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ARTICLE

# Relative consumption, relative wealth, and long-run growth: when and why is the standard analysis prone to incorrect conclusions?

Franz X. Hof[1] and Klaus Prettner[2,3,*]

[1]Institute of Statistics and Mathematical Methods in Economics, Research Unit Economics (105-3), Vienna University of Technology, Wiedner Hauptstr. 8–10, 1040 Vienna, Austria, [2]Department of Economics, Vienna University of Economics and Business, Welthandelsplatz 1, 1020 Vienna, Austria and [3]Wittgenstein Centre for Demography and Global Human Capital (University of Vienna, IIASA, VID/ÖAW), Vordere Zollamtsstraße 3, 1030 Vienna, Austria
*Corresponding author. Email: klaus.prettner@wu.ac.at.

**Abstract**

We employ a novel approach for analyzing the effects of relative consumption and relative wealth preferences on economic growth. In the pertinent literature, these effects are usually assessed by examining the dependence of the growth rate on the two parameters of the utility function that *seem* to measure the strength of the relative consumption and the relative wealth motives. Applying our fundamental factor approach, we identify specifications in which the traditional approach yields incorrect qualitative conclusions. The problematic specifications have the common unpleasant property that the parameter that seems to determine the strength of the relative consumption motive actually also affects the elasticity of intertemporal substitution of absolute consumption (and the strength of the relative wealth motive). Since the standard approach is unaware of the additional effect(s), it attributes the total change in the growth rate incorrectly to the change in the strength of the relative consumption motive.

**Keywords:** Quest for status, Economic growth, Social optimality, Deep factors

**JEL Classifications:** D31, D62, O10, O30

## 1. Introduction

We propose a novel approach to reexamine the implications of both relative consumption and relative wealth preferences within the context of an *AK* model of endogenous growth with homogeneous agents.[1] In the pertinent literature, it is common practice to analyze the implications of such preferences in the following way: First, a functional form of the instantaneous utility function is chosen that (i) allows for the existence of a balanced growth path (BGP) and (ii) contains as few parameters as possible for mathematical convenience. Second, the effects of relative consumption and relative wealth preferences are assessed by analyzing the dependence of the BGP growth rate on the two parameters of the instantaneous utility function that *seem* to be the appropriate measures of the strength of the relative consumption and the relative wealth motive. The aim of this paper is to identify well-known and widely used specifications of the instantaneous utility function in which this standard method of analysis yields incorrect conclusions and, in addition, to give extensive mathematical and economic explanations for the misleading inferences. In doing so, we focus on the case of exogenous labor supply because (i) this allows us to obtain explicit analytical solutions and (ii) the widely used specifications of the standard literature that we address usually abstract from endogenous labor supply.

Applying the standard method of analysis, Tournemaine and Tsoukis (2008) obtain the following results for the case of exogenous labor supply:

> (a) In the absence of consumption-related status ..., wealth-related status increases growth. (b) In the absence of wealth-related status ..., consumption-related status does not affect growth. (c) If both motives coexist, consumption-related status unambiguously harms growth. (Proposition 1, p. 315)

Although the instantaneous utility function employed by Tournemaine and Tsoukis (2008) is rather general, it does not encompass one of the most prominent specifications of status preferences. This widely used instantaneous utility function is obtained by applying an isoelastic (CRRA-type) transformation to a *geometric-weighted average* of absolute consumption and relative consumption, while relative wealth is irrelevant for utility. With this specification, the derivative of the BGP growth rate with respect to the parameter that represents the weight of relative consumption may be of either sign. In our paper, we show that this property of the BGP growth rate must not be used to call into question the robustness of part (b) of the assertion made by Tournemaine and Tsoukis (2008). We explain in detail why the prominent geometric-weighted average specification of pure relative consumption preferences is one of the cases in which the standard approach is prone to incorrect conclusions. The extension of this specification to the general case in which also relative wealth matters yields additional properties of the BGP growth rate that seem to challenge the robustness of further assertions made by Tournemaine and Tsoukis (2008) in Proposition 1. However, we show that also in the general case, the application of the standard approach gives rise to inferences that are clearly incorrect, and hence do not provide valid contradictions to Tournemaine and Tsoukis (2008).

To obtain robust results with respect to the effects of relative consumption and relative wealth preferences and to give both mathematical and economic explanations for the potential fallacies of the standard approach, we extend the framework employed by Tournemaine and Tsoukis (2008) in the following respects: (1) We impose weaker restrictions on the specification of the instantaneous utility function. Hence, our approach encompasses also utility functions widely used in the literature that are not covered by the specification employed by Tournemaine and Tsoukis (2008). (2) We go beyond the consideration of parameters by putting special emphasis on the identification of the *fundamental factors* that ultimately determine both the decentralized and the socially optimal long-run growth rates. While these fundamental factors always have an unambiguous economic meaning, this is, in general, not true for parameters. (3) We offer an alternative representation of the equations that govern the dynamic behavior of the model and, in addition, provide alternative economic interpretations. This alternative presentation allows for the identification of the various channels through which the fundamental factors work. The resulting insights also serve as the basis for the explanation of the pitfalls of the traditional approach with respect to the effects of relative consumption and relative wealth preferences on the decentralized solution and its welfare properties. (4) While the note of Tournemaine and Tsoukis (2008) could only touch on the welfare properties of the decentralized solution, we offer a thorough analysis.

In the following, we sketch the details of our approach. The fundamental factors are connected to technology and preferences. In our analysis, we focus on the three fundamental factors that are linked to the specification of the instantaneous utility function because they are appropriate measures of the household's willingness to substitute (i) relative consumption for absolute consumption, (ii) relative wealth for absolute consumption, and (iii) future absolute consumption for current absolute consumption. In our approach, it becomes possible to analyze the effects of *ceteris paribus changes* in (i) the strength of the relative consumption motive, (ii) the strength of the relative wealth motive, and (iii) the elasticity of intertemporal substitution of absolute consumption by considering a change in the corresponding fundamental factor holding all else equal. In the standard approach, however, such thought experiments cannot be carried out if specifications of utility functions are used in which the crucial parameters affect more than one out of these three fundamental factors. In such instances, the standard approach is prone to incorrect conclusions. This problem is most easily illustrated by means of the prominent geometric-weighted average specification of pure relative consumption preferences mentioned above. The standard approach assesses the implications of the strength of the relative consumption motive by calculating the decentralized BGP growth rate and analyzing its dependence on the weight of relative consumption. Obviously, since the sum of the weights is equal to unity by definition, any rise in the weight of relative consumption is inevitably associated with a fall in the weight of absolute consumption. The standard approach fails to notice that the latter reaction exerts an ambiguous effect on the elasticity of intertemporal substitution of absolute consumption (provided that the parameter of the CRRA-type transformation is unequal to unity). In other words, it is unaware of the fact that changes in the weight of relative consumption affect the BGP growth rate not only *directly* via the change in the strength of the relative consumption motive, but also *indirectly* via a change in the willingness to substitute absolute consumption intertemporally. The punch line of our fundamental factor approach is that the direct effect is zero, that is, due to the absence of the relative wealth motive, the BGP growth rate is independent of the strength of the relative consumption motive. Instead, the unintended indirect effect explains 100% of the reaction of the BGP growth rate. Thus, the traditional approach misinterprets the effect on the growth rate as the result of a change in the strength of the relative consumption motive. Things become even more difficult if the geometric-weighted average specification involves both relative consumption and relative wealth. In this case, changes in the weight of relative consumption affect all three fundamental factors, in particular, also the strength of the relative wealth motive. Consequently, there are two unintended indirect effects that are unnoticed by the traditional approach.

Our analysis draws heavily on the Euler equation that governs the dynamic evolution of aggregate consumption in the presence of relative consumption and relative wealth preferences. The willingness to save and the corresponding common BGP growth rate depend positively on both the *effective* elasticity of intertemporal substitution and the *effective* rate of return. The latter is defined as the sum of the market rate and an extra return that results from social comparisons based on both relative wealth and relative consumption. The weak restrictions on the utility function that are sufficient for the existence of a BGP imply that both relative consumption and relative wealth preferences affect the common growth rate exclusively via the channel of the comparison-induced extra return (CIER). By contrast, the erroneous conclusions of the standard approach are mainly due to unintended and unnoticed side effects of changes in parameters that work via the channel of the *effective* elasticity of intertemporal substitution.

The paper is organized as follows. In Section 2, we describe the assumptions of the model and study the optimal behavior of households and firms. In Section 3, we consider the macroeconomic equilibrium of the decentralized economy. We derive conditions for the existence of a BGP and analyze the long-run effects of relative consumption and relative wealth preferences by means of the corresponding fundamental factors. In Section 4, we discuss widely used specifications of the instantaneous utility function and identify those in which the ignorance of the fundamental factors is most likely to lead to incorrect conclusions. In Section 5, we analyze the socially optimal solution and illustrate the pitfalls of ignoring the fundamental factors. In addition, we study the gap between the decentralized BGP and its socially optimal counterpart. Finally, in Section 6, we conclude and outline the scope of further research.

## 2. The model

### 2.1 Households

Consider a continuum of infinitely lived identical households with mass 1. The flow budget constraint of the representative household is given by

$$\dot{a} = ra + wl - c, \tag{1}$$

where $a$ refers to net assets, $r$ denotes the real interest rate, which is equal to the real rental rate of physical capital because we abstract from depreciation, $w$ is the real wage, $l$ refers to hours worked, and $c$ denotes consumption.

Instantaneous utility depends not only on absolute consumption $c$, but also on relative consumption $c/C$ and/or on relative wealth $a/A$, where $C$ denotes average consumption, while $A$ is average wealth. We restrict our attention to the case in which labor supply is exogenously given so that the appropriate general specification of the instantaneous utility function takes the form $u = u(c, c/C, a/A)$. We allow for the possibility that either relative consumption or relative wealth is irrelevant for utility and assume that the representative household derives positive and diminishing marginal utility from all arguments that matter:

$$\begin{aligned} &u_c > 0, \quad u_{cc} < 0; \qquad u_{c/C} \geq 0, \quad u_{a/A} \geq 0, \quad u_{c/C} > 0 \vee u_{a/A} > 0; \\ &\text{if } u_{c/C} > 0, \text{ then } u_{(c/C)(c/C)} < 0; \qquad \text{if } u_{a/A} > 0, \text{ then } u_{(a/A)(a/A)} < 0. \end{aligned} \tag{2}$$

Below some derivations and interpretations exploit the fact that instantaneous utility can be also expressed as a function of own consumption, average consumption, own wealth, and average wealth: $V = V(c, C, a, A) \equiv u(c, c/C, a/A)$. To ensure a well-behaved intertemporal optimization problem, we assume that $V(c, C, a, A)$ is strictly concave in $c$ and jointly strictly concave in $c$ and $a$ if relative wealth matters for utility:

$$V_{cc} < 0; \qquad \text{if } u_{a/A} > 0, \text{ then } V_{cc}V_{aa} - (V_{ca})^2 > 0. \tag{3}$$

The expressions for $V_{cc}$ and $V_{cc}V_{aa} - (V_{ca})^2$ that are given in Online Appendix A.1 also involve the mixed partial derivatives $u_{c(c/C)}$, $u_{c(a/A)}$, and $u_{(c/C)(a/A)}$ that are not included in (2).

The representative household maximizes overall utility as given by $\int_0^\infty e^{-\rho t} u(c, c/C, a/A)dt$, where $\rho > 0$ denotes the subjective discount rate, subject to the flow budget constraint (1) and the initial condition $a(0) = a_0$ by choosing the time path of own consumption $c$. In doing so, the representative household takes not only the time paths of the real wage $w$ and the real interest rate $r$, but also the time paths of average consumption $C$ and average wealth $A$ as given. The current-value Hamiltonian of the optimization problem is given by $H = u(c, c/C, a/A) + \lambda(ra + wl - c)$, where the costate variable $\lambda$ denotes the shadow price of absolute wealth. The necessary optimality conditions for an interior equilibrium, $H_c = 0$ and $\dot{\lambda} = \rho\lambda - H_a$, can be written as

$$\lambda = u_c(c, c/C, a/A) + u_{c/C}(c, c/C, a/A)C^{-1}, \tag{4}$$

$$\dot{\lambda} = -[r\lambda + u_{a/A}(c, c/C, a/A)A^{-1} - \rho\lambda]. \tag{5}$$

The assumptions given in (3) ensure that, if the transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\lambda a = 0 \tag{6}$$

is satisfied, then the necessary optimality conditions (4) and (5) are also sufficient. From the first-order conditions (FOCs), the growth rate of the shadow price of absolute wealth follows as

$$\dot{\lambda}/\lambda = -\left[r + \frac{u_{a/A}(c, c/C, a/A)A^{-1}}{u_c(c, c/C, a/A) + u_{c/C}(c, c/C, a/A)C^{-1}} - \rho\right]. \tag{7}$$

For the following economic interpretation of the FOCs, it is helpful to make use of the fact that equivalent representations of (4) and (7) are given by

$$\lambda = V_c(c, C, a, A), \qquad \dot{\lambda}/\lambda = -\{r + [V_a(c, C, a, A)/V_c(c, C, a, A)] - \rho\}. \tag{8}$$

First, the introduction of relative consumption preferences implies that the well-known standard FOC for the optimal choice of consumption, $\lambda = u_c$, is replaced by $\lambda = u_c + u_{c/C}C^{-1} = V_c$,

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where $V_c$ is the *total* marginal utility of own consumption. The interpretation of the additional term $u_{c/C}C^{-1}$ is straightforward: Since the household takes average consumption $C$ as given, a variation in own consumption $c$ leads to a change in relative consumption by $d(c/C) = C^{-1}dc$. The resulting change in utility equals $u_{c/C}C^{-1}dc$.

