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BOOK REVIEWS

GILES, J. R., *Convex analysis with application in the differentiation of convex functions* (Research Notes in Mathematics No. 58, Pitman, 1982), £9.95.

Like other terms used to denote areas of Mathematics, "convex analysis" means different things to different people. The core of J. R. Giles's book is a systematic account of the connections between certain properties that Banach spaces (or their duals) may have and Krein–Milman type statements for convex subsets of these spaces. The Krein–Milman statements may be formulated using either extreme, exposed or strongly exposed points. The main Banach space properties in question are (1) the Radon–Nikodym property, defined as: every non-empty, bounded set has "slices" of arbitrarily small diameter, and (2) the Asplund property: every continuous, convex real function on an open, convex subset A is Fréchet-differentiable on a dense G_δ-subset of A.

During the last fifteen years, an extensive theory has been developed by Phelps, Asplund, Stegall, Kenderov and others. For example, the Radon–Nikodym property is equivalent to each bounded, closed, convex set being equal to the closed, convex hull of its strongly exposed points. A space has the Asplund property if and only if its dual has the Radon–Nikodym property. A space has the Radon–Nikodym property if and only if its dual has the "weak-star Asplund property". Many other characterisations of the Asplund property are known, for example, every separable subspace has a separable dual or (an apparent weakening of the definition) every equivalent norm is Fréchet-differentiable on a dense G_δ-subset.

A detailed treatment is given to Ekeland's theorem on lower semi-continuous functions on complete metric spaces. Though clearly not "convex analysis" in any possible sense of the phrase, this theorem has important applications to the subject. It is used here to establish one of the equivalents of the Asplund property, and also to prove the Brønsted–Rockafellar generalisation of the Bishop–Phelps theorem, in which support functionals of convex sets are replaced by "subdifferentials" of convex functions.

Two other topics included "as an epilogue" are the relationship between denting points and the Mazur intersection property, and the problem of convexity of Chebyshev sets.

The author has performed a useful task in assembling and organising this material, in an area in which results have often been rapidly superseded by stronger ones. Proofs are clear and complete, and the original sources are always mentioned. There is a generous scattering of examples and exercises throughout. Prerequisites are minimal, since the first 100 pages are largely devoted to an outline of the relevant background material on convex sets, locally convex spaces and General Topology. The reviewer's only complaint is that the author misses the opportunity to publicise Léger's beautiful proof of the Krein–Milman theorem.

G. J. O. JAMESON

FUHRMANN, PAUL A. *Linear Systems and Operators in Hilbert Space* (McGraw-Hill, 1981), x+325 pp. £19.50.

Must even electrical engineering succumb to the ethos of Bourbaki? Had the illustrious master taken employment in Philips Research Laboratories he might have captured linear systems for pure mathematics as does this book.

In fact there are those in both mathematics and engineering departments who have applied the abstract modern approach to linear algebra (using modules over a principal ideal domain) to linear systems. The axiomatic study of these systems dates from the 1950's. A system transforms a sequence (u_j) of inputs into a sequence (y_j) of outputs: the u_j and y_j are taken to be vectors