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EXISTENTIAL GRAPHS AS AN INSTRUMENT OF LOGICAL ANALYSIS: PART I. ALPHA

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Abstract. Peirce considered the principal business of logic to be the analysis of reasoning. He argued that the diagrammatic system of Existential Graphs, which he had invented in 1896, carries the logical analysis of reasoning to the furthest point possible. The present paper investigates the analytic virtues of the Alpha part of the system, which corresponds to the sentential calculus. We examine Peirce's proposal that the relation of illation is the primitive relation of logic and defend the view that this idea constitutes the fundamental motive of philosophy of notation both in algebraic and graphical logic. We explain how in his algebras and graphs Peirce arrived at a unifying notation for logical constants that represent both truth-function and scope. Finally, we show that Shin's argument for multiple readings of Alpha graphs is circular.

§1. Introduction. According to Peirce, the principal business of logic is the analysis of reasoning (CP 2.532, 1893; CP 4.134, 1893; MS 1147, pp. 13–14, c.1900). Mathematics is the *practice* of deduction, logic its investigation (CP 4.239, 1902). All deduction is mathematical in the sense that it is constructive or diagrammatic (NEM 4, pp. 47–48, 1902). But mathematical deductions or deduction *tout court* is the matter of investigation of deductive or formal logic. Logic cannot ground mathematics: deductions are in the first place *mathematically*, rather than *logically*, valid (CP 4.234, 1902). What logic can do is to *describe* and *analyze* mathematical reasoning (CP 2.192, 1902). Peirce was primarily a logician, and as a logician he felt that his talent was in logical analysis: "my strong point is my power of logical analysis" (Peirce to Carus, July 1908).

Peirce took analysis to be the process of decomposing something into its constituent parts: "if one concept can be accurately defined as a combination of others, and if these others are not of more complicated structure than the defined concept, then the defined concept is regarded as *analyzed* into these others" (MS 284, p. 45; CP 1.294, c.1905). A satisfactorily complete analysis is one in which the compound is decomposed into

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homogeneous parts, that is, into elements that are not in themselves composed of *other* elements and which therefore remain unanalyzed. “No analysis”, he wrote, “whether in logic, in chemistry, or in any other science, is satisfactory, unless it be thorough, that is, unless it separates the compound into components each entirely homogeneous in itself, and therefore free from the smallest admixture of any of the others” (CP 4.548, 1906). For if that which is unanalyzable were *not* homogeneous in itself, then it would be mixed with other components — but then it *would* be analyzable, for analysis is exactly what separates the different components that are mixed in a heterogeneous compound.

Since the 1870s, Peirce’s logical analyses had been algebraic. During 1896 he invented a graphical notation later named Entitative Graphs, which appeared in print the following January (Peirce 1897). Within a month from the invention of Entitative Graphs, he had created the system of Existential Graphs (EGs; see MSS 481–484). Examples of the latter system reached print in 1901 in the *Dictionary of Philosophy and Psychology* edited by J. M. Baldwin (Vol. 1, entry “Symbolic logic”, pp. 640-651), in the *Syllabus* for the Lowell Lectures of 1903, and in the 1906 *Monist* article “Prolegomena to an Apology for Pragmaticism” (Peirce 1906). Peirce continued working on EGs for the rest of his life. He wrote to William James on Christmas Day of 1909 that these graphs “ought to be the logic of the future” (NEM 3, p. 874).