Second, according to the rule of optimal saving given by (7) and (8), the *effective* rate of return to saving $r^e$ is given by the sum of the market rate of return $r$ and the term

$$u_{a/A}A^{-1}/(u_c + u_{c/C}C^{-1}) = V_a/V_c = \left| (dc/da)\big|_{dV=0,dC=dA=0} \right| \tag{9}$$

that measures the extra return resulting from social comparisons based on relative wealth and relative consumption (henceforth, *comparison-induced extra return*, *CIER*).[2] From (9), it is obvious that the CIER is the marginal rate of substitution (MRS) of own wealth $a$ for own consumption $c$ for given values of $C$ and $A$. The equivalence of the two alternative representations of the MRS is easily verified: Since the household takes average wealth $A$ as given, a variation in own wealth $a$ leads to a change in relative wealth by $d(a/A) = A^{-1}da$. Hence, the marginal utility of own wealth is $V_a = u_{a/A}A^{-1}$. The interpretation of the total marginal utility of own consumption $V_c = u_c + u_{c/C}C^{-1}$ is known from above. Obviously, in the absence of relative wealth preferences the CIER is identical to zero. If relative wealth matters for utility, the MRS of $a$ for $c$ given by (9) is used to express the comparison-induced increase in future instantaneous utility, $V_a = u_{a/A}A^{-1}$, which results from an increase in future own wealth $a$ by a marginal unit, in terms of the future consumption equivalent given by $dc = V_a/V_c$. From $V_c = u_c + u_{c/C}C^{-1}$, it follows that the presence of relative consumption preferences ($u_{c/C} > 0$) exerts a negative effect on the value of this future consumption equivalent, and hence on the CIER. The consumer is willing to forgo a smaller amount of own consumption $c$ in exchange for an additional unit of own wealth $a$ because the fall in $c$ now decreases utility not only via the decrease in absolute consumption but also via the fall in relative consumption.

It is essential for the rest of the paper to represent the CIER given in (9) as a product of the consumption–wealth ratio $c/a$ and an expression that we denote as CIER factor so that the effective rate of return can be written as

$$r^e = r + \ [m^{a/A}/(1 + m^{c/C})] \times (c/a) = r + [(a/c) \times (V_a/V_c)] \times (c/a), \tag{10}$$

$$\text{where} \quad m^x = m^x(c, c/C, a/A) \equiv (x/c) \times [u_x(c, c/C, a/A)/u_c(c, c/C, a/A)], \tag{11}$$

for $x = c/C$ and $a/A$. While $u_x/u_c$ is the *standard* MRS of $x$ for absolute consumption $c$, where $x$ is either relative consumption or relative wealth, $m^x \geq 0$ is the corresponding *percentage* MRS of $x$ for $c$. More precisely, $m^{c/C}$ refers to the percent of absolute consumption $c$ the consumer would be willing to forgo to raise relative consumption $c/C$ by one percent for a given value of relative wealth $a/A$. The term $m^{a/A}$ refers to the percent of absolute consumption $c$ the household would be willing to sacrifice to raise relative wealth $a/A$ by one percent for a given value of relative consumption $c/C$. Analogously, $(a/c) \times (V_a/V_c)$ is the percentage MRS of $a$ for $c$. The assumptions that we introduce below to ensure the existence of a BGP imply that—in contrast to $u_{c/C}/u_c$ and $u_{a/A}/u_c$—both $m^{c/C}$ and $m^{a/A}$ are constant along the BGP. Due to this pleasant property, we henceforth employ the percentage MRS $m^{c/C}$ and $m^{a/A}$ instead of their standard counterparts as measures of the strength/intensity of the relative consumption motive and the relative wealth motive, respectively. In the rest of the paper, the CIER factor $m^{a/A}/(1 + m^{c/C})$ plays a crucial role. Using $m^{c/C}$ we can express the FOC (4) in a more instructive way:

$$\lambda = u_c(c, c/C, a/A)[1 + m^{c/C}(c, c/C, a/A)]. \tag{12}$$

This equation shows that the total marginal utility of own consumption, $V_c = u_c + C^{-1}u_{c/C}$, can be represented as the product of the marginal utility of absolute consumption $u_c$ and the factor $(1 + m^{c/C})$.

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Finally, the introduction of relative wealth preferences implies that in the common representation of the transversality condition, the market rate of return $r$ is replaced by the effective rate of return $r^e$. Integrating (7) and using the representation of $r^e$ given in (10) it can be shown that the transversality condition (6) is equivalent to

$$\lim_{t\to\infty} \exp\left[-\int_0^t \left(r(v)+\{m^{a/A}(v)/[1+m^{c/C}(v)]\}\times[c(v)/a(v)]\right)dv\right]a(t)=0, \tag{13}$$

where $m^x(v)=m^x(c(v), c(v)/C(v), a(v)/A(v))$ for $x=c/C$ and $a/A$. Hence, if relative wealth matters for utility ($m^{a/A}>0$), the CIER is strictly positive so that $r^e$ exceeds $r$. This situation allows for the possibility that the rate of growth of own wealth $\dot{a}/a$ is excessive in the sense that it violates the transversality condition of the standard model. However, in the absence of relative wealth preferences ($m^{a/A}=0$), the CIER is identical to zero. In this case, $r^e=r$ holds so that the transversality condition equals its counterpart of the standard model and, consequently, excessive saving is ruled out.

### 2.2 Production

There is a continuum of firms with mass 1. To allow for endogenous growth in the simplest way possible, we follow Barro and Sala-i-Martin (1995, Subsection 4.3), which is inspired by Arrow (1962) and Romer (1986). The production function of the representative firm is given by $y=f(k, Bl)$, where $y$ is output, $k$ refers to input of physical capital, $l$ denotes labor input, $B$ is an index of knowledge available to the firm, and $Bl$ denotes effective labor input. The assumptions made by Barro and Sala-i-Martin (1995) allow them to set $B=K$, where $K$ is the aggregate capital stock of the economy. This simple production structure leads to a model that is—with respect to all aspects that are relevant for our analysis—isomorphic to more sophisticated models in which long-run economic growth is endogenously explained by purposeful R&D investments (see, e.g. Romer (1990)).[3]

The production function has the standard neoclassical properties of (i) positive and diminishing marginal products with respect to each input, (ii) constant returns to scale, and (iii) the fulfillment of the Inada conditions. There is perfect competition in all markets and the representative firm maximizes its profit by optimally choosing capital input $k$ and labor input $l$, with the services of these two production factors being rented from households. Since there is a continuum of firms, the representative firm takes not only the rental rate of capital $r$ and the real wage $w$, but also the available stock of knowledge $B(=K)$ as given. The corresponding first order conditions for a profit maximum can be written as

$$r=f_k(k, Kl), \qquad w=f_{(Bl)}(k, Kl)K, \tag{14}$$

where $f_k$ and $f_{(Bl)}$ denote the marginal products of $k$ and effective labor input $Bl$, respectively. Thus, $f_{(Bl)}\times B$ gives the marginal product of $l$. The conditions given by (14) require that each input is utilized up to the point at which its marginal product equals its real price.

## 3. The decentralized solution—general results of the fundamental factor approach

### 3.1 General features of the symmetric macroeconomic equilibrium

We follow the status literature with homogeneous individuals in which it is common practice to proceed with the analysis of the symmetric macroeconomic equilibrium.[4] Since both the mass of households and the mass of firms are normalized to unity, aggregate values of output, capital, labor, consumption, and wealth equal the corresponding average values. Hence, aggregate and average values can be used interchangeably. Their common notation is given by $Y$, $K$, $L$, $C$, and $A$. In a macroeconomic equilibrium in which all markets clear, the rental rate (= real interest rate)

$r$ and the real wage $w$ are endogenously determined by the following equations (for a detailed proof, see Online Appendix B.1):

$$r = f_k(1, L), \qquad w = f_{(Bl)}(1, L)K. \tag{15}$$

Please note that $L$ is treated as given, since we restrict our attention to the case in which labor supply is exogenous. By assumption, private households are identical in every respect. Hence, in any symmetric macroeconomic equilibrium, they make identical choices. Net loans of any household to other households and to firms are zero so that physical capital is the only store of households' wealth. Consequently, we have

$$c = C, \qquad a = A = K, \qquad l = L. \tag{16}$$

The equations that govern the dynamic evolution of aggregate consumption $C$ and aggregate capital $K$ are obtained by substituting (15) and (16) into the FOCs and the transversality condition and eliminating the shadow price of wealth $\lambda$ by appropriate substitution (for a detailed proof, see the Online Appendix B.2).

First, the Euler equation for aggregate consumption can be written as

$$\dot{C}/C = \sigma^D(C)[f_k(1, L) + \eta^D(C) \times (C/K) - \rho], \qquad \text{where} \tag{17}$$

$$\sigma^D(C) \equiv -\left[\varepsilon^{u_c,c}(C, 1, 1) + \frac{m^{c/C}(C, 1, 1)}{1 + m^{c/C}(C, 1, 1)} \times \varepsilon^{m^{c/C},c}(C, 1, 1)\right]^{-1}, \tag{18}$$

$$\eta^D(C) \equiv m^{a/A}(C, 1, 1)/[1 + m^{c/C}(C, 1, 1)] \tag{19}$$

denote the *effective* elasticity of intertemporal substitution of own consumption and the CIER factor in symmetric situations in which $(c/C) = (a/A) = 1$ holds. The superscript "$D$" stands for "Decentralized". Throughout the paper, we use $\varepsilon^{z,x_i} \equiv (\partial z/\partial x_i) \times (x_i/z)$ to denote the elasticity of $z$ with respect to $x_i$, where $z = z(x_1, \ldots, x_n)$ is an arbitrary function of arbitrary variables $x_i$, $i = 1, \ldots, n$. Hence, in (18), $\varepsilon^{u_c,c} \equiv u_{cc} \times (c/u_c)$ and $\varepsilon^{m^{c/C},c} \equiv m_c^{c/C} \times (c/m^{c/C})$ are the elasticities of the marginal utility of absolute consumption $u_c$ and of the percentage MRS $m^{c/C}$ (with respect to absolute consumption $c$).[5] Here, both elasticities are evaluated at $(c, c/C, a/A) = (C, 1, 1)$. The expressions $m^{c/C}(C, 1, 1)$ and $m^{a/A}(C, 1, 1)$ measure the strength of the relative consumption and the relative wealth motives in symmetric situations.

For the economic interpretation of the *effective* elasticity of intertemporal substitution $\sigma^D(C)$, it is essential to note that in a symmetric equilibrium, the FOC with respect to the optimal choice of own consumption (12) simplifies to $\lambda = u_c(C, 1, 1)[1 + m^{c/C}(C, 1, 1)]$. The right-hand side gives the total marginal utility of own consumption $V_c = u_c + C^{-1}u_{c/C}$ in symmetric situations. It is easily verified that the elasticity of $u_c(C, 1, 1)[1 + m^{c/C}(C, 1, 1)]$ with respect to $C$ equals the expression within brackets in (18). Consequently, $\sigma^D$ is the reciprocal of the magnitude of the elasticity of the total marginal utility of own consumption, $\sigma^D(C) = [-\varepsilon^{\{u_c(C,1,1)\times[1+m^{c/C}(C,1,1)]\},C}]^{-1}$. While the strength of the relative consumption motive obviously exerts a positive effect on the total marginal utility of own consumption, it does not necessarily affect its elasticity for the following reason. All specifications of $u = u(c, c/C, a/A)$ in which $m^{c/C}(C, 1, 1)$ is a constant function of $C$ have the property that the total marginal utility of own consumption is given by $u_c(C, 1, 1) \times (1 + \hat{m}^{c/C})$, where $\hat{m}^{c/C}$ denotes a constant. This feature—that below plays a crucial role with respect to the existence of a BGP—implies that the second term within brackets in (18) vanishes so that $\sigma^D$ simplifies to the elasticity of intertemporal substitution of *absolute* consumption, $\sigma^D(C) = -1/\varepsilon^{u_c,c}(C, 1, 1)$.

In (17), the expression $f_k(1, L) + \eta^D(C) \times (C/K)$ gives the effective rate of return $r^e$. The CIER factor $\eta^D(C)$ defined by (19) captures the direct effects of both relative consumption and relative wealth preferences on the CIER given by $\eta^D(C) \times (C/K)$. Below, we show that these preferences also exert an indirect effect by influencing the equilibrium level of the consumption–capital ratio

$C/K$. Please note that there are no effects on the market rate of return $f_k(1, L) = r$, since we restrict our attention to the case in which labor supply $L$ is exogenous.

