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FOURTH ORDER GEOMETRIC EVOLUTION EQUATIONS

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In this thesis the chief objects of study are hypersurface flows of fourth order, with the speed of the flow varying from the Laplacian of the mean curvature, to the more general constrained flows which include a function of time in the speed, and satisfy various conditions. Our aim is to instigate a study of the regularity of these flows, answering questions of local and global existence, and some preliminary singularity analysis. Among our results are positive lower bounds for smooth and regular existence, classification of stationary solutions, interior estimates, and blowup asymptotics. Applying these results to a certain class of constrained surface diffusion flows, we obtain long time existence and exponential convergence to spheres for initial surfaces with small L^2 norm of tracefree curvature. We present one application of this theorem, using it to deduce the isoperimetric inequality with optimal constant for 2-surfaces satisfying the above smallness condition. The long time existence theorem can be thought of as a stability of spheres result, as the smallness condition is an averaged distance from a standard round sphere to the initial manifold in L^2. This strengthens a related earlier result specialized to the surface diffusion and Willmore flows [17], where the distance is small in $C^{2,\alpha}$, obtained through a completely different method. Our techniques have more in common with [9–11], from which we have drawn much inspiration. The results throughout this thesis are new contributions for both surface diffusion flow, which has been considered by many authors [1–8, 12, 16, 17], and the constrained flows, which have only recently been considered [13–15, 18, 19].

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