 139–148.

difficulties posed by the field configuration in the outer penumbra and finally I shall comment on the global structure of a sunspot.

2. Energy Transport in Sunspot Umbrae.

It has been accepted for almost 50 years that sunspots are cool owing to partial suppression of convection by a strong magnetic field (Biermann 1941; Cowling 1976). As more detailed sunspot models were computed (Schlüter & Temesvary 1957; Chitre 1963; Deinzer 1965; Chitre & Shaviv 1967; Yun 1970) it became apparent that even the reduced energy flux emitted by the umbra cannot be supplied by radiative transport alone. Hence it is essential to establish the nature of convection in an imposed vertical magnetic field – a topic to which Indian astrophysicists have made important contributions.

Boussinesq magnetoconvection has been studied in considerable detail. For a fluid with magnetic diffusivity η and thermal diffusivity κ behaviour depends critically on the ratio

$$\zeta = \eta/\kappa\ ;$$

in a star the radiative diffusivity κ is typically large ($\zeta \approx 10^{-6}$ in the solar photosphere). Linear theory shows that for $\zeta < 1$ and a sufficiently strong magnetic field convection sets in at an oscillatory (Hopf) bifurcation (Chandrasekhar 1961) and the development of nonlinear oscillations has been followed with analytical and numerical techniques (Proctor & Weiss 1982; see also Rudraiah, Kumudini & Unno 1985a, b).

Recent work has been more concerned with compressible magnetoconvection (Hughes & Proctor 1988). The linear instability of a polytropic atmosphere in a vertical magnetic field has been investigated by Antia & Chitre (1979) and Cattaneo (1984). In the regime of interest there is a Hopf bifurcation from the static solution, giving rise to branches of standing wave and travelling wave solutions, followed eventually by a stationary (pitchfork) bifurcation which gives rise to a branch of steady solutions. The steady solutions are initially unstable but may ultimately gain stability. The effects of compressibility and stratification on steady nonlinear magnetoconvection were studied by Hurlburt & Toomre (1988). Subsequently, Hurlburt *et al.* (1989) investigated time-dependent convection in a weakly stratified layer. The behaviour found depends on the parameter $\beta = 2\mu_0 p/B_0^2$, where B_0 is the mean field strength and p is the gas pressure; in a sunspot β increases from unity at the photosphere to 30 at a depth of 1000 km. Although convection sets in as periodic oscillations, stability is transferred to travelling waves for $32 \geq \beta \geq 6$ if $\zeta < 0.3$. The travelling waves have strong prograde velocities but carry very little heat. If they were excited in the umbra we might expect to observe them but no behaviour of this type has yet been seen (except perhaps at the umbral-penumbral boundary). Convection in a deep stratified layer apparently takes a different form, though locally generated travelling waves, at depths of only 100 km below the photosphere, may still be able to excite umbral oscillations (e.g. Evans & Roberts

1990).

Within a sunspot the density increases rapidly with depth. Moreno Insertis & Spruit (1989) have studied the stability of a model atmosphere in the absence of diffusion. In the sun the opacity increases beneath the photosphere owing to ionization and the radiative diffusivity consequently falls so that $\zeta > 1$ at depths between 2000 and 20000 km (Meyer *et al.* 1974). This raises the following question: what happens in a layer where $\zeta > 1$ at the bottom, favouring steady overturning, but $\zeta < 1$ at the top, where oscillations are locally preferred? In sunspot umbrae magnetic fields are most effective in hindering convection immediately below the photosphere. Umbral dots have been ascribed to overshooting from below (Obridko 1974; Parker 1979a). Knobloch & Weiss (1984), extrapolating from Boussinesq convection, suggested that oscillations would be decoupled from overturning motion, while Parker (1979a, b) and Choudhuri (1986) favoured oscillatory convection in field-free inclusions.