Second, the aggregate resource constraint is given by $C + \dot{K} = Y = f(1, L)K$. Hence, the dynamic evolution of aggregate capital $K$ is governed by the differential equation

$$\dot{K}/K = f(1, L) - C/K. \tag{20}$$

Third, in a symmetric macroeconomic equilibrium, the transversality condition (13) becomes

$$\lim_{t\to\infty} \exp\left(-\int_0^t \{f_k(1, L) + \eta^D(C(v)) \times [C(v)/K(v)]\}dv\right) K(t) = 0, \tag{21}$$

where the expression within curly brackets gives the effective rate of return $r^e$. Finally, the initial condition is given by $K(0) = K_0$.

The differential equations (17) and (20) contain the terms $f(1, L)$ and $f_k(1, L)$. For various results derived in the rest of the paper, it is of crucial importance that

$$f(1, L) > f_k(1, L). \tag{22}$$

For given employment $L$, $f_k(1, L)$ gives the *constant* value of the private marginal product of capital in the decentralized equilibrium. In the decentralized economy, the expression $f(1, L) = Y/K$ has a single meaning: it describes the *constant* average product of capital (the ratio of aggregate production $Y$ to aggregate capital $K$). In the socially planned economy discussed in Section 5, $f(1, L)$ also represents the social marginal product of capital, that is, the marginal product as perceived by the social planner that internalizes the knowledge spillovers resulting from the capital accumulation of individual firms.

### *3.2 Balanced growth path (BGP)—existence and properties*

Next, we provide sufficient conditions for the existence of an economically meaningful BGP in the decentralized economy and analyze its properties. We use the term “economically meaningful BGP” to describe a BGP in which (1) the growth rate is strictly positive, (2) the consumption–capital ratio is strictly positive, and (3) the transversality condition is satisfied.

**Proposition 1** (Conditions for the existence of an economically meaningful decentralized BGP). *If (i) the specification of the instantaneous utility function $u = u(c, c/C, a/A)$ has the property that in symmetric situations both the effective elasticity of intertemporal substitution and the CIER factor are independent of $C$ such that*

$$\sigma^D(C) = \hat{\sigma}, \qquad \eta^D(C) = \hat{\eta}, \qquad \forall C > 0, \tag{23}$$

*where $\hat{\sigma} > 0$ and $\hat{\eta} \geq 0$ are constants and (ii) the condition*

$$[1 - (1/\hat{\sigma})](1 + \hat{\eta})^{-1}\rho^g < \rho < \rho^g, \qquad \rho^g \equiv f_k(1, L) + \hat{\eta} f(1, L) \tag{24}$$

*is satisfied in addition to $\rho > 0$, then an economically meaningful BGP exists in the decentralized economy. Along the BGP, the common growth rate of consumption and capital $g^D = (\dot{C}/C)^D = (\dot{K}/K)^D$, the consumption–capital ratio $(C/K)^D$, and the saving rate $(\dot{K}/Y)^D$ are given by*

$$g^D = [(1/\hat{\sigma}) + \hat{\eta}]^{-1}[f_k(1, L) - \rho + \hat{\eta} f(1, L)] > 0, \tag{25}$$

$$(C/K)^D = f(1, L) - g^D, \qquad (\dot{K}/Y)^D = g^D/f(1, L). \tag{26}$$

In Online Appendix B.6, we give an extended proof of Proposition 1 in which we also show that the decentralized solution has no transitional dynamics and, in addition, give conditions for the occurrence of excessive wealth accumulation in the sense that the transversality condition of the *standard* model is violated. At this point, we nevertheless sketch the calculation of the results given in (25) and (26) because the resulting insight is very helpful for the interpretation of the properties

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of these solutions. If (23) holds, then the Euler equation of consumption (17) and the differential equation of aggregate capital (20) (that results from the economy-wide resource constraint) imply that, along the BGP, the common growth rate $g^D = (\dot{C}/C)^D = (\dot{K}/K)^D$ and the consumption–capital ratio $(C/K)^D$ satisfy the equations

$$g^D = \hat{\sigma}\,[f_k(1,L) + \hat{\eta} \times (C/K)^D - \rho] \quad \text{and} \quad (C/K)^D = f(1,L) - g^D. \tag{27}$$

The saving rate is given by $(\dot{K}/Y)^D = (\dot{K}/K)^D/(Y/K)^D = g^D/f(1,L)$. Solving the system (27) for $g^D$, we obtain (25). It is obvious from (25) and (26) that $g^D$, $(C/K)^D$, and $(\dot{K}/Y)^D$ are completely determined by the following five mathematical expressions/parameters: $f(1,L)$, $f_k(1,L)$, $\rho$, $\hat{\sigma}$, and $\hat{\eta}$. Hence, the same is true for the CIER $\hat{\eta} \times (C/K)^D$. Since labor supply $L$ is exogenously given by assumption, relative consumption and relative wealth preferences affect the BGP, if at all, only via the $\hat{\sigma}$-channel and/or the $\hat{\eta}$-channel of the Euler equation. For this reason, a thorough understanding of the operation of these two channels is essential.

Before we provide the corresponding analysis in the next proposition, we briefly comment on the conditions given in (24): On the one hand, the upper bound for the subjective discount rate given by $\rho < \rho^g$ implies that the representative household is sufficiently patient so that the common growth rate $g^D$ and the saving rate $(\dot{K}/Y)^D$ are strictly positive. On the other hand, the lower bound $[1 - (1/\hat{\sigma})](1+\hat{\eta})^{-1}\rho^g < \rho$ requires a sufficient degree of impatience to ensure that (i) the consumption–capital ratio $(C/K)^D$ is strictly positive and the saving rate is less than unity, and (ii) the transversality condition is satisfied. Please note that if $\hat{\sigma} < 1$ holds, then the lower bound is negative, and hence is redundant since $\rho > 0$ has to hold by assumption anyway.

**Proposition 2** (The dependence of the decentralized BGP on $\hat{\sigma}$ and $\hat{\eta}$). *The decentralized growth rate $g^D$ given by equation (25) and the corresponding saving rate $(\dot{K}/Y)^D$ depend positively on both the effective elasticity of intertemporal substitution $\hat{\sigma}$ and the CIER factor $\hat{\eta}$, while the opposite results obtain for the consumption–capital ratio $(C/K)^D$:*

$$\partial g^D/\partial\hat{\sigma} > 0, \qquad \partial(\dot{K}/Y)^D/\partial\hat{\sigma} > 0, \qquad \partial(C/K)^D/\partial\hat{\sigma} < 0, \tag{28}$$

$$\partial g^D/\partial\hat{\eta} > 0, \qquad \partial(\dot{K}/Y)^D/\partial\hat{\eta} > 0, \qquad \partial(C/K)^D/\partial\hat{\eta} < 0. \tag{29}$$

*The CIER depends positively on $\hat{\eta}$. If $\hat{\eta} > 0$, then it depends negatively on $\hat{\sigma}$. However, if $\hat{\eta} = 0$, then it is independent of $\hat{\sigma}$ because $\hat{\eta} \times (C/K)^D = 0$ holds for all $\hat{\sigma} > 0$:*

$$\partial[\hat{\eta} \times (C/K)^D]/\partial\hat{\eta} > 0, \qquad sgn(\partial[\hat{\eta} \times (C/K)^D]/\partial\hat{\sigma}) = -sgn(\hat{\eta}). \tag{30}$$

For a proof of the mathematical assertions made in (28)–(30), see Online Appendix B.7. The following economic interpretation is mainly based on the steady-state versions of the Euler equation and the aggregate resource constraint that are both given in (27).

First, we interpret the dependence of the BGP on the effective elasticity of intertemporal substitution $\hat{\sigma}$. According to the Euler equation, a ceteris paribus increase in $\hat{\sigma}$ exerts a direct effect on the common growth rate of consumption and capital $g^D = (\dot{C}/C)^D = (\dot{K}/K)^D$ and, if $\hat{\eta} > 0$ holds, also an indirect effect via the reaction of the consumption–capital ratio $(C/K)^D$. The direct effect results from the fact that a rise in $\hat{\sigma}$ increases the willingness of private households to substitute future absolute consumption for present absolute consumption. In other words, there is an increase in the willingness to save, which, in turn, causes $g^D$ to rise. If $\hat{\eta} > 0$ holds, there is also an indirect effect. Since labor supply $L$ is exogenously given, it follows from the aggregate resource constraint that any rise in the growth rate of capital $(\dot{K}/K)^D$ requires a fall in $(C/K)^D = f(1,L) - (\dot{K}/K)^D$. The decrease in $(C/K)^D$ causes the CIER $\hat{\eta} \times (C/K)^D$, and hence the effective rate of return, $f_k(1,L) + \hat{\eta} \times (C/K)^D$, to fall. The latter effect dampens the incentives to save and thus exerts a negative effect on the accumulation of capital. Since the positive direct effect exceeds the negative indirect effect, the saving rate $(\dot{K}/Y)^D$ and the decentralized growth rate $g^D$ depend positively on $\hat{\sigma}$.

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Second, we explain the dependence of the BGP on the CIER factor $\hat{\eta}$. A ceteris paribus rise in $\hat{\eta}$ exerts a positive direct effect on the CIER. The resulting rise in the effective rate of return, $r^e = f_k(1, L) + \hat{\eta} \times (C/K)^D$, enhances the incentives to save and thus boosts the accumulation of capital and economic growth. There is also an indirect effect, because due to the aggregate resource constraint, the rise in the growth rate of capital $(\dot{K}/K)^D$ is associated with a fall in $(C/K)^D$. However, since the increase in $\hat{\eta}$ is only partially offset by the decrease in $(C/K)^D$, there is a positive net effect on $\hat{\eta} \times (C/K)^D$ and $r^e$. Consequently, the saving rate $(\dot{K}/Y)^D$ and the decentralized growth rate $g^D$ depend positively on $\hat{\eta}$.

In the next two propositions that build on the results given above, we dig deeper by considering explicitly both the strength of the relative consumption motive and the strength of the relative wealth motive. In this context, the definitions of $\sigma^D(C)$ and $\eta^D(C)$ given by (18) and (19) play a crucial role.

**Proposition 3** (Alternative conditions for the existence of a decentralized BGP). *If the instantaneous utility function $u = u(c, c/C, a/A)$ has the property that in symmetric situations $m^{c/C}$, $m^{a/A}$, and $\varepsilon^{u_c,c}$ are constant functions of $C$ so that*

$$m^{c/C}(C, 1, 1) = \hat{m}^{c/C}, \quad m^{a/A}(C, 1, 1) = \hat{m}^{a/A}, \quad \varepsilon^{u_c,c}(C, 1, 1) = \hat{\varepsilon}^{u_c,c}, \quad \forall C > 0, \tag{31}$$

*where $\hat{m}^{c/C} \geq 0$, $\hat{m}^{a/A} \geq 0$ (with $\hat{m}^{c/C} > 0 \vee \hat{m}^{a/A} > 0$), and $\hat{\varepsilon}^{u_c,c} < 0$ are constants, then the two conditions given in (23) [Proposition 1] are satisfied, since*

$$\sigma^D(C) = 1/|\hat{\varepsilon}^{u_c,c}| \equiv \hat{\sigma} > 0, \quad \eta^D(C) = \hat{m}^{a/A}/(1 + \hat{m}^{c/C}) \equiv \hat{\eta} \geq 0, \quad \forall C > 0. \tag{32}$$

*If, in addition, the condition (24) given in Proposition 1 is satisfied for the values of $\hat{\sigma}$ and $\hat{\eta}$ defined by (32), then an economically meaningful decentralized BGP exists. The corresponding BGP growth rate is given by*

$$g^D = \frac{f_k(1, L) - \rho + [\hat{m}^{a/A}/(1 + \hat{m}^{c/C})] \times f(1, L)}{|\hat{\varepsilon}^{u_c,c}| + [\hat{m}^{a/A}/(1 + \hat{m}^{c/C})]}. \tag{33}$$

For a proof of Proposition 3, see Online Appendix B.8. In general, the constant CIER factor $\hat{\eta}$ defined in (32) depends on both $\hat{m}^{c/C}$ and $\hat{m}^{a/A}$, where the latter two constants measure the strength of the relative consumption motive and the relative wealth motive, respectively, in *symmetric situations*.[6] In contrast to $\hat{\eta}$ and $r^e$, the constant effective elasticity of intertemporal substitution $\hat{\sigma}$ depends neither on $\hat{m}^{c/C}$ nor on $\hat{m}^{a/A}$. It equals the elasticity of intertemporal substitution of absolute consumption, $\hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}|$. From the discussion of equation (18), we already know that the independence of $\sigma^D$ on the strength of the relative wealth motive is a general property of the model, while the irrelevance of relative consumption preferences results from an assumption that we made in (31) to ensure the existence of a BGP, namely that $m^{c/C}(C, 1, 1)$ is a constant function of $C$.