Why so? The graphs, as had later crystallized to Peirce, are first and foremost an instrument of logical analysis:

> [T]he system of Existential Graphs is designed to afford a sort of geometrical *παρασκευή*,—or diagram,—for logical analysis, i.e. for illustrating and facilitating the same. (MS 300, p. 34, 1908)
>
> [T]he system of Existential Graphs alone enables us to carry the logical analysis of terms, propositions, and arguments to the furthest point possible in the nature of things. (MS 296, pp. 7–8, 1908)
>
> [T]here is no organ of definition and logical analysis that is at all equal to [EGs]. (Peirce to Carus, 18 Sept. 1908)

Not only is the analysis carried out through EGs the most complete one; it is also necessarily correct:

> [A]ll its represented analyses must be logically correct, since to say that an analysis is logically correct only means that it will be so represented in such a system that is, as this will then be, stripped of all superfluities. (MS 296, p. 9, 1908)
>
> [I]f a graph expresses a concept analytically, its analysis must be logically correct, and the only logically correct analysis from elements all of which are expressed in the graph. This is plain, since all that we mean by a logically correct analysis is one in which the elements are so put together as precisely to express the concept to be analyzed (Ibid.)

Peirce’s suggestion is that once the notational system has reached a maximum of analyticity, any particular analysis performed by it will prove to be a correct analysis. A system, constructed so as to employ the least amount of logical machinery and the least number of logical objects, forces us to the correct analysis of propositions. To say that an analysis is correct can in the first place mean nothing more than this.

Now, how does the system of EGs yield correct logical analyses? This is our task to explain in the present paper. EGs divide into three parts: Alpha, corresponding to

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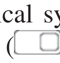
propositional calculus; Beta, corresponding to a fragment of quantificational logic with identity; and Gamma, which consists of modal logic, higher-order notions, abstraction, and logics for nondeclarative assertions (Roberts 1973; Pietarinen 2011). Roberts has elucidated different aspects of the analyticity of EGs, especially concerning the rules of transformation and the analysis of the logical structure of propositions. The rules allow dividing each piece of reasoning into its smallest steps, namely either insertions or omissions, which can hardly be considered complex operations (CP 4.564, 1906; MS 490, 1906). Peirce's favorite example here is how by the rules of EGs a syllogism in Barbara is divided into no less than seven distinct logical steps (CP 4.571, 1906). Concerning the Beta part or quantificational logic, Peirce claimed that EGs provide "the only method by which all connections of relatives can be expressed by a single sign" (MS 482, 1897), as "the System of Existential Graphs recognizes but one mode of combination of ideas" (MS 490, 1906; cf. MS 296, 1908). The Beta line of identity performs the office of predication, identity, existence, and class-inclusion, all in one single sign (Pietarinen 2011), thus answering the puzzle of the composition of concepts (MS 498-499, 1906; Pietarinen 2005). Zeman (1968) has suggested that the continuity of the Beta lines provides an analysis of the conception of identity. Shin (2002, 2011), whose proposal we shall discuss below, has argued that EGs have multiple readings *notwithstanding* their being analytic.

What about the first part of the system corresponding to propositional calculus, the Alpha part? What it is that makes Alpha more analytic (in Peirce's sense) than other systems of propositional logic? According to Peirce, Alpha is more analytic than other systems because, at bottom, it employs one single logical conception, that of consequence *de inesse*, or material implication. To express the material conditional, the system employs one single logical symbol, the so-called "scroll", constituted by two closed lines one inside the other (), forming two compartments with the antecedent placed in the outer compartment and the consequent in the inner one. Peirce discovered the functional completeness of the joint denial for Boolean algebra in 1880, which was rediscovered and proved by H. M. Sheffer in 1913.[1] In his 1885 work, "On the Algebra of Logic", he uses the "copula of inclusion" (⤙)[2] as the primitive, functionally complete operator of his nonrelative logic. This was to become Peirce's basic idea in the philosophy of propositional logic, and his experiments to properly express inclusion/implication since the 1880s directly contributed to the invention of the graphical systems of the late 1890s.

However, if Alpha's analyticity consisted simply in having a minimal functionally complete set of logical operations, then the Boolean algebra with one constant of 1880 and the algebras of nonrelative logic of the 1880s and 1890s would be as analytical as Alpha.

