# BOOK REVIEWS

CHEVALLEY, CLAUDE, *Fundamental Concepts of Algebra* (Academic Books Ltd., London, 1956), 241 pp., 54s. 6d.

In this book some of the fundamental concepts of algebra are treated with great generality from an abstract point of view. Adopting the rigorously deductive mode of presentation of the modern French school, the author begins with monoids (associative systems with a neutral element). From there he proceeds to a brief discussion of groups, including free groups. This is followed by a substantial chapter on rings and modules, in which the idea of a tensor product plays a prominent part. A short chapter on general algebras is the prelude to a detailed study of some aspects of associative algebras. Here the reader will find applications of most of the concepts previously introduced, and he will meet some familiar concepts in a new guise; for example, determinants, which are not mentioned until p. 174 is reached, are defined within the framework of Grassmann algebras. The same ideas lead to results about derivations and to a definition of the Pfaffian. There are few illustrative examples in the text, but each chapter is followed by a large number of exercises, some of a fairly theoretical nature.

Although based on a first-year graduate course the book will be found difficult by most students. Inexorably theorem follows upon theorem (there are 73 on rings and modules in a space of 80 pages), and the inexperienced reader will need further guidance if he wants to understand the motivation for some of the abstract concepts, as distinct from the routine development of each concept once it has been introduced.

Nevertheless, Professor Chevalley's penetrating power and striking originality as a mathematician makes itself felt throughout the book. He has given us an authoritative account of some of the most vital and viable algebraical ideas.

WALTER LEDERMANN

ZARISKI, O., AND SAMUEL, P., with the cooperation of I. S. COHEN, *Commutative Algebra*, vol. i (D. van Nostrand Co., London, 1958), 320 pp., 32s. 6d.

According to the Preface, Professor Zariski, having contemplated writing a volume on Algebraic-Geometry, felt constrained first to write a two-volume work on the underlying algebraic theory. Volume 1, written with the collaboration of Professor Samuel, is the book under consideration. Volume 2 is in preparation. Instead of writing an encyclopædic account of the subject the authors " have preferred to write a self-contained book which could be used in a basic graduate course of modern algebra." British readers should be careful to translate " graduate " as " post-graduate " for this is not a book for the novice. Bearing this in mind the volume is an excellent exposition of commutative rings, fields and ideals. The Chapter headings are I Introductory Concepts, II Elements of Field Theory, III Ideals and Modules, IV Noetherian rings, V Dedekind Domains, Classical Ideal Theory. There are valuable indexes of definitions and notations but the lack of a more comprehensive index impairs considerably the usefulness of the book as a reference work. The authors state that they are unaware of any systematic account of the subject since Krull's *Idealtheorie* published in 1935, but surely they have overlooked Gröbner's *Moderne Algebraische Geometrie die idealtheoretischen Grundlagen* (1949) and McCoy's *Rings and Ideals* (1948). The present volume does, however, contain some important material not to be found in either of these works and it should form a valuable addition to the library of an Algebraic Geometer or Algebraist.

The mathematical exposition in this work is usually admirable although the



https://doi.org/10.1017/S0013091500021684 Published online by Cambridge University Press

reviewer feels that the clarity might have been enhanced by the inclusion of more illustrations. Italics, we feel, are used to excess, sometimes for definitions and sometimes for emphasis, and the reader is not always certain for which purpose they are employed. We would have welcomed also introductory paragraphs outlining the purpose and chief results of each chapter, for, as is common in many works on algebra, the reader often finds it difficult to decide which results are of prime importance and which results are secondary. The printing is excellent.

D. E. RUTHERFORD

PICCARD, SOPHIE, *Sur les bases des groupes d'ordre fini* (Neuchatel, Secrétariat de l'Université, 1957), pp. xxiv + 242.

Mlle. Professor Piccard is known not only as an enthusiast but also as the expert on the bases of finite permutation groups. It follows that those who are interested in this topic will give her book a warm welcome. To describe the subject matter to the uninitiated it is perhaps best to illustrate it with an example. The symmetric group $\mathscr{S}_4$ of all permutations on four letters which we may designate simply as 1, 2, 3, 4, is generated by the three transpositions (12), (23), (34). These three independent generators form a system of order 3. Quite apart from equivalent systems of generators obtained from the above merely by renumbering the letters, other systems of independent generators are possible and those of the smallest possible order are called bases. In $\mathscr{S}_4$ the bases are of order 2 and indeed this group can be generated by any one of the following pairs of permutations (123), (34); (1234), (12); (1234), (123); (1234), (132); (1234), (1324). Thus $\mathscr{S}_4$ has five different types of base. For any given base there are a set of characteristic relations which define the group. For instance if $S = (1234)$ and $T = (1324)$ the relations which define $\mathscr{S}_4$ are $S^4 = TS^2T^{-3}S^2 = TSTS^3T^{-1}S^3 = 1$. The book under review is a study of such bases and characteristic relations, especially those of the symmetric and alternating groups but also of many other permutation groups with special types of base. The value of the subject is that it throws some light on the difficult problem of classifying all finite groups of a given order according to their structure.

Parts of this volume are necessarily in the nature of a catalogue of bases of those groups which have been fully investigated. For instance there are 2308320 bases of the alternating group $\mathscr{A}_7$. As might be expected Hölder's theorem that $\mathscr{S}_6$ is the only symmetric group possessing outer automorphisms can be established by considering characteristic relations.

The book is carefully composed and excellently printed. A very useful feature of the work is a preliminary synopsis defining all those group concepts with which the reader is expected to be familiar, and including some results established in Mlle. Piccard's two previous works on the symmetric group. These features make the book self-contained. It is a pity that there is no index in addition to the Table of Contents.

D. E. RUTHERFORD

KHINCHIN, A. Y., *Three Pearls of Number Theory* (Graylock Press, Rochester, N.Y., 1956), $2.00, 64 pp., 16s.

This little book contains an account of three very beautiful results in Number Theory. They are typical of the subject in three respects : the deceptive simplicity of statement; the complication of their proofs (which even Khinchin's lucid exposition cannot entirely hide); and the fact that each one was first proved by a young man on the threshold of his career, after defeating the efforts of learned mathematical scholars.

The book is thus not merely a lifeless exposition of abstractions, but a human document, full of challenge and stimulation. It is excellent reading for the

https://doi.org/10.1017/S0013091500021684 Published online by Cambridge University Press