Equation (33) plays an essential role in the rest of the paper. It shows that the decentralized growth rate $g^D$ can be *ultimately* represented as a function of $f(1, L)$, $f_k(1, L)$, $\rho$, $\hat{m}^{a/A}$, $\hat{m}^{c/C}$, and $|\hat{\varepsilon}^{u_c,c}|$. Henceforth, these six terms are called the "fundamental factors" of growth in the decentralized economy. Obviously, these fundamental factors are also the *ultimate* determinants of the saving rate $(\dot{K}/Y)^D = g^D/f(1, L)$, the consumption–capital ratio $(C/K)^D = f(1, L) - g^D$, and the resulting CIER $\hat{\eta} \times (C/K)^D$. In the following proposition, we analyze the implications of *ceteris paribus* changes in the three fundamental factors that have a connection with the specification of the instantaneous utility function, namely $\hat{m}^{a/A}$, $\hat{m}^{c/C}$, and $|\hat{\varepsilon}^{u_c,c}|$.

**Proposition 4** (The dependence of the decentralized growth rate $g^D$ on the fundamental factors $\hat{m}^{a/A}$, $\hat{m}^{c/C}$, and $|\hat{\varepsilon}^{u_c,c}|$).

*(i) $g^D$ depends positively on the strength of the relative wealth motive (in symmetric situations) as measured by $\hat{m}^{a/A}$, where a rise in $\hat{m}^{a/A}$ affects $g^D$ exclusively via its positive effect on the*

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*CIER factor $\hat{\eta}$:*

$$\frac{\partial g^D}{\partial \hat{m}^{a/A}} = \frac{\partial g^D}{\partial \hat{\eta}} \times \frac{\partial \hat{\eta}}{\partial \hat{m}^{a/A}} = \frac{\partial g^D}{\partial \hat{\eta}} \times \frac{1}{1+\hat{m}^{c/C}} > 0. \tag{34}$$

(ii) *If relative wealth matters for utility so that $\hat{m}^{a/A} > 0$, then $g^D$ depends negatively on the strength of the relative consumption motive (in symmetric situations) as measured by $\hat{m}^{c/C}$, where an increase in $\hat{m}^{c/C}$ affects $g^D$ exclusively via its negative effect on the CIER factor $\hat{\eta}$. However, if relative wealth is irrelevant for utility so that $\hat{m}^{a/A} = 0$, and hence $\hat{\eta} = 0$ for $\hat{m}^{c/C} \geq 0$, then $g^D$ is independent of $\hat{m}^{c/C}$:*

$$\begin{aligned}\frac{\partial g^D}{\partial \hat{m}^{c/C}} &= \frac{\partial g^D}{\partial \hat{\eta}} \times \frac{\partial \hat{\eta}}{\partial \hat{m}^{c/C}} = -\frac{\partial g^D}{\partial \hat{\eta}} \times \frac{\hat{m}^{a/A}}{(1+\hat{m}^{c/C})^2} \\ &\Rightarrow \frac{\partial g^D}{\partial \hat{m}^{c/C}} \begin{cases} < 0, \text{ for } \hat{m}^{a/A} > 0, \\ = 0, \text{ for } \hat{m}^{a/A} = 0. \end{cases}\end{aligned} \tag{35}$$

(iii) *$g^D$ depends negatively on the magnitude of the elasticity of the marginal utility of absolute consumption $u_c$ (in symmetric situations), $|\hat{\varepsilon}^{u_c,c}|$, where a rise in $|\hat{\varepsilon}^{u_c,c}|$ affects $g^D$ solely via its negative effect on the effective elasticity of intertemporal substitution $\hat{\sigma}$:*

$$\frac{\partial g^D}{\partial |\hat{\varepsilon}^{u_c,c}|} = \frac{\partial g^D}{\partial \hat{\sigma}} \times \frac{\partial \hat{\sigma}}{\partial |\hat{\varepsilon}^{u_c,c}|} = -\frac{\partial g^D}{\partial \hat{\sigma}} \times \frac{1}{|\hat{\varepsilon}^{u_c,c}|^2} < 0. \tag{36}$$

The mathematical results given in (34)–(36) are easily obtained by using the chain rule of differentiation and making use of the fact that $\partial g^D/\partial \hat{\sigma} > 0$ and $\partial g^D/\partial \hat{\eta} > 0$ hold according to Proposition 2. The statements made in Proposition 4 confirm, generalize, and extend the results obtained by Tournemaine and Tsoukis (2008) that are stated above at the beginning of the introduction. With respect to the implications of relative wealth preferences on economic growth, item (i) extends the corresponding statement of Tournemaine and Tsoukis (2008) by claiming that there is a positive effect not only in the absence, but also in the presence of relative consumption preferences. Item (ii) confirms the assertion that relative consumption preferences affect economic growth only in the presence of relative wealth preferences, where the effect is unambiguously negative. Most importantly, our results are not only valid for the particular instantaneous utility function employed by Tournemaine and Tsoukis (2008), but hold for all specifications $u = u(c, c/C, a/A)$ that satisfy the rather weak conditions given in (31) [Proposition 3]. Proposition 4 yields the additional information that both the relative wealth and the relative consumption motive affect economic growth exclusively via the $\hat{\eta}$-channel (for which we gave a detailed economic interpretation above in the context of Proposition 2). Since $\hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}|$, there is no effect via the $\hat{\sigma}$-channel. In the absence of relative wealth preferences, we have $\hat{\eta} = 0$ so that the CIER is identical to zero. Hence, in this situation, there is obviously no pathway by which relative consumption preferences could affect the growth rate. Changes in the fundamental factor $|\hat{\varepsilon}^{u_c,c}|$ that affect the growth rate via the $\hat{\sigma}$-channel are incorporated into the proposition because below they play an essential role for the explanation of the erroneous conclusions of the standard approach.

The crucial property of our Proposition 4 is that it allows studying the effects of *ceteris paribus* changes in the strength of the relative consumption and relative wealth motives. We show below that there are instances in which such ceteris paribus thought experiments cannot be carried out within the standard approach that does not identify the fundamental factors but restricts attention to the parameters of the utility function. Consequently, the standard analysis is prone to incorrect conclusions whenever it employs utility functions in which the change in a single parameter does not constitute a change in a single fundamental factor. The problematic quite standard specifications of the utility function that we give in Section 4 have the common feature that the parameter

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that *seems* to determine the strength of the relative consumption motive *actually* affects not only $\hat{m}^{c/C}$, but also $|\hat{\varepsilon}^{u_c,c}|$ (and $\hat{m}^{a/A}$).

After having presented these four propositions, the crucial question is whether there exist specifications of $u = u(c, c/C, a/A)$ which satisfy the sufficient conditions for the existence of a BGP given by (23) in Proposition 1 and (31) in Proposition 3. The following proposition answers this question in the affirmative.

**Proposition 5** (A general multiplicatively separable specification of $u(c, c/C, a/A)$ that ensures the existence of an economically meaningful BGP). *Let the instantaneous utility function have the form*

$$u(c, c/C, a/A) = (1-\theta)^{-1}\{[c^{\xi_1} Q(c/C, a/A)]^{1-\theta} - 1\}. \tag{37}$$

*By assumption, the parameters $\theta$ and $\xi_1$ satisfy the conditions*

$$\xi_1 > 0, \qquad \theta > 0, \qquad 1 + (\theta - 1)\xi_1 > 0, \tag{38}$$

*and the function $Q(c/C, a/A)$ has the property that*

$$\begin{aligned}
&Q > 0, \quad Q_{c/C} \geq 0, \quad Q_{a/A} \geq 0, \quad Q_{c/C} > 0 \vee Q_{a/A} > 0;\\
&\textit{if } Q_{c/C} > 0, \textit{ then } \varepsilon^{Q_{c/C},c/C} - \theta\varepsilon^{Q,c/C} < 0;\\
&\textit{if } Q_{a/A} > 0, \textit{ then } \varepsilon^{Q_{a/A},a/A} - \theta\varepsilon^{Q,a/A} < 0.
\end{aligned} \tag{39}$$

*(A) The assumptions made in (38) and (39) imply that the specification of $u(c, c/C, a/A)$ given by (37) is well behaved in the sense that all assumptions made in (2) are satisfied.*

*(B) $m^{c/C}(C, 1, 1)$, $m^{a/A}(C, 1, 1)$, and $\varepsilon^{u_c,c}(C, 1, 1)$ are constant functions of $C$, with*

$$\hat{m}^{c/C} = \hat{\varepsilon}^{Q,c/C}/\xi_1 \geq 0, \quad \hat{m}^{a/A} = \hat{\varepsilon}^{Q,a/A}/\xi_1 \geq 0, \quad \hat{\varepsilon}^{u_c,c} = -[1 + (\theta - 1)\xi_1] < 0, \tag{40}$$

*and $\hat{m}^{c/C} > 0 \vee \hat{m}^{a/A} > 0$, where the constants $\hat{\varepsilon}^{Q,c/C}$ and $\hat{\varepsilon}^{Q,a/A}$ denote the values that the elasticities $\varepsilon^{Q,c/C}(c/C, a/A)$ and $\varepsilon^{Q,a/A}(c/C, a/A)$ take in symmetric situations, so that, $\hat{\varepsilon}^{Q,c/C} \equiv \varepsilon^{Q,c/C}(1, 1)$ and $\hat{\varepsilon}^{Q,a/A} \equiv \varepsilon^{Q,a/A}(1, 1)$.*

*Consequently, also $\sigma^D(C)$ and $\eta^D(C)$ are constant functions of $C$, with*

$$\hat{\sigma} = 1/[1 + (\theta - 1)\xi_1] > 0, \quad \hat{\eta} = (\hat{\varepsilon}^{Q,a/A}/\xi_1)/[1 + (\hat{\varepsilon}^{Q,c/C}/\xi_1)] \geq 0. \tag{41}$$

*If the constants $\hat{\sigma}$ and $\hat{\eta}$ defined in (41) satisfy the condition (24) given in Proposition 1, then an economically meaningful decentralized BGP exists. The corresponding constant common growth rate is given by*

$$g^D = \frac{f_k(1, L) - \rho + (\hat{\varepsilon}^{Q,a/A}/\xi_1)[1 + (\hat{\varepsilon}^{Q,c/C}/\xi_1)]^{-1} \times f(1, L)}{1 + (\theta - 1)\xi_1 + (\hat{\varepsilon}^{Q,a/A}/\xi_1)[1 + (\hat{\varepsilon}^{Q,c/C}/\xi_1)]^{-1}} > 0. \tag{42}$$

In Online Appendix B.9, we provide a proof for an extended version of Proposition 5 that also contains the complicated conditions for the well behavedness of $V(c, C, a, A) \equiv u(c, c/C, a/A)$ given by (3). Moreover, we show that $u$ given by (37) is *general* in the sense that it covers *all* specifications of $u$ that result from a transformation of a multiplicatively separable function in the form of $u(c, c/C, a/A) = T[P(c)Q(c/C, a/A)]$.[7] In this paper, we focus our attention on this class of utility functions because it is predominant in the literature.[8] Several prominent specifications are even encompassed by a simplified version of (37) that is obtained by assuming that $Q(c/C, a/A) = (c/C)^{\xi_2}(a/A)^{\xi_3}$. Please note that, in this special case, the elasticities of the function $Q$ are constant functions, that is, $\varepsilon^{Q,c/C}(c/C, a/A) = \xi_2$ and $\varepsilon^{Q,a/A}(c/C, a/A) = \xi_3$ hold over the domain of $Q$.

**Corollary 1** (A simple version of the multiplicatively separable utility function (37)). *Let the instantaneous utility function take the form*

$$u(c, c/C, a/A) = (1-\theta)^{-1}\{[c^{\xi_1}(c/C)^{\xi_2}(a/A)^{\xi_3}]^{1-\theta} - 1\}, \qquad \textit{where} \tag{43}$$

$$\theta > 0, \quad \xi_1 > 0, \xi_2 \geq 0, \xi_3 \geq 0, \xi_2 > 0 \vee \xi_3 > 0, \quad (1-\theta)(\xi_1 + \xi_2 + \xi_3) < 1. \tag{44}$$

*(A) Both the instantaneous utility function $u(c, c/C, a/A)$ given by (43) and the resulting representation of preferences given by $V(c, C, a, A) \equiv u(c, c/C, a/A)$ are well behaved in the sense that all assumptions made in (2) and (3) are satisfied.*

*(B) The expressions for $\hat{m}^{c/C}$, $\hat{m}^{a/A}$, and $\hat{\eta}$ given in Proposition 5 simplify to*

$$\hat{m}^{c/C} = \xi_2/\xi_1, \quad \hat{m}^{a/A} = \xi_3/\xi_1, \quad \hat{\eta} = (\xi_3/\xi_1)/[1 + (\xi_2/\xi_1)] \geq 0, \tag{45}$$

*while the expressions for $\hat{\varepsilon}^{u_c,c}$ and $\hat{\sigma}$ remain unchanged:*

$$\hat{\varepsilon}^{u_c,c} = -[1 + (\theta-1)\xi_1] < 0, \quad \hat{\sigma} = 1/[1 + (\theta-1)\xi_1] > 0. \tag{46}$$

*(C) The expression for the common growth rate given in Proposition 5 simplifies to*

$$g^D = \frac{f_k(1, L) - \rho + (\xi_3/\xi_1)[1 + (\xi_2/\xi_1)]^{-1} \times f(1, L)}{1 + (\theta-1)\xi_1 + (\xi_3/\xi_1)[1 + (\xi_2/\xi_1)]^{-1}}. \tag{47}$$

The equations (45) and (47) given in (B) and (C) are easily obtained by substituting $\hat{\varepsilon}^{Q,c/C} = \xi_2$ and $\hat{\varepsilon}^{Q,a/A} = \xi_3$ into (40)–(42) given in Proposition 5. By contrast, the proof of (A) requires tedious calculations that are provided in Online Appendix B.10. Please note that, under the specification (43), the constants $\hat{m}^{c/C} (= \xi_2/\xi_1)$ and $\hat{m}^{a/A} (= \xi_3/\xi_1)$ measure the strength of the relative consumption motive and the relative wealth motive not only locally in symmetric situations, but *globally*. This follows from the fact that both $m^{c/C}(c, c/C, a/A)$ and $m^{a/A}(c, c/C, a/A)$ are constant functions over their whole domains.