[1] This unpublished manuscript (MS 378), entitled "A Boolian Algebra with One Constant", was according to Irving Anellis still in 1926 tagged "to be discarded" at Harvard University's philosophy department. In the manuscript Peirce reduces the number of logical operations to one constant. He states that "this notation ... uses the minimum number of different signs ... shows for the first time the possibility of writing both universal and particular propositions with but one copula" (W4, p. 221). Peirce's notation was later termed the Sheffer stroke and is well-known as the NAND operation. In Peirce's terms it is one in which "[t]wo propositions written in a pair are considered to be both denied" (W4, p. 218). In the same manuscript, he also discovers what is the expressive completeness of the NOR operation, indeed today rightly known as the Peirce arrow.

[2] In the 1885 article he uses the claw "—<". The "cursive" form we use in the present paper (⤙) was introduced by Peirce in his later writings (see e.g. MS 530, pp. 31-32, 1904).

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Therefore, Peirce's own emphasis on the *greater* analyticity of EGs remains in need of an explanation:

> We now have an apparatus capable of analytically expressing every proposition which can be analytically expressed by the "general algebra of logic" [. . .] this system is far more perfect than logical algebra in being more analytical, and *analysis* is the chief thing in logic. ("On Logical Graphs", MS 481, pp. 9–10, 1896)

> [J]ust as logical algebra "frees us", as Schröder says,"from the trammels of language", so this system frees us from the trammels of algebra. (MS 1147, p. 24, c. 1900)

Why, then, did Peirce resort to Alpha, if he already had experimented upon analytically equivalent algebraic systems? Does not the copula of inclusion yield a complete analysis of nonrelative logic? The reason, we claim, is *that functional minimality does not exhaust the analysis*. In the mid-1890s Peirce took a step further in the analysis of propositional logic. The idea was to have, one may say, truth-functional signs that at the same time represent their own scope. The recognition that there are two functions that symbols may have in the analysis of logic, namely (i) truth-function (the "meaning" of a logical operation) and (ii) collectional function (the "scope" of a logical constant, including the representation of the order of operations) is a straightforward one to be made and we do not claim that to be Peirce's original discovery. But the idea of a notation in which the two functions are merged into one single notational devise is typical of his work. We claim that it is this merging of the truth-function and collectional function in a unifying notation that constitutes the core meaning of Peirce's claim that the system of Alpha graphs gives us the most complete analysis of propositional logic. This idea was to have significant consequences also to how he understood quantification to operate in the Beta part of the system (Pietarinen 2015a).

The neglect of these crucial aspects of Peirce's EGs has caused two sorts of misunderstandings. The first is the claim that EGs represent almost a complete break with Peirce's earlier logics and philosophy of logic:

> It is now usual to think of this later work as merely being an "iconic" treatment of his earlier discoveries in the algebra of logic. It is also common to think of the graphs as intended merely as a tool for the visual representation or manipulation of logical propositions, still understood mentally as linearly notated. However, I will argue that it instead represents almost a complete break with most central motifs in his earlier work in the Algebraic Period and is based upon a philosophy of logic that even Peirce himself did not have the opportunity fully to develop. (Dipert 2006, p. 293)

Dipert is certainly right that no appeal to sheer iconicity is really of use here. Algebra itself is iconic (Peirce 1885; 1906; MS 595, 1893; MS 303, 1903; MS 292, 1906; MS 634, 1909), and iconic thinking in general comes in two varieties, the geometric and the algebraic (MS 616, 1906). The algebraic and the diagrammatic are the two sides of the same iconic coin, and therefore the reference to the iconic character of the graphs can by no means adequately explain their emergence as an autonomous system of notation. However, contrary to what Dipert maintains, we will show that under some important respects Alpha is *not* a break with Peirce's earlier algebraic work. Rather, the Alpha graphs are the *development* of logical algebras, the pushing of them to the extreme.

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The philosophical motivation behind the algebraic and graphical systems is ultimately the same: that of analyzing the reasoning process and elements into their smallest and simplest components. The philosophy of graphs is a more developed and sophisticated version of the philosophy of algebra.