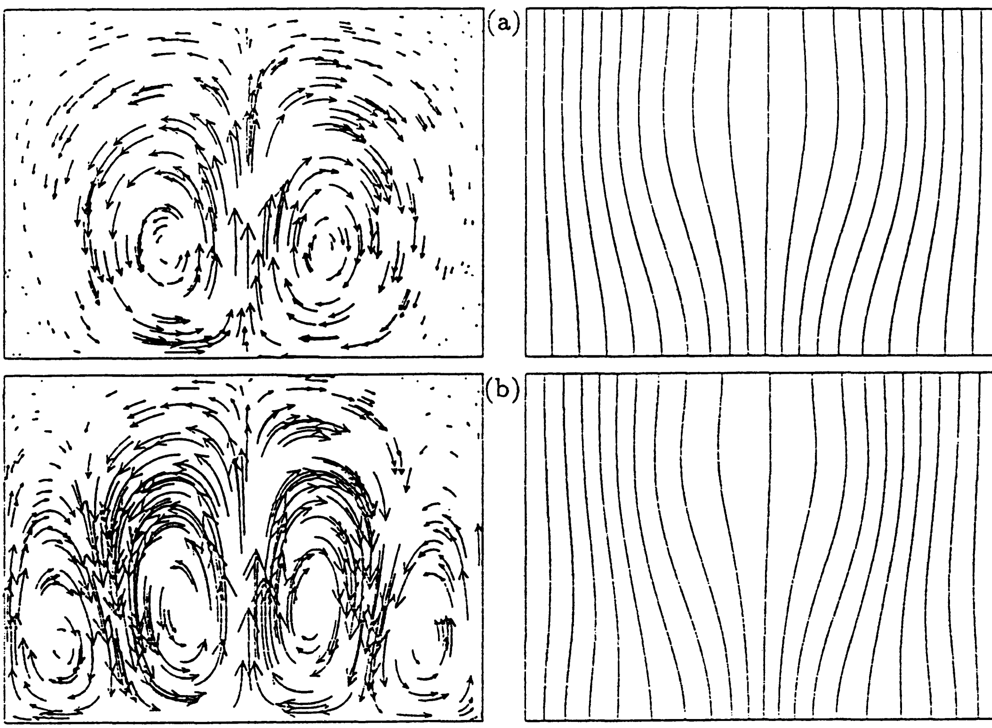

Figure 1. Nonlinear convection in a stratified compressible layer ($0.2 \leq \zeta \leq 2.2$). Streaklines (left) and fieldlines (right) for steady solutions with (a) $R = 1.07\ R_0$ and (b) $R = 1.39\ R_0$. Note the development of countercells in the corners.

Weiss *et al.* (1990) have recently investigated fully compressible nonlinear magnetoconvection in a stratified two-dimensional model, where $\zeta \propto \rho$ and increases continuously with depth. The most interesting results are obtained when $0.2 \leq \zeta \leq 2.2$, so that oscillations are favoured in the upper half of the layer. A linear stability analysis shows, however, that convection sets in at a stationary (pitchfork) bifurcation when the Rayleigh number $R = R_0$, giving rise to overturning motion. Numerical experiments yield weakly nonlinear solutions that are steady. At first motion barely penetrates to the top of the layer, as shown in Figure 1(a) for $R = 1.07\ R_0$. As R is increased countercells develop in the region where the magnetic field is strong, as illustrated in Figure 1(b) for $R = 1.39\ R_0$. These steady solutions eventually lose stability in a second Hopf bifurcation at $R = 1.47\ R_0$ and (after various vicissitudes) all trajectories are attracted to a branch of periodic solutions that exists for $R \geq 1.44\ R_0$. Figure 2 shows the behaviour of the velocity and magnetic field for an oscillatory solution at $\mathrm{R} = 2.59\ \mathrm{R}_0$. The horizontal scale has changed so that there are four rolls with rising plumes at the centres and sides of the region. These plumes are modulated periodically so that they alternate in vigour. Motion is now more vigorous at the top of the layer, where the velocity reverses its direction in each half of the domain, though the sense of motion at the base of the layer is unchanged. These oscillations are robust and persist over a wide range of parameter values. Note that the time-dependence as well as the underlying horizontal scale (with four rolls) is controlled by the upper part of the layer, where the magnetic field is most effective. On the other hand, the increase in ζ towards the base eliminates any travelling waves.

It is easy to generalize from the results in Figure 2 to the behaviour of an infinite layer. There one might expect to find aperiodically modulated rolls with rising plumes that sporadically burst through to the top of the layer. Similar behaviour in three dimensions, with an irregularly tesselated pattern of convection, would give rise to spatiotemporal chaos. The associated timescale is comparable with the Alfvén transit time for the layer, around 2 hours in the sun. This process provides a natural explanation for umbral dots in the centre of a sunspot. Observers have distinguished between central and peripheral umbral dots (e.g. Grossmann-Doerth *et al.* 1986). The latter occur near the umbral-penumbral boundary and may connect with bright filaments or even form linear features that develop into light bridges separating different portions of the umbra (García de la Rosa 1987; Scharmer 1989). The central umbral dots can be regarded as a convective phenomenon within a more or less homogeneous flux rope, where the field strength varies owing to the motion, but behaviour near the inner edge of the penumbra suggests that field-free plasma is intruding from outside the flux tube, as proposed by Parker (1979a, b) and Choudhuri (1986).