In the next section, we reexamine various specifications of relative consumption and relative wealth preferences that are employed in the literature. The corresponding utility functions can be expressed as special cases of (37) and (43). We identify the specifications in which the ignorance of the fundamental factors is problematic and illustrate the resulting erroneous conclusions within the context of the *AK* growth model. To avoid misunderstandings, it is essential to issue the following warning: Since various papers cited below do not employ a deterministic model with endogenous growth of the *AK*-type, our critical insights cannot be applied directly and without any qualifications to these alternative frameworks. Obviously, a detailed reexamination of these alternative models is beyond the scope of our paper.

## 4. The decentralized solution—utility functions used in the literature and potential fallacies of the standard analysis

### 4.1 Preliminaries

One of the main goals of this section is to show that the traditional method of analysis involves the great risk of drawing incorrect conclusions. For instance, it will become obvious that one of the most prominent specifications of the instantaneous utility function used in the literature seems to allow for the possibility that relative consumption preferences enhance long-run growth. Such a conclusion is clearly at variance with our fundamental factor approach results given in Proposition 4. For the explanation of the traditional method's fallacy, the following properties of (37) [resp. (43)], which follow directly from Proposition 5 and Corollary 1, are essential:

**Corollary 2** (Properties of the multiplicatively separable utility functions (37) and (43)).

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(A) *Ceteris paribus changes in* $\hat{\varepsilon}^{Q,c/C} \equiv \varepsilon^{Q,c/C}(1,1)$ *[resp. the exponent of relative consumption* $\xi_2$*] affect only the percentage MRS of relative consumption* $\hat{m}^{c/C}$.

(B) *Ceteris paribus changes in* $\hat{\varepsilon}^{Q,a/A} \equiv \varepsilon^{Q,a/A}(1,1)$ *[resp. the exponent of relative wealth* $\xi_3$*] exert solely an effect on the percentage MRS of relative wealth* $\hat{m}^{a/A}$.

(C) *Ceteris paribus changes in* $\theta$ *influence exclusively the magnitude of the elasticity of the marginal utility of absolute consumption* $u_c$, $|\hat{\varepsilon}^{u_c,c}|$.

(D) *Ceteris paribus changes in the exponent of absolute consumption,* $\xi_1$*, affect (i)* $\hat{m}^{c/C}$*, provided that* $\hat{\varepsilon}^{Q,c/C} \neq 0$ *[resp.* $\xi_2 \neq 0$*], (ii)* $\hat{m}^{a/A}$*, provided that* $\hat{\varepsilon}^{Q,a/A} \neq 0$ *[resp.* $\xi_3 \neq 0$*] and, (iii)* $|\hat{\varepsilon}^{u_c,c}|$*, provided that* $\theta \neq 1$.

(E) *In the special case in which* $\xi_1 = 1$ *holds, we have*

$$\hat{m}^{c/C} = \hat{\varepsilon}^{Q,c/C} \text{ [resp. } \xi_2], \quad \hat{m}^{a/A} = \hat{\varepsilon}^{Q,a/A} \text{ [resp. } \xi_3], \quad |\hat{\varepsilon}^{u_c,c}| = \theta,$$

*which, in turn, implies that there is a one-to-one correspondence between the three fundamental factors* $\hat{m}^{c/C}$*,* $\hat{m}^{a/A}$*, and* $|\hat{\varepsilon}^{u_c,c}|$ *and the three parameters given by* $\hat{\varepsilon}^{Q,c/C}$*,* $\hat{\varepsilon}^{Q,a/A}$*, and* $\theta$ *[resp.* $\xi_2$*,* $\xi_3$*, and* $\theta$*].*

Item D) implies that ceteris paribus changes in the exponent of absolute consumption $\xi_1$ may alter the willingness to substitute (i) relative consumption for absolute consumption, (ii) relative wealth for absolute consumption, and (iii) absolute consumption intertemporally.

The exponents $\xi_1$, $\xi_2$, and $\xi_3$ that are included in the simple specification of the utility function given by (43) are either parameters or functions of several parameters. In the following, we denote the parameters by $p_1, \ldots, p_n$, where without loss of generality, we choose $p_1 = \theta$. Consequently, the common growth rate $g^D$ depends on these parameters, too: $g^D = g^D(\theta, p_2, \ldots, p_n)$. The traditional approach studies the effects of relative consumption [resp. relative wealth] preferences by examining the dependence of $g^D$ on a parameter $p_i$ [resp. $p_j \neq p_i$] that seems to be the main determinant of the exponent of relative consumption [resp. relative wealth]. As long as $\xi_2$ depends on the single parameter $p_i$, while $\xi_3$ depends only on $p_j$, there is no problem of choosing the appropriate parameters. Corollary 2 implies that this method yields correct results—in spite of its unawareness of the fundamental factors—if the utility function involves three parameters $(\theta, p_2, p_3)$ and the three exponents $\xi_1$, $\xi_2$, and $\xi_3$ have the following properties:

$$\xi_1 = 1, \qquad \xi_2 = \xi_2(p_2), \qquad \xi_3 = \xi_3(p_3). \tag{48}$$

The reason for the correctness of the results is that a ceteris paribus change in the parameter $p_2$ [resp. $p_3$] leads to a ceteris paribus change in the strength of the relative consumption [resp. wealth] motive as measured by the fundamental factor $\hat{m}^{c/C} = \xi_2(p_2)$ [resp. $\hat{m}^{a/A} = \xi_3(p_3)$]. The conditions given in (48) are, for instance, violated if a *dependence* between the three exponents $\xi_1$, $\xi_2$, and $\xi_3$ is introduced, while the number of parameters remains unchanged:

$$\xi_1 = 1 - \xi_2 - \xi_3 \qquad \xi_2 = \xi_2(p_2), \qquad \xi_3 = \xi_3(p_3).$$

These properties imply that $\xi_1 + \xi_2 + \xi_3 = 1$ so that instantaneous utility depends on a geometric-weighted average of absolute consumption, relative consumption, and relative wealth. Below we show that the geometric-weighted average specification plays a prominent role in the literature. Its crucial feature is that any rise in the exponents of either relative consumption, $\xi_2$, or relative wealth, $\xi_3$, is inevitably associated with a fall in the exponent of absolute consumption, $\xi_1 = 1 - \xi_2 - \xi_3$. The resulting property that the exponent of absolute consumption depends on both parameters, that is, $\xi_1 = \xi_1(p_2, p_3)$, implies that it is impossible to analyze ceteris paribus changes in the strength of the relative consumption or the relative wealth motive by considering ceteris paribus variations in the parameters $p_2$ or $p_3$. This problem is obvious from the following mathematical representations:

$$\hat{m}^{c/C} = \frac{\xi_2(p_2)}{\xi_1(p_2, p_3)}, \quad \hat{m}^{a/A} = \frac{\xi_3(p_3)}{\xi_1(p_2, p_3)}, \quad |\hat{\varepsilon}^{u_c,c}| = 1 + (\theta - 1)\xi_1(p_2, p_3). \tag{49}$$

Changes in the parameter $p_2$ affect not only the strength of the relative consumption motive as measured by $\hat{m}^{c/C}$, but also the strength of the relative wealth motive as measured by $\hat{m}^{a/A}$, and the willingness to substitute absolute consumption intertemporally as measured by $1/|\hat{\varepsilon}^{u_c,c}|$ (provided that $\theta \neq 1$). An analogous statement holds for changes in the parameter $p_3$. Since the standard approach does not decompose the total effect of a change in $p_2$ on $g^D$ into the direct effect via the $\hat{m}^{c/C}$-channel and the two indirect effects via the $\hat{m}^{a/A}$- and the $|\hat{\varepsilon}^{u_c,c}|$-channels, it cannot be ruled out that even its *qualitative* conclusion with respect to the implications of relative consumption preferences is incorrect. In the next subsections, we give specifications in which the results are actually qualitatively erroneous. We show that the literature seems to be unaware of the problem mentioned above even in the pure relative consumption case in which $\xi_3 = 0$ and $\xi_1 = 1 - \xi_2$ holds.

For the mathematically minded reader, we finally add the following presentation: From (33) and (49) it follows that

$$\frac{\partial g^D}{\partial p_j} = \frac{\partial g^D}{\partial \hat{m}^{c/C}} \cdot \frac{\partial \hat{m}^{c/C}}{\partial p_j} + \frac{\partial g^D}{\partial \hat{m}^{a/A}} \cdot \frac{d\hat{m}^{a/A}}{\partial p_j} + \frac{\partial g^D}{\partial |\hat{\varepsilon}^{u_c,c}|} \cdot \frac{d|\hat{\varepsilon}^{u_c,c}|}{\partial p_j}, \quad j = 2, 3. \tag{50}$$

Recall that according to our fundamental factor approach [see Proposition 4, equations (34) and (35)], the correct measure to assess the qualitative effect of a ceteris paribus change in the strength of the relative consumption [resp. relative wealth] motive on the decentralized growth rate $g^D$ is given by the sign of $\partial g^D/\partial \hat{m}^{c/C}$ [resp. $\partial g^D/\partial \hat{m}^{a/A}$]. By contrast, the standard approach bases its assessment on the partial derivative $\partial g^D/\partial p_2$ [resp. $\partial g^D/\partial p_3$]. In this context, it is unaware of the right-hand side of (50), which gives the decomposition of the total effect into the direct effect and the two indirect effects.

### *4.2 The pure relative consumption case*

In this subsection, we consider four specifications of pure relative consumption preferences in which relative wealth does not matter at all for utility. We show that two of the four specifications seem to yield contradictory results if the analysis is carried out by means of the standard approach. Throughout this subsection, we assume that the condition (24) is satisfied so that an economically meaningful BGP exists.

Before presenting the four specifications, it is convenient to recall our own results. In the absence of the relative wealth motive, the percentage MRS of relative wealth is identical to zero, $\hat{m}^{a/A} = 0$, so that the CIER factor and the CIER are identical to zero, too. Consequently, according to Proposition 5 and Corollary 1, the common growth rate is given by

$$g^D = \hat{\sigma}[f_k(1, L) - \rho], \qquad \hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}| = 1/[1 + (\theta - 1)\xi_1], \tag{51}$$

irrespective of whether the instantaneous utility function is of the general or the simple type given by (37) or (43). Equation (51) and the following facts are essential for the results and interpretations given in the rest of the subsection:

$$\frac{\partial g^D}{\partial \hat{m}^{c/C}} = 0 \quad \Rightarrow \quad \frac{\partial g^D}{\partial \hat{m}^{c/C}} \cdot \frac{\partial \hat{m}^{c/C}}{\partial p_i} = 0 \quad \Rightarrow \quad \frac{\partial g^D}{\partial p_i} = \frac{\partial g^D}{\partial |\hat{\varepsilon}^{u_c,c}|} \cdot \frac{d|\hat{\varepsilon}^{u_c,c}|}{\partial p_i}, \tag{52}$$

where $p_i$ denotes an arbitrary parameter of the utility function. The interpretation of (52) is straightforward: From Proposition 4, we know that in the absence of the relative wealth motive, the common growth rate $g^D$ is independent of the strength of the relative consumption motive as measured by $\hat{m}^{c/C}$, $\partial g^D/\partial \hat{m}^{c/C} = 0$. Consequently, changes in an arbitrary parameter $p_i$ cannot exert any direct effect on $g^D$ via the $\hat{m}^{c/C}$-channel, $(\partial g^D/\partial \hat{m}^{c/C}) \cdot (\partial \hat{m}^{c/C}/\partial p_i) = 0$. Hence, if there is any effect at all, then it is clearly an indirect effect via the $|\hat{\varepsilon}^{u_c,c}|$-channel. In other words, if relative wealth does not matter for utility, then the variation in a parameter leads to a change in the

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growth rate if and only if it affects the willingness to substitute absolute consumption intertemporally as measured by $1/|\hat{\varepsilon}^{u_c,c}|$. The change in the strength of the relative consumption motive is completely irrelevant for growth.