The second, and a lot more consequential misunderstanding likewise derives from a misconception of the relation between the graphs and the algebra of logic. It is to maintain, as Shin (2002, 2011) does, that EGs differ from symbolic notations because they are, unlike symbolic notations, capable of "multiple readings". We show in Section 4 that Shin's argument is circular, for it presupposes what it is supposed to prove, namely that Alpha differs from standard symbolic notations. What really distinguishes Alpha from symbolic notations is, as Peirce emphasized, that the Alpha signs are the fewest with which one can express the propositional calculus.

Our investigation belongs to that essential doctrine of logic that Peirce named the "philosophy of notation" (Peirce 1885). The philosophy of notation is in itself an important part of the philosophy of logic. It has been pursued by many in the modern era, from Leibniz to Frege and Peano. Peirce was, among his fellow logicians, the more attentive one towards the notational aspects of logic, more perceptively discerning the precise nature of the problems involved in the invention, modification, and adoption of logical notations. Given Peirce's antipsychologistic approach, one should not be surprised to discover that notions such as *cognitive efficiency*, *visual clarity*, and *persuasive efficacy* are in principle extraneous to the philosophy of notation.[3] But unlike psychological considerations, ethical considerations on the notation fully enter the logical scene. One of the first teachings of the Ethics of Notation (MS 253, 1903, MS 530, 1904)—which does for notations what the Ethics of Terminology (MS 478, 1903) does for language—is that a new notation is to be adopted instead of an older and more established one only upon *valid* and *justifiable* grounds. If no justification for a notational change can be provided, then the older and more established form is to be maintained.[4] If one accepts the Ethics of Notation as a methodological principle to be followed in the historiography of logic, one is thereby committed to *justify* Peirce's shifts and improvements in notation—in the nonrelative department the shift from the algebra of inclusion to that of consequence and from Entitative to Existential Graphs. Any such justification, we claim, must fit with the notion that animates Peirce's entire philosophy of deductive logic: namely, analysis. By tracing Peirce's "pursuit of analysis" from the algebras to the graphs, and by explaining the reasons for his notational moves, we hope to contribute a new chapter to the history of the philosophy of notation.

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[3] We do not deny that these notions (cognitive efficiency, visual clarity, and persuasive efficacy) do sometimes play a role even in Peirce's philosophy of logic. Nor do we underestimate the importance of modern cognitive approaches to logical and mathematical notations (see e.g. Dutilh Novaes 2012; De Cruz & De Smedt 2013). What we deny is that the evolution of Peirce's logics and the birth of the graphs can be explained in terms of such notions alone. In this paper, we argue that another notion – analysis – has precedence. For further criticism and counter-criticism concerning issues such as visuality, free rides, or generality of logical diagrams, see Pietarinen (2015b), Pietarinen & Bellucci (2015a, 2015b).

[4] Peirce's statement of the maxim of the ethics of notation is this: "*The person who introduces a conception into science has both the right and the duty of prescribing a terminology and a notation for it; and his terminology and notation should be followed except so far as it may prove positively and seriously disadvantageous to the progress of science. If a slight modification is sufficient to remove the objection, a much greater one should be avoided*" (MS 530, p. 1, 1904, emphasis in the original).

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The present paper is the first part of a series of papers investigating EGs as an instrument of logical analysis. It is divided as follows. Section 2 examines Peirce's idea that the relation of inclusion is the primitive relation of logic and defends the view that this idea constitutes the fundamental motive of his notational researches, both algebraic and graphical. Section 3 explains how Peirce managed to represent scope in his algebras and graphs. Sections 2 and 3 provide a more accurate picture of the philosophy of logic behind Alpha than what we find in contemporary discussions on EGs. Having cleared up the precise meaning of Alpha's analyticity, Section 4 discusses Shin's argument for multiple readings of Alpha graphs.

