

P. Samuel, Anneaux Factoriels, edited by A. Micali.  
Sociedade de Matematica de Sao Paulo, 1963. 102 pages. \$3.00.

This delightful little book pursues unique factorization domains with a singleness of purpose. Let  $A$  be any commutative integral domain with unity and let  $B$  be (a) the polynomial ring over  $A$ , (b) the ring of quotients of  $A$  with respect to a multiplicative subset of  $A$ , (c) the  $M$ -adic completion of  $A$  with respect to an ideal  $M$  of  $A$ , (d) the formal power series ring over  $A$ . In most of these cases the question is considered whether from the fact that  $A$  is noetherian or a unique factorization domain one can deduce the same property for  $B$  and conversely. In addition to well-known classical results one finds here a number of important theorems by Nagata, Mori and Samuel.

The first five chapters are completely elementary and can be read by an undergraduate honours student. In fact, the book is said to comprise the lecture notes of a one-semester undergraduate course at the University of Paris. The last (and longest) chapter utilizes homological methods and culminates in the Auslander-Buchsbaum theorem that every regular local ring is a unique factorization domain.

One will be forgiven for doubting that the material of this last chapter was digested by undergraduates, even in Paris. At any rate, the four hour examination, reproduced on page 98, does not seem to refer to it.

On the whole, it is an intriguing idea to build an undergraduate course on a single theme like the present one, letting general concepts be introduced and illuminated as they are needed.

The booklet has some minor faults that can easily be remedied. The notation for formal power series rings is found on page 3 without explanation; maximal ideals are defined, but local rings seem to appear suddenly on page 40 without definition; and, finally, the spelling of many French words seems slightly suspect to this untutored reviewer.

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M. A. A. Studies in Mathematics, Volume 2, Studies in Modern Algebra. A. A. Albert, Editor. 1963. 190 pages.

This modest looking volume contains six articles, and is supposed to present the broad aspects of modern algebraic thought.

The most exciting article is by C. W. Curtis. It traces the

history (1840-1958) of the problem: "Find all division algebras (not necessarily associative) over the field of real numbers", and it concludes with the KERVARE, BOTT, MILNOR solution that the only ones are essentially the Cayley numbers, quaternions, the complex numbers and the real numbers. Curtis goes on to connect up, briefly, the Cayley numbers with Jordan and Lie Algebras.

But what audience is this for? It begins with the definition of a field and thus seems aimed at undergraduates, but the last section gives Jacobson's proof of HURWICZ's problem on normed algebras and uses alternative algebras with involutions. This is perhaps beyond the scope of the undergraduate.

Nevertheless this is very useful. Mathematics is in a constant state of flux and someone must summarize results from time to time. This is a clear, well written summary of an interesting and important problem.

L. J. PAIGE's article on Jordan Algebras is a clear exposition of the general theory of Jordan Algebras, with examples, a discussion of the radical, semi-simplicity, idempotents, simple algebras and the exceptional simple Jordan Algebras. For anyone who has had the associative Wedderburn theory, this is an excellent introduction to nonassociative work.

E. KLEINFELD's article on Cayley numbers proves that alternative division rings of characteristic not two are either Cayley-Dickson division algebras or associative division rings. We would have preferred a survey of alternative algebras a la Paige's Jordan section.

R. H. BRUCK's article on loops we felt was somewhat patronizing. Nevertheless it is a good introduction to groupoids, semigroups, quasi-groups and loops. The inclusion of some thirty exercises seemed somewhat out of place in a book of this nature.

S. MacLane has two articles, both on associative algebra. The first one was first published in 1939, and the second was written in 1961. The second article deals with finite groups and homological algebra. MacLane points out that algebra has moved from studying structure theory to an involvement with all of mathematics - topology, differential geometry etc. We were somewhat surprised that no mention was made of Jacobson's or Goldie's structure theorems, even though structure theory is no longer the central algebraic theme.

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