**Specification #1**: The specification

$$V(c, H) = (1-\theta)^{-1}[(c/H^{\beta})^{1-\theta} - 1], \qquad 0 < \beta < 1, \quad \theta > 0, \tag{53}$$

where $H$ denotes the household's reference level, is widely used in the status and in the habit persistence literature. The simplest version of (53) results from the assumption that the reference level is given by average consumption in the economy, $H = C$. The resulting version of (53), $V(c, C)$, corresponds to the following specification of pure relative consumption preferences:[9]

$$u(c, c/C) = (1-\theta)^{-1}\{[c^{1-\beta}(c/C)^{\beta}]^{1-\theta} - 1\}. \tag{54}$$

According to (54), the representative household derives utility from a geometric-weighted average of absolute and relative consumption. In case of $\beta = 0$, relative consumption is irrelevant so that only absolute consumption matters. The greater the parameter $\beta$, the more important is relative consumption as compared to absolute consumption. Obviously, (54) is a special case of our specification (43) that is obtained by setting $\xi_2 = \beta$, $\xi_1 = 1 - \xi_2 = 1 - \beta$, and $\xi_3 = 0$. Hence, using (45) and (46), we obtain

$$\hat{m}^{c/C} = \beta/(1-\beta), \qquad \hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}| = 1/[1 + (\theta - 1)(1-\beta)]. \tag{55}$$

The resulting decentralized growth rate $g^D = [1 + (\theta-1)(1-\beta)]^{-1}[f_k(1, L) - \rho]$ depends positively (resp. negatively) on $\beta$ if $\theta > 1$ (resp. $\theta < 1$). Employing the standard approach and restricting the analysis to the mere calculation of $\partial g^D/\partial\beta$, one could come to the conclusion that the specification of $u$ given by (54) allows for the possibility that the decentralized BGP growth rate is affected by the strength of the relative consumption motive although relative wealth is irrelevant for utility. Even more surprisingly, one could conclude that relative consumption preferences enhance GDP growth. The fallacy of the traditional approach is obvious from (52) and (55): Due to the dependence $\xi_1 = 1 - \xi_2 = 1 - \beta$, variations in the parameter $\beta$ affect not only $\hat{m}^{c/C}$, but also $|\hat{\varepsilon}^{u_c,c}|$ provided that $\theta \neq 1$. Consequently, if $\theta \neq 1$ holds, then changes in $\beta$ do not represent ceteris paribus changes in the strength of the relative consumption motive. We know from above that the direct effect of a rise in $\beta$ on the common growth rate $g^D$ via the $\hat{m}^{c/C}$-channel is zero due to the absence of the relative wealth motive. In case that $\theta \neq 1$ holds, there is an ambiguous indirect effect via the $|\hat{\varepsilon}^{u_c,c}|$-channel and the resulting reaction of the effective elasticity of intertemporal substitution $\hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}|$, where $\mathrm{sgn}(\partial\hat{\sigma}/\partial\beta) = -\mathrm{sgn}(\partial\hat{\sigma}/\partial\xi_1) = \mathrm{sgn}(\theta - 1)$. Hence, if $\theta > 1$, then the indirect effect of a rise in $\beta$ on $g^D$ (that results exclusively from the fall in $\xi_1 = 1 - \beta$) is positive. The rise in $1/|\hat{\varepsilon}^{u_c,c}|$ causes the willingness of private households to substitute future absolute consumption for present absolute consumption to increase. The resulting rise in the propensity to save stimulates the decentralized BGP growth rate $g^D$. According to our analysis, this indirect effect that is an unintended consequence of the dependence $\xi_1 = 1 - \xi_2$ explains 100% of the reaction of $g^D$. By contrast, the traditional approach erroneously attributes the total reaction of $g^D$ to the change in the strength of the relative consumption motive. In this context, it is unaware of the fact that the reaction of the growth rate (that occurs if $\theta \neq 1$) does not result from the change in the exponent of relative consumption, $\xi_2 = \beta$, but is caused by the associated change in the exponent of absolute consumption, $\xi_1 = 1 - \beta$.

**Specification #2**: The specification of $V(c, C)$ that is employed by Galí (1994) is equivalent to the following representation: $V(c, C) = (1-\theta)^{-1}c^{1-\theta}C^{\gamma\theta}$, where $\theta > 0$, $\theta \neq 1$, and $\gamma < 1$. It corresponds to the following specification of pure relative consumption preferences:

$$u(c, c/C) = (1-\theta)^{-1}\left[c^{[(\theta-1)-\gamma\theta]/(\theta-1)}(c/C)^{\gamma\theta/(\theta-1)}\right]^{1-\theta}. \tag{56}$$

Obviously, (56) is obtained by setting $\xi_2 = \gamma\theta/(\theta - 1)$, $\xi_1 = 1 - \xi_2$, and $\xi_3 = 0$ in (43) and ignoring the constant term "$-1$". Please note that, in contrast to specification #1, the exponent $\xi_2$ now depends on two parameters, namely $\gamma$ and $\theta$. To ensure that $\xi_1 > 0$ and $\xi_2 > 0$ so that $u$ is well behaved, we have to introduce the additional assumption that $0 < \gamma\theta/(\theta - 1) < 1$. According to this restriction, the parameter $\gamma$ may be of either sign, with $\mathrm{sgn}(\gamma) = \mathrm{sgn}(\theta - 1)$. The dependence of $\xi_2$ on $\gamma$ is ambiguous, too: If $\theta > 1$ so that $\gamma > 0$ (resp. if $\theta < 1$ so that $\gamma < 0$), then a rise in $\xi_2$ requires an increase (resp. a decrease) in $\gamma$. Using (45) and (46), we obtain $\hat{m}^{c/C} = -(\gamma\theta)/[1 - (1 - \gamma)\theta]$ and $\hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}| = 1/[(1 - \gamma)\theta]$. Substitution of the result for $\hat{\sigma}$ into (51) yields $g^D = [(1 - \gamma)\theta]^{-1}[f_k(1, L) - \rho]$. The dependence $\xi_1 = 1 - \xi_2$ implies that changes in the parameter $\gamma$ affect both $\hat{m}^{c/C}$ and $|\hat{\varepsilon}^{u_c,c}|$. Hence, for the detailed reasons given above, it is erroneous to conclude from $\partial g^D/\partial\gamma > 0$ that the strength of the relative consumption motive affects BGP growth in spite of the absence of relative wealth preferences.

**Specification #3**: The problems that are inherent in specifications #1 and #2 do not arise if one uses the specification

$$u(c, c/C) = (1 - \theta)^{-1}\{[c(c/C)^{\kappa/(1-\kappa)}]^{1-\theta} - 1\}, \qquad 0 < \kappa < 1, \quad \theta > 0,$$

which is equivalent to the functional form employed in Wendner (2010) [equation (10)]. This specification implies that $\hat{m}^{c/C} = \kappa/(1 - \kappa)$ and $\hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}| = 1/\theta$ [see (45) and (46)]. Since the BGP growth rate $g^D = (1/\theta)[f_k(1, L) - \rho]$ is independent of $\kappa$, the standard approach yields the correct result. The reason for the absence of erroneous conclusions is clear: Setting $\xi_1 = 1$ implies that $\left|\hat{\varepsilon}^{u_c,c}\right| = \theta$. Since $\left|\hat{\varepsilon}^{u_c,c}\right|$ is independent of $\kappa$, the occurrence of an unintended and misleading indirect effect that works via the $\left|\hat{\varepsilon}^{u_c,c}\right|$-channel (by affecting the effective elasticity of intertemporal substitution, $\hat{\sigma} = 1/\left|\hat{\varepsilon}^{u_c,c}\right|$) is ruled out. The direct effect of a rise in $\kappa$ on the growth rate $g^D$ via the $\hat{m}^{c/C}$-channel is zero due to the absence of the relative wealth motive.

**Specification #4**: From the following, it becomes clear that the simple form (43) does not cover all relevant specifications used in the literature. Hence, it is essential that our analysis also includes the general representation given by (37). One of the illustrations employed by Liu and Turnovsky (2005) [see equation (14a), p. 1110] is equivalent to

$$V(c, C) = (1 - \theta)^{-1}\left\{\left[\left(\frac{c^{\varphi} - \kappa C^{\varphi}}{1 - \kappa}\right)^{1/\varphi}\right]^{1-\theta} - 1\right\}, \qquad 0 < \kappa < 1, \quad 0 < 1 - \varphi < \theta. \tag{57}$$

Considering the limiting case in which $\varphi \to 1$ and ignoring the irrelevant expression $1 - \kappa$ in the denominator, we get the specification that would obtain in Ljungqvist and Uhlig (2000) [see p. 357] if—in contrast to the authors' assumption—work effort were not treated as endogenously determined but as exogenously given: $V(c, C) = (1 - \theta)^{-1}[(c - \kappa C)^{1-\theta} - 1]$. In terms of the pure relative consumption approach, the more general Liu and Turnovsky (2005) version (57) corresponds to the following specification of the instantaneous utility function:

$$u(c, c/C) = (1 - \theta)^{-1}\left\{\left[c \times \left(\frac{1 - \kappa(c/C)^{-\varphi}}{1 - \kappa}\right)^{1/\varphi}\right]^{1-\theta} - 1\right\}. \tag{58}$$

Obviously, (58) is obtained by setting $Q = Q(c/C) = \{[1 - \kappa(c/C)^{-\varphi}]/(1 - \kappa)\}^{1/\varphi}$ and $\xi_1 = 1$ in (37). The assumptions $0 < \kappa < 1$ and $0 < 1 - \varphi < \theta$ ensure that both $u = u(c, c/C)$ and $V = V(c, C)$ are well behaved (for a proof, see Online Appendix C.1). In symmetric situations, the elasticity $\varepsilon^{Q,c/C}(c/C) = \kappa(c/C)^{-\varphi}/[1 - \kappa(c/C)^{-\varphi}]$ ceases to depend on the parameter $\varphi$, $\hat{\varepsilon}^{Q,c/C} \equiv \varepsilon^{Q,c/C}(1) = \kappa/(1 - \kappa)$. Taking into account that $\xi_1 = 1$, we obtain $\hat{m}^{c/C} = \hat{\varepsilon}^{Q,c/C}/\xi_1 = \kappa/(1 - \kappa)$ and $\hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}| = 1/\theta$. The common growth rate $g^D = (1/\theta)[f_k(1, L) - \rho]$ depends neither on $\kappa$ nor on $\varphi$. Hence, the standard approach does not run the risk of making incorrect conclusions

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regardless of whether it uses $\partial g^D/\partial\kappa$ or $\partial g^D/\partial\varphi$ to assess the implications of relative consumption preferences in the absence of the relative wealth motive. Just as in specification #3, erroneous conclusions are ruled out by setting $\xi_1 = 1$ so that $|\hat{\varepsilon}^{u_c,c}| = \theta$. Since $|\hat{\varepsilon}^{u_c,c}|$ is independent of both $\kappa$ and $\varphi$, variations in these parameters do not lead to unintended and misleading side effects on the effective elasticity of intertemporal substitution, $\hat{\sigma} = 1/\left|\hat{\varepsilon}^{u_c,c}\right|$.

### 4.3 Specifications in which both relative consumption and relative wealth matter for utility

In this subsection, we consider two specifications in which both relative consumption and relative wealth matter for utility. According to our Proposition 4, the introduction of the relative wealth motive implies that in contrast to the preceding subsection, the decentralized BGP growth rate is no longer independent of the strength of the relative consumption motive, but now depends negatively on it, $\partial g^D/\partial\hat{m}^{c/C} < 0$. Moreover, the strength of the relative wealth motive exerts a positive effect on the growth rate, $\partial g^D/\partial\hat{m}^{a/A} > 0$, irrespective of whether relative consumption matters for utility.

**Specification #5**: Setting

$$\xi_1 = 1, \qquad Q(c/C, a/A) = [\Omega(c/C)]^{\gamma}[\Psi(a/A)]^{\delta},$$

in our general representation of the instantaneous utility function given by (37), where $\Omega(\cdot)$ and $\Psi(\cdot)$ denote functions, we obtain a specification that is equivalent to the utility function used in the seminal paper by Tournemaine and Tsoukis (2008) for the case of exogenous labor supply. Our presentation differs only with respect to the notation. It is easily verified that $\hat{\varepsilon}^{Q,c/C} \equiv \varepsilon^{Q,c/C}(1,1) = \gamma\varepsilon^{\Omega,c/C}(1)$ and $\hat{\varepsilon}^{Q,a/A} \equiv \varepsilon^{Q,a/A}(1,1) = \delta\varepsilon^{\Psi,a/A}(1)$, where $\varepsilon^{\Omega,c/C}(1)$ denotes the elasticity of the function $\Omega$ with respect to $c/C$, evaluated at $c/C = 1$, while $\varepsilon^{\Psi,a/A}(1)$ denotes the elasticity of the function $\Psi$ with respect to $a/A$, evaluated at $a/A = 1$. Substituting these two results and $\xi_1 = 1$ into (40), we obtain

$$\hat{m}^{c/C} = \gamma\varepsilon^{\Omega,c/C}(1), \quad \hat{m}^{a/A} = \delta\varepsilon^{\Psi,a/A}(1), \quad |\hat{\varepsilon}^{u_c,c}| = \theta. \tag{59}$$

Since Tournemaine and Tsoukis (2008) set $\xi_1 = 1$ and, in addition, use the implicit assumption that the two expressions $\gamma\varepsilon^{\Omega,c/C}(1)$ and $\delta\varepsilon^{\Psi,a/A}(1)$ are independent of each other, there is a one-to-one correspondence between the three fundamental factors $|\hat{\varepsilon}^{u_c,c}|$, $\hat{m}^{c/C}$, and $\hat{m}^{a/A}$, and the three expressions $\theta$, $\gamma\varepsilon^{\Omega,c/C}(1)$, and $\delta\varepsilon^{\Psi,a/A}(1)$, where $\gamma$ and $\delta$ are obviously parameters, while $\varepsilon^{\Omega,c/C}(1)$ and $\varepsilon^{\Psi,a/A}(1)$ can be interpreted as parameters, too. Changes in $\gamma\varepsilon^{\Omega,c/C}(1)$ affect only the strength of the relative consumption motive, while changes in $\delta\varepsilon^{\Psi,a/A}(1)$ alter solely the intensity of the relative wealth motive. Hence, it is legitimate to assess the implications of the relative consumption [resp. relative wealth] preferences by applying the standard approach, that is, by analyzing the dependence of $g^D$ on $\gamma\varepsilon^{\Omega,c/C}(1)$ [resp. $\delta\varepsilon^{\Psi,a/A}(1)$], where $g^D$ is obtained by substituting (59) into (42).