The second and the third part (forthcoming) address the Beta and the Gamma systems of EGs as instruments of logical analysis.

§2. The Relation of Inclusion and the Emergence of Negation. Since 1865, Peirce identified categorical propositions with hypotheticals,[5] and since 1880 he identified both with the relation of illation expressed by the *ergo*.[6] In his "Description of a Notation for the Logic of Relatives" (Peirce 1870), he had already moved away from Boole's *equational* system and adopted an *implicational* one. His argument against the primacy of identity was that inclusion is a simpler notion than identity. Inclusion analyzes identity, that is, "$x = y$" is analyzed as "$x \prec y \wedge y \prec x$ (Peirce 1870, W2, p. 360).[7] To express the paramount relation of logic, which comprises at once class-inclusion, hypotheticals or conditionals, and illation, he uses the "copula of inclusion": $\prec$. From "the identity of the relation expressed by the copula with that of illation, springs an algebra" (Peirce 1880; W4, p. 173),which he calls the "algebra of the copula":

> This identification, by means of which all that is found true of term, proposition, or inference is at once known to be true of all three, is a most important engine of reasoning, which we have gained by beginning with a consideration of the genesis of logic. [...] In consequence of the identification in question, in S $\prec$ P, I speak of S indifferently as *subject*, *antecedent*, or *premise*, and of P as *predicate*, *consequent*, or *conclusion*. (Peirce 1880; W4, p. 170, 170n5)

The reason for the identification of logical consequence with material implication is that "logic supposes inferences not only to be drawn, but also to be subjected to criticism; and therefore we not only require the form $P \therefore C$ to express an argument, but also a form $P_i \prec C_i$ to express the truth of its leading principle" (W4, p. 166, 1880). When the leading principle of an argument is stated in a conditional proposition, the premises become the antecedent and the conclusion the consequent. This much Peirce had learnt from the medieval doctors, who "always called the minor premise the antecedent and the conclusion the consequent" (NEM 4, p. 178, 1898).[8] Peirce was not confused about material impli-

[5] Cf. Logic Notebook, 1865, W1, p. 337.
[6] Cf. W4, p. 421.
[7] Such an "analysis" of equality through inclusion was later imitated by Schröder (1890, p. 147).
[8] For Peirce's discussion of the medieval doctrine of consequences see W2, pp. 431–432 (1870), MS 594, pp. 62–64 (c. 1893), MS 408, p. 121, MS 411, pp. 177–178 (1894). See also Bellucci (2015).

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cation and logical consequence.[9] In fact, he informally used a deduction theorem in his 1880s algebras,[10] and subsequently proved it as a meta-theorem of the system of graphs.[11] Inclusion is *transitive* (if $A \prec B$ and $B \prec C$, then $A \prec C$),[12] *antisymmetric* (if $A \prec B$, the reverse $B \prec A$ does not hold) and *reflexive* ($A \prec A$). It is what we nowadays call a partial order. In 1881 "On The Logic of Number" Peirce calls any such relation a "fundamental relative of quantity" and the systems of objects having a fundamental relation of quantity a "system of quantity" (W4, pp. 299–300). The fundamental relative of quantity is crucial in the construction of an axiomatic base for arithmetic that Peirce undertook in that important paper.[13] But the transitivity and antisymmetry of this relation was important for Peirce because it mirrors inference, namely the passing from premises to conclusion. Inference is a transitive, antisymmetric process, and the basic operation in formal logic must reflect these properties as far as possible. The idea that material implication is primary because it mirrors inference remains a constant theme throughout his logical thought.[14]

