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Unsteady viscous axisymmetric flows associated with rotating surfaces

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In this thesis solutions of the Navier-Stokes equations are studied, from a mathematical viewpoint, for some unsteady axisymmetric flows which are associated with surfaces of revolution rotating with non-constant angular velocity.

The major part of the thesis is concerned with flows associated with plane disks of infinite extent. For these a similarity transformation enables the number of spatial coordinates to be reduced to one. The resulting equations are studied in three separate sections of the work.

In the first section some general properties of the equations are obtained. A general similarity solution of the equations is found, the resulting ordinary differential equations being similar to those which arise from unsteady two dimensional boundary layers. Next flow at large distances is considered, for bounded values of the time, and some possible asymptotic forms of the solutions are obtained. One of these asymptotic forms is found to be an exact solution and it is studied in more detail, particular attention being paid to the form the solution takes for large values of the time.

In the next section flows associated with impulsive rotary motion of two parallel disks are treated. The particular flow studied in detail is that set up when two disks, initially rotating as a rigid body with the fluid, are given small impulsive changes in angular velocity in opposite directions. The exact solution of the linearised equations is obtained in terms of elliptic θ functions. This is examined in order to determine

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the dimensionless time which elapses until the new steady state (which is not rigid body rotation) is established. It is found to be of the order of the Reynolds number (assumed large) associated with the initial state. Thus it is much longer than the time required when both velocity increments are of the same sign, and the final state is rigid body rotation.

The flow between two disks performing torsional oscillations of equal amplitude and frequency, but out of phase, is studied in the next section, for high Reynolds numbers. The flow field is found to consist of shear wave layers immediately adjacent to the disks, intermediate layers in which the angular velocity is identically zero, and an inner inviscid core. In the intermediate layers the dominant part of the axial-radial flow is a steady streaming, satisfying the same equation as that arising in the flow near a single oscillating disk.

In most flows of physical interest the velocity components must satisfy some boundary conditions at a finite value of the radial coordinate, so these flows cannot satisfy the similarity relations used earlier. The second part of the thesis, which may be read independently, is a tentative step towards the mathematical study of unsteady flows associated with more realistic body shapes. The flow induced by high Reynolds number torsional oscillations of a certain class of bodies of revolution is studied. The bodies are restricted to being semi-infinite in extent, since the flow appears to become quite complicated in the neighbourhood of points where the tangent plane to the body is parallel to the axis of symmetry. The flow has the same general form with three regions of different types, as occurs in flows associated with oscillating disks, and the steady streaming in the intermediate region is studied in some detail.

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