answer, and concluding that this principle is synthetic *a priori*. Finally a table summarizing the preceding generalizations and reductions is given.

As suggesting the generality introduced into logic by developments in symbolic logic, this paper may be of some value to the lay reader. The independent discussions and constructive suggestions are, however, woefully inadequate. I mention only two examples. (1) M. Picard rejects material implication as usually defined, because of its so-called paradoxes. He holds that only true propositions imply, that whenever p implies q there is a necessary relation between them, and that such an implication holds only if it be a case of a true formal implication. Though he does not give a new definition of implication, he asserts, in comparing disjunction with implication, that $p \vee q$ is equivalent to the "simultaneous affirmation" of both $\sim p$ implies q and $\sim q$ implies p but to neither separately; though elsewhere in the paper he accepts that p implies q is equivalent to $\sim q$ implies $\sim p$. The reviewer is unable to make sense of this. And (2) in trying to make conversion applicable to all propositions, the author finds the converse of "Some a is not b" to be "Some b is not a, or all b is a." That this is a failure hardly need be remarked.

EVERETT J. NELSON

ORRIN FRINK, JR. *New algebras of logic.* ***The American mathematical monthly,*** vol. 45 (1938,), pp. 210–219.

This paper first presents the properties of Boolean algebra, and then takes up in turn the Łukasiewicz-Tarski many-valued algebras (407*1*), Heyting's intuitionist logic (385*2, 3, 10*), and the quantum logic of Birkhoff and von Neumann (II 44), pointing out the properties in which each agrees with or differs from the Boolean algebra and from the others.

Since certain different formulae are provable in Łukasiewicz-Tarski algebras having different numbers of elements, there arises the question as to which formulae are provable in all such algebras. This Professor Frink answers by presenting a new set of postulates, in which the primitive ideas are a class of elements a, b, c, $\cdots$, of which one is 0, and the operation $\supset$. By definition, $a \vee b = (a \supset b) \supset b$, $a' = a \supset 0$, and $1 = 0'$. His postulates are: $a \supset (b \supset c) = b \supset (a \supset c)$; $a \vee b = b \vee a$; $a \vee 0 = a$; either $a \supset b = 1$ or $b \supset a = 1$; there are exactly $n+1$ elements. Frink outlines a proof that the consequences of this set are the formulae provable in all Łukasziewicz-Tarski algebras having n values, where $1 < n < \aleph_0$.

In commenting on Heyting's algebra, Frink asserts that it "serves the purpose of defending the intuitionists from the charge . . . that a consistent logic which denies the law of excluded middle is impossible." He points out too that in the Łukasiewicz-Tarski algebra this law fails to hold for logical sum. ($a \vee b = (a \supset b) \supset b$.) Now it is true that the Boolean form $a + a' = 1$ does not have an analogue in either of these systems. E.g., $a \vee a' = 1$ fails in the Łukasiewicz-Tarski algebra when a is neither 0 nor 1. It is true too that if any structure analogous to the Boolean form $a + a' = 1$ may be called *the* law of excluded middle, then, of course, that law does not hold in these systems. But with this admission even the staunchest protagonist of the Law of Excluded Middle would find no quarrel. He might, however, point out that what he means by the "Law of Excluded Middle" is a certain *proposition* and not an uninterpreted structure. On the level of abstract structures, the question of the acceptance or rejection of what he means by this law does not arise. It arises only when an interpretation is assigned, and its answer depends upon considerations going beyond those structures; namely, upon the subject-matter of the interpretation—in the case before us, upon the nature of logic. For a clear statement of this point of view, I refer to C. A. Baylis (I 66). Precisely similar remarks apply to Heyting's "rejection" of the law of double negation, to the Birkhoff-von Neumann "rejection" of the distributive law, to the failure of the Law of Contradiction for logical product in the Łukasiewicz-Tarski system, etc. According to a recent report (M. Kokoszyńska in this JOURNAL, vol. 3, p. 44), Łukasiewicz himself holds that the law of contradiction is an unconditional truth common to all logical systems. Whether a *proposition* is true is one thing. Quite another is whether an abstract system contains a *structure* having a certain form. And still another is whether the operation- and term-variables in that system may be assigned values such that some structure in that system becomes the proposition in question. Decision on the first and third points rests on considerations transcending the abstract system. Hence, it would seem impossible to adjudicate the case of the Law of Excluded Middle, or of any other challenged principle of logic, by the construction of abstract systems.

Besides his contributing the new set of postulates referred to above, Professor Frink has done a genuine service in presenting and comparing in a clear and instructive manner these new algebras. EVERETT J. NELSON

GR. C. MOISIL. *Sur le syllogisme hypothétique dans la logique intuitioniste.* ***Journal de mathématiques pures et appliquées,*** ser. 9 vol. 17 (1938), pp. 197–202.