While the specification used by Tournemaine and Tsoukis (2008) has the property that the standard procedure yields correct results, it does not encompass specifications in which instantaneous utility depends on a geometric-weighted average of absolute consumption, relative consumption, and relative wealth. Hence, it covers neither the specifications #1 and #2 for the pure relative consumption case nor the following specification for the general case.

**Specification #6**: We assume that

$$u(c, c/C, a/A) = (1-\theta)^{-1}\{[c^{1-\beta_2-\beta_3}(c/C)^{\beta_2}(a/A)^{\beta_3}]^{1-\theta} - 1\}, \tag{60}$$

where the constant parameters $\beta_2$ and $\beta_3$ are strictly positive and satisfy the condition that $\beta_2 + \beta_3 < 1$. This specification has the property that

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$$\hat{m}^{c/C} = \frac{\beta_2}{1-\beta_2-\beta_3}, \quad \hat{m}^{a/A} = \frac{\beta_3}{1-\beta_2-\beta_3}, \quad |\hat{\varepsilon}^{u_c,c}| = 1 + (\theta-1)(1-\beta_2-\beta_3), \tag{61}$$

$$g^D = \frac{f_k(1,L) - \rho + \hat{\eta} f(1,L)}{(1/\hat{\sigma}) + \hat{\eta}} = \frac{f_k(1,L) - \rho + [\beta_3/(1-\beta_3)] f(1,L)}{1 + (\theta-1)(1-\beta_2-\beta_3) + [\beta_3/(1-\beta_3)]}. \tag{62}$$

The standard approach is unaware of both the fundamental factors given in (61) and the first general representation of $g^D$ given in (62) that involves both the effective elasticity of intertemporal substitution $\hat{\sigma} \equiv 1/|\hat{\varepsilon}^{u_c,c}|$ and the CIER factor $\hat{\eta} \equiv \hat{m}^{a/A}/(1+\hat{m}^{c/C})$. Instead, it restricts its attention to the calculation of the growth rate and thereby obtains the second representation of $g^D$ directly. It is easily verified that (i) $\partial g^D/\partial \beta_2 > 0$ holds for $\theta > 1$ and (ii) $\partial g^D/\partial \beta_2 = 0$ for $\theta = 1$. By pointing out these properties, the standard analysis would question the validity of our assertion that in the presence of the relative wealth motive, $g^D$ depends negatively on the strength of the relative consumption motive. The flaw of such a criticism is obvious from (61): Due to the linear dependence of the exponents as given by $\xi_1 = 1 - \xi_2 - \xi_3$, an increase in the parameter $\beta_2$ causes not only $\hat{m}^{c/C}$ but also $\hat{m}^{a/A}$ to rise, and, in addition, affects the third fundamental factor $|\hat{\varepsilon}^{u_c,c}|$ provided that $\theta \neq 1$. Consequently, a ceteris paribus change in $\beta_2$ does not represent a ceteris paribus change in the strength of the relative consumption motive as measured by $\hat{m}^{c/C}$. In the following, we show in detail what a variation in $\beta_2$ really means.

A special property of the specification (60) follows from the fact that $\hat{\eta} = \beta_3/(1-\beta_3)$: Changes in $\beta_2$ do not affect the CIER factor $\hat{\eta} \equiv \hat{m}^{a/A}/(1+\hat{m}^{c/C})$, since the two effects that result from the rise in $\hat{m}^{c/C}$ and $\hat{m}^{a/A}$ offset each other perfectly. Consequently, changes in $\beta_2$ affect the decentralized growth rate $g^D$, if at all, only via the resulting change in the effective elasticity of intertemporal substitution $\hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}|$, where $\text{sgn}(\partial\hat{\sigma}/\partial\beta_2) = -\text{sgn}(\partial\hat{\sigma}/\partial\xi_1) = \text{sgn}(\theta-1)$. Hence, if $\theta > 1$ holds, then the increase in $\beta_2$ causes $\hat{\sigma}$ to rise. From Proposition 2 and its thorough economic interpretation we know that the increase in $\hat{\sigma}$ leads to a rise in $g^D$, since the positive direct effect on the willingness to save more than offsets the negative indirect effect that results from the decrease in $(C/K)^D$. The latter causes the CIER $\hat{\eta} \times (C/K)^D$ to fall, where the CIER factor $\hat{\eta} = \beta_3/(1-\beta_3) > 0$ remains unchanged.[10] Finally, if $\theta = 1$, then changes in $\beta_2$ affect neither $\hat{\eta}$ nor $\hat{\sigma}$ so that $g^D$ is independent of $\beta_2$.

In Online Appendix C.2, we analyze a further problematic geometric-weighted average specification (specification #7) in which the presence of relative consumption and relative wealth in the instantaneous utility function results not only implicitly, but explicitly from status considerations in the sense that $u(c, c/C, a/A) \equiv \tilde{u}(c, s(c/C, a/A))$, where $s$ stands for status.

## 5. The socially planned solution—general results and potential fallacies of the standard analysis

The benevolent social planner internalizes not only the knowledge spillovers in the production sector, but also the externalities that result from relative consumption and relative wealth preferences. Since, by assumption, households are identical in every respect, the social planner will assign identical consumption paths to the individual households so that $c = C$ holds. This, in turn, implies that the resulting time paths of individual wealth are also identical so that $a = A = K$. Using these aspects, the optimization problem of the social planner can be reduced to the following simple problem: Maximize overall utility of the representative household as given by $\int_0^\infty e^{-\rho t} u(C,1,1)dt$, subject to the economy's resource constraint $\dot{K} = f(1,L)K - C$ and the initial condition $K(0) = K_0$ by choosing the time path of aggregate (= average) consumption $C$ optimally. In Online Appendix D.1, we show that the Euler equation of aggregate consumption and the transversality condition are given by

$$\dot{C}/C = \sigma^P(C)[f(1,L) - \rho], \qquad \sigma^P(C) \equiv -1/\varepsilon^{u_c,c}(C,1,1), \tag{63}$$

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$$\lim_{t\to\infty} e^{-f(1,L)t}K(t) = 0, \tag{64}$$

where $\sigma^P(C)$ is the effective elasticity of intertemporal substitution in the socially planned economy (the superscript "$P$" stands for "Planner").

**Proposition 6** (The socially optimal BGP—existence and properties).

*(A) If the instantaneous utility function $u = u(c, c/C, a/A)$ satisfies the conditions given in (31) [Proposition 3] so that $m^{c/C}(C, 1, 1) = \hat{m}^{c/C} \geq 0$, $m^{a/A}(C, 1, 1) = \hat{m}^{a/A} \geq 0$, and $\varepsilon^{u_c,c}(C, 1, 1) = \hat{\varepsilon}^{u_c,c} < 0$ holds $\forall C > 0$, then the effective elasticities of intertemporal substitution in both the socially planned economy and the decentralized economy are constant functions of C, where the corresponding constant levels are identical:*

$$\sigma^P(C) = \sigma^D(C) = 1/|\hat{\varepsilon}^{u_c,c}| \equiv \hat{\sigma} \qquad \forall C > 0. \tag{65}$$

*If the condition*

$$(\hat{\sigma} - 1)\hat{\sigma}^{-1}f(1, L) < \rho < f(1, L) \tag{66}$$

*is also satisfied (in addition to $\rho > 0$), then an economically meaningful BGP exists in the socially planned economy. Along the BGP, the common growth rate of consumption and capital $g^P = (\dot{C}/C)^P = (\dot{K}/K)^P$, the consumption–capital ratio $(C/K)^P$, and the saving rate $(\dot{K}/Y)^P$ are given by*

$$g^P = \hat{\sigma}[f(1, L) - \rho] > 0, \tag{67}$$

*$(C/K)^P = f(1, L) - g^P$, and $(\dot{K}/Y)^P = g^P/f(1, L)$.*

*(B) The socially optimal growth rate $g^P$ has the following properties:*

$$\frac{\partial g^P}{\partial \hat{\sigma}} > 0, \quad \frac{\partial g^P}{\partial \hat{\eta}} = 0, \quad \frac{\partial g^P}{\partial \hat{m}^{c/C}} = 0, \quad \frac{\partial g^P}{\partial \hat{m}^{a/A}} = 0, \quad \frac{\partial g^P}{\partial |\hat{\varepsilon}^{u_c,c}|} < 0. \tag{68}$$

*(i) It depends positively on the elasticity of intertemporal substitution $\hat{\sigma}$, while it is independent of the CIER factor $\hat{\eta} = \hat{m}^{a/A}/(1 + \hat{m}^{c/C})$.*

*(ii) It is independent of the fundamental factors $\hat{m}^{c/C}$ and $\hat{m}^{a/A}$ so that neither relative consumption preferences nor relative wealth preferences matter.*

*(iii) It depends negatively on the fundamental factor $|\hat{\varepsilon}^{u_c,c}|$, where a rise in $|\hat{\varepsilon}^{u_c,c}|$ influences $g^P$ exclusively via its negative effect on $\hat{\sigma} = 1/\,|\hat{\varepsilon}^{u_c,c}|$.*

In Online Appendix D.2, we give an extended proof in which we also show that the socially optimal solution has no transitional dynamics. Please note that irrespective of whether the instantaneous utility function is of the general type (37) or the simple type (43), we have

$$g^P = \hat{\sigma}[f(1, L) - \rho], \qquad \hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}| = 1/[1 + (\theta - 1)\xi_1]. \tag{69}$$

Consequently, a variation in a parameter $p_i$ leads to a change in the socially optimal growth rate $g^P$ if and only if it affects the willingness to substitute absolute consumption intertemporally as measured by $1/|\hat{\varepsilon}^{u_c,c}|$. The standard approach is unaware of (69) and might therefore question the assertions made in B-ii) by, for instance, using the geometric-weighted average specification #6 in which $\xi_2 = \beta_2$, $\xi_3 = \beta_3$, and $\xi_1 = 1 - \beta_2 - \beta_3$ and pointing out that

$$g^P = \frac{f(1, L) - \rho}{1 + (\theta - 1)(1 - \beta_2 - \beta_3)} \Rightarrow \mathrm{sgn}(\partial g^P/\partial \beta_i) = \mathrm{sgn}(\theta - 1), \quad i = 2, 3.$$

Our analysis makes it clear that the (ambiguous) dependence of $g^P$ on $\beta_2$ and $\beta_3$ that exists for $\theta \neq 1$ cannot be used to reject our results. From (61), it is obvious that changes in $\beta_2$ or $\beta_3$ affect all three fundamental factors $\hat{m}^{c/C}$, $\hat{m}^{a/A}$, and $|\hat{\varepsilon}^{u_c,c}|$. The crucial point is that only the change in

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$|\hat{\varepsilon}^{u_c,c}| = 1 + (\theta - 1)\xi_1$ exerts an effect on $g^P$, namely via the (ambiguous) reaction of the effective elasticity of intertemporal substitution $\hat{\sigma} = 1/|\hat{\varepsilon}^{u_c,c}|$. In other words, there are actually changes in the strength of the relative consumption and the relative wealth motive as measured by $\hat{m}^{c/C}$ and $\hat{m}^{a/A}$, but these changes are irrelevant for $g^P$. The (unintended) variation in the willingness to substitute absolute consumption intertemporally that results from the change in the exponent of absolute consumption $\xi_1 = 1 - \beta_2 - \beta_3$ explains 100% of the reaction of $g^P$. Erroneous conclusions of the standard approach are ruled out if a specification of the general or the simple type is used in which $\xi_1 = 1$ holds so that $g^P = (1/\theta)[f(1, L) - \rho]$.