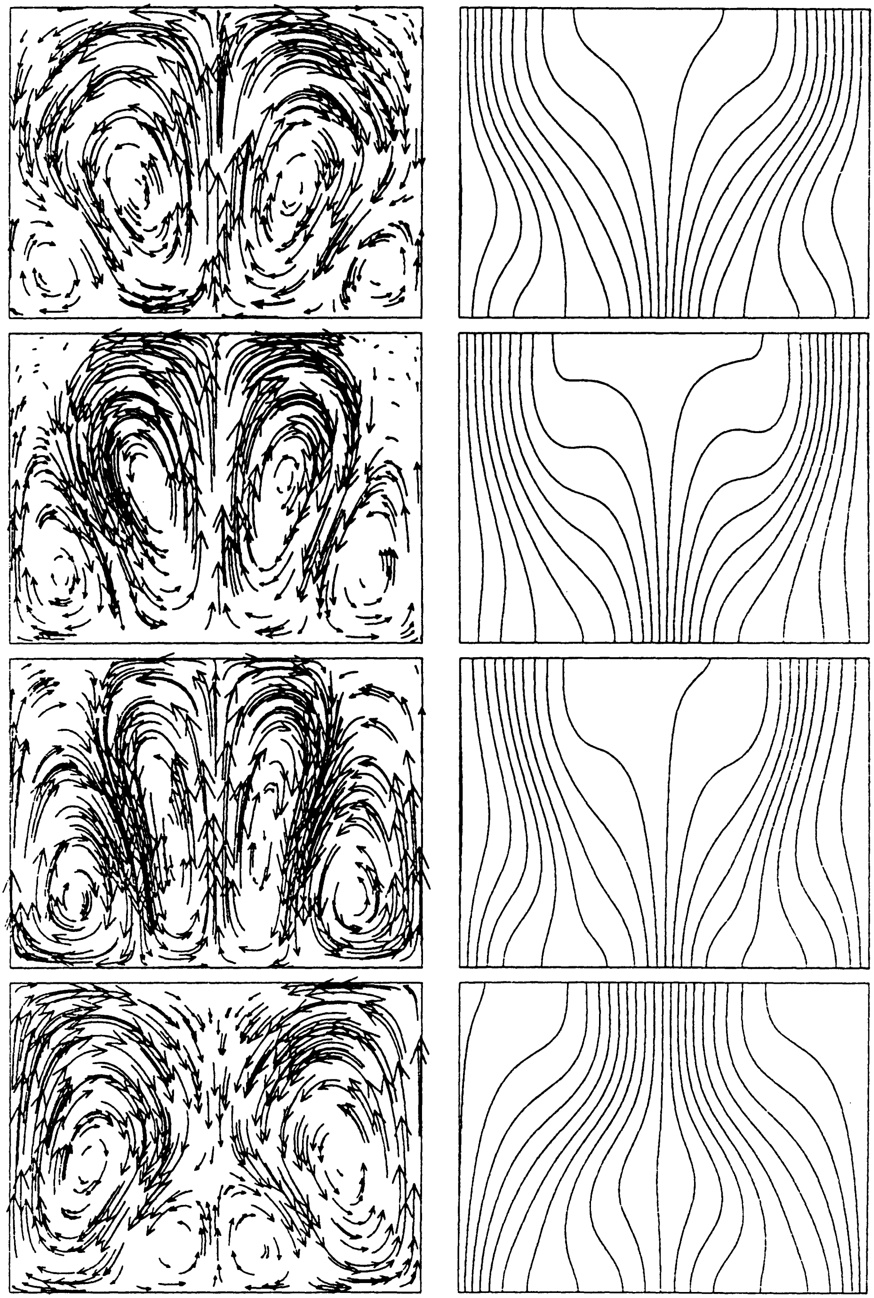

Figure 2. Oscillatory convection in a stratified layer for $R = 2.59\ R_0$. Streaklines and fieldlines at successive intervals during half a period of the oscillation.