The author gives formal proofs within the Heyting propositional calculus (385*2*) of

$$\neg\neg p \wedge \neg\neg (p \supset q) \supset \neg\neg q \quad \text{and} \quad \neg\neg (p \supset q) \wedge \neg\neg (q \supset r) \supset \neg\neg (p \supset r).$$

Intepreting double negation as expressing possibility, he regards these (or the associated derived rules of inference) as two new forms of the hypothetical syllogism, "la forme problématique simple" and "la forme problématique complète."

The latter half of §4 contains a number of troublesome typographical errors, including an incorrect reference to Heyting. ALONZO CHURCH

JERZY KUCZYŃSKI. *O twierdzeniu Gödla* (Über den Satz von Gödel). Polnisch mit französischer Zusammenfassung. ***Kwartalnik filozoficzny,*** Bd. 14 (1938), S. 74–80.

Verf. behauptet, der Satz von Gödel über die Existenz unentscheidbarer Sätze sei bis jetzt nur im Gebiet der arithmetisierten Metamathematik bewiesen worden. Dieses rein arithmetische Ergebnis sei nicht interessant, erst seine metamathematische Interpretation hätte einen Wert. Verf. versucht also, eine Interpretation des Satzes von Gödel in einem formalen System "Meta I" der Metaarithmetik durchzuführen und, da er dabei auf einen Widerspruch gestoßen zu haben glaubt, kommt er zu dem Schluß, daß der Satz von Gödel nur eine Antinomie sei—freilich nicht in der Arithmetik selbst, wohl aber im System "Meta I." Den Widerspruch leitet aber Verf. aus einer falschen Voraussetzung ab: er hat nämlich übersehen, daß in der Gödelschen Abhandlung 418*3* nicht der Satz $(x)\ \overline{x\ B_\kappa\ (17\ \mathrm{Gen}\ r)}$ sondern der Satz $\mathrm{Wid}(\kappa) \rightarrow (x)\ \overline{x\ B_\kappa\ (17\ \mathrm{Gen}\ r)}$ bewiesen wurde ($\mathrm{Wid}(\kappa)$ ist eine arithmetische Interpretation des Satzes "κ ist widerspruchsfrei"). Fügt man diese Korrektur in den Beweis des Verf. ein, so erhält man anstatt des Widerspruchs das Ergebnis, daß der arithmetische Satz $\mathrm{Wid}(\kappa)$ im System "Meta I" nicht beweisbar ist, es sei denn "Meta I" wäre widerspruchsvoll; dies wurde ja schon von Gödel (a. a. O. Satz XI) festgestellt. Es sei noch darauf hingewiesen, daß "Meta I" eine Interpretation in der Arithmetik hat, so daß es keinen Widerspruch im System "Meta I" geben kann, falls die Arithmetik widerspruchsfrei ist. Dem Ref. sind auch die Schlußbemerkungen des Verf. unverständlich, in denen es heißt, es scheine unmöglich zu sein, eine nicht-arithmetisierte Metamathematik formal so aufzubauen, daß sie nicht intensional wäre und dabei die Gödelschen Schlußweisen zuließe: der Satz von Gödel läßt sich z. B. in der axiomatisierten Metamathematik von Tarski (285*13* S. 100 und 285*16* S. 289 f.) ohne weiteres beweisen, obgleich diese Systeme durchaus extensional sind. ANDRZEJ MOSTOWSKI

W. V. QUINE. *Completeness of the propositional calculus.* ***The journal of symbolic logic,*** Bd. 3 (1938), S. 37–40.

Es wird die Vollständigkeit eines (auf Tarski, Bernays und Wajsberg zurückgehenden) Systems des Aussagenkalküls bewiesen, worin der Wahrheitswert "falsch" ("F") und die Operation der Implikation ("$\supset$") als Grundbegriffe auftreten. Die Negation "$\sim p$" wird als "$(p \supset \mathrm{F})$", der Wahrheitswert "wahr" ("T") als "$(\mathrm{F} \supset \mathrm{F})$" ausgedrückt. Die Vollständigkeit wird zunächst in dem Sinne bewiesen, daß sämtliche (aus Variablen und "F" durch "$\supset$" aufgebauten) tautologischen, d. h. identisch wahren Formeln aus den Axiomen

$$((p \supset q) \supset ((q \supset r) \supset (p \supset r))),$$
$$(((p \supset q) \supset p) \supset p),$$
$$(p \supset (q \supset p)),$$
$$(\mathrm{F} \supset p)$$