Finally, we compare the decentralized BGP with the socially planned one. The aggregate resource constraints are identical in the two economies. In contrast to the resulting common differential equation for aggregate capital, $\dot{K} = f(1, L)K - C$, the Euler equations for aggregate consumption that are given by

$$\dot{C}/C = \hat{\sigma}[f_k(1, L) + \hat{\eta} \times (C/K) - \rho] \quad \text{and} \quad \dot{C}/C = \hat{\sigma}[f(1, L) - \rho],$$

differ. According to Proposition 6, assumption (31) implies that the effective elasticities of intertemporal substitution of the decentralized and the socially planned economies are identical, $\sigma^P(C) = \sigma^D(C) = \hat{\sigma}$, $\forall C > 0$ [see (65)]. Hence, it is verified at first glance that the decentralized growth rate $g^D$ deviates from its socially optimal counterpart $g^P$ if and only if the decentralized effective rate of return $f_k(1, L) + \hat{\eta} \times (C/K)^D$ deviates from the social marginal product of capital $f(1, L)$. The following property of the growth rate gap $g^P - g^D$ is easily verified. If the CIER factor equals zero, $\hat{\eta} = 0$, then the decentralized effective rate of return simplifies to the private marginal product of capital $f_k(1, L)$. Taking into account that $f_k(1, L) < f(1, L)$ [see (22)] holds because individual firms do not internalize knowledge spillovers, we obtain that $g^D < g^P$ holds for $\hat{\eta} = 0$. Hence, if relative wealth does not matter for utility so that $\hat{m}^{a/A} = 0$ and $\hat{\eta} = \hat{m}^{a/A}/(1 + \hat{m}^{c/C}) = 0$, then the growth rate in the decentralized economy is inefficiently low. Obviously, to obtain a complete picture of the properties of the growth rate gap $g^P - g^D$, we have to dig deeper and offer results that also obtain in the presence of the relative wealth motive, that is, for $\hat{m}^{a/A} > 0$. In the following proposition, we assume that (i) the instantaneous utility function $u$ satisfies the conditions (31) given in Proposition 3 and (ii) the subjective discount rate $\rho$ satisfies the conditions given by (24) and (66) so that in both the decentralized economy and the socially planned economy, an economically meaningful BGP exists.

**Proposition 7** (The dependence of the growth rate gap $g^P - g^D$ on $\hat{m}^{a/A}$ and $\hat{m}^{c/C}$).

*(i)* *$g^P - g^D$ decreases monotonically as $\hat{m}^{a/A}$ increases, $\partial(g^P - g^D)/\partial\hat{m}^{a/A} < 0$.*
*(ii)* *There exists a threshold given by*

$$(\hat{m}^{a/A})^{crit} \equiv \frac{[f(1, L) - f_k(1, L)](1 + \hat{m}^{c/C})}{[1 - (1/|\hat{\varepsilon}^{u_c,c}|)]f(1, L) + (1/|\hat{\varepsilon}^{u_c,c}|)\rho} > 0$$

*such that* $sgn(g^P - g^D) = sgn[(\hat{m}^{a/A})^{crit} - \hat{m}^{a/A}]$. *Hence, if* $0 \leq \hat{m}^{a/A} < (\hat{m}^{a/A})^{crit}$, *then* $g^D < g^P$, *that is, the decentralized growth rate is inefficiently low. If* $\hat{m}^{a/A} = (\hat{m}^{a/A})^{crit}$, *then* $g^D = g^P$. *Finally, if* $\hat{m}^{a/A} > (\hat{m}^{a/A})^{crit}$, *then there is excessive growth in the decentralized economy,* $g^D > g^P$.

*(iii)* *If relative wealth matters for utility so that* $\hat{m}^{a/A} > 0$, *then* $g^P - g^D$ *depends positively on* $\hat{m}^{c/C}$. *However, if* $\hat{m}^{a/A} = 0$, *that is, in the absence of the relative wealth motive,* $g^P - g^D$ *is independent of* $\hat{m}^{c/C}$. *Mathematically, we have* $sgn[\partial(g^P - g^D)/\partial\hat{m}^{c/C}] = sgn(\hat{m}^{a/A})$.

The validity of (i) and (iii) is easily verified by (1) taking into account that, according to Proposition 6 [see (68)], $g^P$ is independent of both $\hat{m}^{c/C}$ and $\hat{m}^{a/A}$ and (2) recalling that $\partial g^D/\partial\hat{m}^{a/A} > 0$ and $\text{sgn}(\partial g^D/\partial\hat{m}^{c/C}) = -\text{sgn}(\hat{m}^{a/A})$ hold according to Proposition 4. A proof of item (ii) is given in Online Appendix D.3.[11]

Since the standard approach makes erroneous conclusions with respect to both $g^D$ and $g^P$ in case that the analysis employs a geometric-weighted average specification of the instantaneous utility function, its analysis of the growth rate gap $g^P - g^D$ is flawed, too. For a simple illustration, we refer the reader to Online Appendix D.4.

## 6. Conclusions

We use a novel approach to reexamine the effects of relative consumption and relative wealth preferences in the context of an otherwise standard *AK* model with homogeneous agents and exogenous labor supply. In the literature, the implications of such preferences are usually assessed by employing a particular instantaneous utility function and analyzing the dependence of the economic growth rate on those parameters that *seem* to determine the strength of the relative consumption and relative wealth motives. In contrast to the standard approach, we put special emphasis on the identification of the fundamental factors that ultimately determine long-run growth, where the results hold for a broad class of utility functions. Our approach allows to analyze *separately* the effects of changes in (i) the strength of the relative consumption motive, (ii) the strength of the relative wealth motive, and (iii) households' willingness to substitute absolute consumption intertemporally by considering *ceteris paribus changes* in the corresponding fundamental factor. We show that a widely used type of the instantaneous utility function has the property that such ceteris paribus thought experiments cannot be carried out in the context of the standard approach. The reason is that the parameter that seems to affect only the strength of the relative consumption motive actually also influences the willingness to substitute absolute consumption intertemporally (and the strength of the relative wealth motive). Since the standard approach is unaware of the additional unintended effects, it attributes the total change in the growth rate that results from a variation in this parameter incorrectly to the change in the strength of the relative consumption motive. It is thus possible that the resulting assertions of the standard approach are not only quantitatively, but also qualitatively incorrect. We also identify prominent specifications of the utility function in which the standard analysis yields correct results. These specifications have the common property that none of the parameters of the utility function affects more than one of the fundamental factors.

To obtain correct general results and to explain the pitfalls of the standard analysis, we draw heavily on the Euler equations for aggregate consumption in the decentralized and the socially planned economy, respectively. In the socially planned economy, neither the relative consumption nor the relative wealth motive affects the growth rate. In the decentralized economy, in the presence of weak restrictions on the utility function that are sufficient for the existence of a balanced growth path (BGP), all effects can be explained by means of the *effective* rate of return. The latter is defined as the sum of the market rate of return and an extra return that results from social comparisons based on both relative wealth and relative consumption. If labor supply is exogenous, as throughout the analysis, relative consumption and relative wealth preferences do not affect the market rate of return. Hence, these preferences influence the decentralized economy only via their effect on the comparison-induced extra return (CIER). Since the willingness to save depends positively on the CIER, the following qualitative properties of the CIER also hold for the decentralized growth rate: (i) The CIER depends positively on the strength of the relative wealth motive, irrespective of the strength of the relative consumption motive. (ii) If relative wealth matters for utility, then a rise in the strength of the relative consumption motive causes the CIER to fall. (iii) In the absence of the relative wealth motive, the CIER is identical to zero. Consequently, if relative wealth does not matter, then the decentralized growth rate does not depend on the strength of the relative consumption motive. Moreover, it is inefficiently low. The positive gap between the socially optimal growth rate and its decentralized counterpart decreases, however, with an increasing strength of the relative wealth motive. There is even a critical level of the latter such that the growth rate gap vanishes.

In contrast to our general results, the traditional approach (that is based on parameters instead of fundamental factors) seems to allow for the possibility that (i) relative consumption preferences enhance growth in the decentralized economy or (ii) both the strength of the relative consumption and the relative wealth motive affect the socially optimal growth rate. Our analysis shows that these incorrect conclusions are due to unnoticed and unintended side effects of parameter changes on the elasticity of intertemporal substitution of *absolute* consumption that have nothing to do with relative consumption or relative wealth preferences.

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**Supplementary materials.** To view supplementary material for this article, please visit http://doi.org/10.1017/S1365100521000389.

## Notes

**1** It is common practice to focus either on relative consumption or on relative wealth. For specifications that employ relative consumption (or more general consumption externalities), see, for instance, Abel (1990, 2005), Galí (1994), Harbaugh (1996), Carroll et al. (1997), Rauscher (1997), Grossmann (1998), Fisher and Hof (2000), Ljungqvist and Uhlig (2000), Liu and Turnovsky (2005), Turnovsky and Monteiro (2007), Fisher and Heijdra (2009), Knell (2009), Barnett et al. (2010), Wendner (2010), Strulik (2015), and Pham (2019). Examples of the relative wealth approach are Corneo and Jeanne (1997, 2001a,b), Futagami and Shibata (1998), Van Long and Shimomura (2004), Fisher and Hof (2005, 2008), García-Peñalosa and Turnovsky (2008), and Fisher (2010). For frameworks that allow for both specifications see Tournemaine and Tsoukis (2008), Riegler (2009), Ghosh and Wendner (2014), Wendner (2015), Klarl (2017), Ghosh and Wendner (2018), and Chang et al. (2018).

**2** While in our paper ($r$+CIER) is interpreted as effective interest rate, Tournemaine and Tsoukis (2008) follow Futagami and Shibata (1998) and dub an expression that is equivalent to ($\rho-$CIER) as the effective discount rate (however, without introducing the concept of the CIER).

**3** We analyze the effects of relative wealth preferences in the Romer (1990) model in Hof and Prettner (2019). In the working paper version (Hof and Prettner, 2016), we include relative consumption preferences, too.

**4** Carroll et al. (1997) and Ljungqvist and Uhlig (2000) are exceptions.

**5** Henceforth, we skip the clarifying formulation "with respect to absolute consumption". There is no risk of confusion, because the elasticities of $u_c$ and $m^{c/C}$ with respect to relative consumption $c/C$ and relative wealth $a/A$ do not play any role in the Euler equation that governs *aggregate* consumption in a symmetric macroeconomic equilibrium. Instead, they appear only in the Euler equation that determines optimal *individual* consumption for arbitrary, and hence also off-symmetric-equilibrium paths of $C$ and $A$. For details, see Online Appendix B.3.

**6** Strictly speaking, $\hat{m}^{c/C}$ and $\hat{m}^{a/A}$ yield only *local* information about the strength of the corresponding motives. Below we discuss six specifications of the instantaneous utility function $u = u(c, c/C, a/A)$ in which the condition (31) is satisfied. In four out of these six illustrations, the functions $m^{c/C}(c, c/C, a/A)$ and $m^{a/A}(c, c/C, a/A)$ are constant functions over their whole domains so that $\hat{m}^{c/C}$ and $\hat{m}^{a/A}$ are also measures of the *global* strength of the relative consumption motive and the relative wealth motive, respectively.

**7** We show that a BGP exists if and only if the transformation $T$ is of the CRRA type and $P(c)$ is a power function.

**8** For the additively separable specification $u(c, c/C, a/A) = T[P(c) + Q(c/C, a/A)]$, it can be shown that a BGP exists if the transformation $T$ is of the CARA type and $P(c) = \varphi_1 \ln c$. We ignore this case, because the resulting utility function $u(c, c/C, a/A) = -(1/\kappa)c^{-\kappa\varphi_1} \exp[-\kappa Q(c/C, a/A)]$ does not play any role in the literature.

**9** Specifications that are equivalent to (54) are, for instance, used in Harbaugh (1996) [Equation (1)], Grossmann (1998) [Equation (7)], Fisher and Hof (2000) [Equation (20)], Liu and Turnovsky (2005) [Equation (14b)], García-Peñalosa and Turnovsky (2008) [Equation (22)], Nakamoto (2009) [Equation (22)] and Pham (2019) [Equation (3)]. In other models that employ (53), $H$ is treated as predetermined stock variable that evolves over time. Carroll et al. (1997) and Alvarez-Cuadrado et al. (2004) distinguish in this context between outward- and inward-looking agents. The case of outward-looking households (external habits) [resp. inward-looking households (internal habits)] is modeled by assuming that $\dot{H} = \gamma(C - H)$ [resp. $\dot{H} = \gamma(c - H)$], which, in turn, implies that the reference stock $H$ is calculated as an exponentially declining weighted average of past *average* levels of consumption in the economy [resp. of her *own* past levels of consumption], $H(t) = \gamma \int_{-\infty}^{t} e^{\gamma(\tau-t)} C(\tau) d\tau$ [resp. $H(t) = \gamma \int_{-\infty}^{t} e^{\gamma(\tau-t)} c(\tau) d\tau$]. Koyuncu and Turnovsky (2010) restrict their attention to external habits. Chen (2007) considers only internal habits, but uses a more complicated differential equation for $H$. Johansson-Stenman et al. (2002)

employs a specification of the utility function that has the functional form given by (54), but incorporates absolute and relative income as arguments instead of absolute and relative consumption. The parameter that corresponds to $\beta$ is called the degree of positionality.

**10** Recall that this indirect effect of changes in $\hat{\sigma}$ on $g^D$ is absent in the pure relative consumption case in which $\hat{\eta} = 0$ holds due to the absence of the relative wealth motive.

**11** Chu (2007) analyzes an alternative phenomenon that affects the difference between the decentralized long-run growth rate and its socially optimal counterpart, namely systematic overconfidence of R&D entrepreneurs in a quality-ladder model of vertical innovation. A rise in overconfidence boosts employment in the R&D sector, which causes the step size of technological innovations and the resulting balanced growth rate to increase. There exists a critical degree of overconfidence such that the decentralized growth rate equals its socially optimal counterpart. Too much overconfidence, however, leads to excessive growth.

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