3. Penumbral Field Configurations

The field becomes increasingly inclined to the vertical towards the outer boundary of the penumbra. Indeed the Evershed flow in dark penumbral filaments is virtually horizontal. Thus it is tempting to suppose that the magnetic field in the outer penumbra is likewise almost horizontal, as implied by Wittmann's (1974) measurements. Now the transition between the magnetic flux tube beneath the sunspot and the external plasma is observed to be sharp. If the field is horizontal this interface reduces the efficiency of convection so drastically that only shallow penumbral models (about 100 km thick) can be constructed (Schmidt, Spruit & Weiss 1986). These models allow one to develop an attractive and consistent picture of the outer penumbra but they have several fatal flaws. First of all, the magnetic flux emerging through the umbra and penumbra is insufficient for a monopole field with a strength of 1500G at the outer edge of the penumbra (Schmidt 1987). Secondly, the most recent precise measurements show that the mean field has an inclination of about 70° at the penumbral boundary (Adam 1990). Finally, high resolution measurements of the line-of-sight field in a spot as it rotates across the disc show that fields in bright and dark filaments have different directions, with the former inclined at $60-70°$ at the edge of the spot (Title *et al.* 1989). Thus one is forced back to considering a two-component field in the penumbra. Unfortunately no-one has yet produced a convincing description of such a configuration.

Some isolated features are understood: for example, the Evershed outflow (discovered at Kodaikanal) can be explained as a siphon effect driven by pressure differences along slightly inclined flux tubes (Meyer & Schmidt 1968; Thomas 1988; Montesinos & Thomas 1989). The inward motion of bright grains (Muller 1973) suggests that the filamentary structure is essentially time-dependent and that the intersection of a bright filament with the photosphere moves towards the umbra as the filament rises (Spruit 1981). The alternation between bright and dark filaments could then be related to some undular instability but it is not clear how this process could be represented by a local model without involving the deep structure of the sunspot.

4. Global Structure

The simplest models of a flux tube in magnetohydrostatic equilibrium assume an axisymmetric force-free poloidal field. This requires a potential field within the flux tube, separated by a current sheet from the external field-free plasma. The boundary condition is that the total pressure must be continuous across the interface, whose position is unknown. Sophisticated procedures for solving the resulting free boundary problem have now been developed (Schmidt & Wegmann 1983; Jahn 1989; Pizzo 1990). These potential field models provide valid descriptions of small flux tubes and pores but fail for sunspots. In a spot with a Wilson depression of 600 km the difference between the gas pressures at the centre of the umbra and in the external plasma is about 10^6 dyne cm^{-2}. This corresponds to a field of 5000 G,

https://doi.org/10.1017/S0074180900087878 Published online by Cambridge University Press

much greater than anything observed in normal sunspots. Hence it is necessary to invoke volume currents within the flux tube in order to explain the equilibrium of a sunspot. Jahn (1989) has made a great step forward in modelling the static structure of an (azimuthally averaged) axisymmetric sunspot. Figure 3 shows details of one of his solutions, which is matched to the observed surface field distribution. The currents are confined to a thick sheath below the penumbra, enclosing a central core with a potential field. This equilibrium model, with a gradually tapering flux tube, seems convincing.

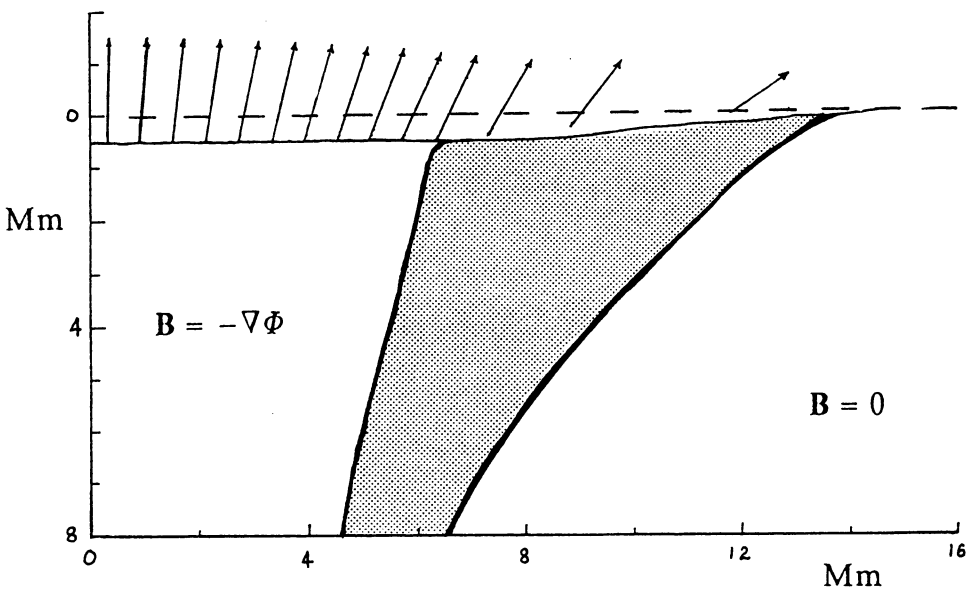

Figure 3. Cross-section through an axisymmetric model of a sunspot in magnetohydrostatic equilibrium (after Jahn 1989). The current sheath (shaded) encloses the potential field underneath the umbra. Arrows indicate the magnitude and direction of the magnetic field at the surface.

The difficulties come when we consider non-axisymmetric fine structure and energy transport mechanisms in the penumbra. Then we must describe dynamical processes in the flux tube. The region below the penumbra has an inclined magnetic field. It is convectively unstable and may also be unstable to interchanges, owing to the curvature of the boundary. Hence we may expect undular instabilities which could lead to the formation of penumbral filaments. In addition they might give rise to intrusions of field-free plasma into the umbra, as suggested by observations made at Tenerife and La Palma. So we are faced with the problem of describing the nonlinear development of instabilities in the current sheath below the penumbra. Here the geometry is awkward, the boundary conditions are uncertain and several effects are simultaneously involved. It is not obvious how to isolate individual processes, while a full simulation would be complicated and difficult to interpret. Yet we cannot claim to understand a sunspot until we are able to analyse this structure.

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DISCUSSION

MONTGOMERY: In our MHD computations, when we have gone from two dimensions to three, we have often seen large changes and qualitatively new effects. Do you expect the same will happen with your convective MHD computations? What can one learn in two dimensions that is likely to remain true in three?

WEISS: Two-dimensional models are obviously over-simplified, though they enable us to identify bifurcations and to investigate transitions from one form of solution to another. Three-dimensional simulations of compressible convection in the absence of a magnetic field show that sinking sheets are focussed into plumes and allow a richer structure. It is likely that our two-dimensional solutions are unstable to three-dimensional perturbations. Nevertheless, we expect travelling waves to persist in the regime where $1 \lesssim \beta \lesssim 10$ and secondary oscillations to survive in stratified layers. We plan to carry out three-dimensional numerical experiments in the near future.

KNEER: Could you make predictions on the timescale, temperature- and magnetic field fluctuations, and velocities involved with the oscillatory convection you simulated?

WEISS: Our model is too idealized to predict the magnitudes of temperature and field perturbations at the umbral photosphere. The timescale is determined by the Alfvén speed over a layer about 3000 Km deep, which is compatible with the observed lifetime of umbral dots.

CHITRE: Is the time ripe now to construct two-dimensional models of sunspots in order to understand problems concerning the depth of a sunspot (*i.e.* location of the base of the spot)?

WEISS: I think there is scope for more detailed models extending the magnetohydrostatic models developed recently by K. Jahn. There is still a difficulty in describing thermodynamic properties of the region with volume currents, which makes it hard to formulate a consistent treatment.

RUDRAIAH: (i) Are there observations to reveal the existence of horizontal magnetic field comparable in magnitude to that of vertical magnetic field?
(ii) How dark is the Sunspot? Is your model capable of explaining this?

WEISS: (i) There is some evidence for azimuthal fields but they do not seem to be significant. The radial field increases from zero at the centre of the umbra to the edge of the spot where the field is almost horizontal. Small-scale variations in field directions are very difficult to measure.
(ii) The mean umbral heat flux is about 20% of the photospheric flux. This is consistent with our model but any quantitative prediction will need detailed simulations like that being carried out by Nordlund and Stein.

AL-KHASHLAN: How dark is the sunspot?

WEISS: The umbra is about 2000K cooler than the normal photosphere so the intensity is reduced by a factor of 6. The isolated umbral dots are small and have photospheric brightness. The mean penumbral intensity is about 25% less than the normal photospheric value but dark filaments are 40% less bright while bright filaments are only slightly cooler.