Besides being iconic of inference itself, inclusion is also the most analytic operation. That the copula of inclusion (together with falsity) alone is sufficient to express the whole logic of propositions is one of the dominant motives of Peirce's philosophy of algebraic notation. Negation is defined in terms of implication (as the implication of what is false). In the 1880 paper, he derives negation from implication as follows: let us take x to be a constant falsehood; $A \prec x$ then amounts to the negation of A (Peirce 1880, W4, p. 176). This roughly corresponds to intuitionistic negation, which gets $\neg A$ from $A \rightarrow \bot$. The "fourth icon" of the 1885 "On the Algebra of Logic" states exactly this (Peirce 1885, W5, p. 172; cf. Prior 1958). Negation is introduced in the system for the sake of easier calculations, but implication is logically primary. When an operation is defined by means of another, the former is dispensable:

> The algebra of the copula, as given in my first paper, is adequate to every problem of nonrelative logic, and it makes use of but two operational signs, the copula $\prec$ and the sign of negation. Accordingly, an algebra of nonrelative logic which contains three signs, say of addition, multiplication, and negation, contains a surplusage of signs. (W5, p. 108, 1884)

A superfluous sign is a sign signifying a logical operation or function that can be signified in terms of other signs. A superfluous concept is one signified by a superfluous sign. In the "Philosophy of Notation" paper of 1885 Peirce is clear on what he means by

9 But Dipert 1981, p. 592 suggests that he was "deliberately ambiguous" between them.

10 Cf. W4, p. 173, 1880; CP 3.380, 1885.

11 "If one graph can be illatively transformed into another an enclosure may be written consisting of an oval enclosing the former graph and an oval enclosing nothing but the latter" (*Logic Notebook*, June 1898, MS 339, p. 118r; cf. MS 339, p. 180r; cf. also Roberts 1973, pp. 120–121; Pietarinen 2015c).

12 In his notes on Peirce's Johns Hopkins logic lectures of 1878-79, Allan Marquand writes that all syllogistic and all logic springs out from the transitiveness of the copula, and that the resemblances between the copula of inclusion and the relation of illation are more important than the differences (December 3, 1878, Marquand 1879, p. 45).

13 On Peirce's 1881 axiomatization of arithmetic see Shields 2012.

14 See NEM 4, p. 277, c. 1895, CP 3.440, 1896, CP 3.472, 1897, NEM 4, p. 174, 1898.

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notational superfluity:

> The forms of Boolian algebra hitherto used, have either two operational signs and a special sign of negation, or three operational signs. One of the operational signs is in that case superfluous. Thus, in the usual notation we have
>
> $$\overline{x+y} = \bar{x}\,\bar{y}$$
> $$\bar{x} + \bar{y} = \overline{xy}$$
>
> showing two modes of writing the same fact. (W 5, pp. 174-175)

In a notation that has both a sign of conjunction and a sign of disjunction it is possible to express the same fact in two different ways. In contemporary notation, $\neg(x \vee y) := \neg x \wedge \neg y$ and $\neg x \vee \neg y := \neg(x \wedge y)$. De Morgan's laws are the effect of a surplus of signs. Signs of addition and multiplication can be introduced by definition, but philosophically speaking they are superfluous.

Wittgenstein wrote to Russell in a similar vein:

> The big question now is, how must a system of signs be constituted in order to make every tautology recognizable as such *in one and the same way*? This is the fundamental problem of logic. (1913/2012, p. 59)

Although $\neg\neg p$ is equivalent to p, the sign "$\neg\neg p$" is not equivalent to the sign "p", for "$\neg\neg p$" contains two occurrences of the sign of negation, while "p" does not; this suggests that "$\neg\neg p$" contains something not contained in p, which is obviously false because the two are equivalent. Something is wrong in the notation, Wittgenstein concludes (*Tr.* 4.0621). That two things are identical, or that two states of things have the same truth-value, should be *shown* in the notation itself. The fact that the alleged primitives of logic are interdefinable shows that they are not the real primitives (*Tr.* 5.42). In Peirce's terms, the possibility of having two modes of writing the same fact, while a virtue for an algebra considered as a calculus, is an imperfection for an algebra considered as an instrument for logical analysis.[15]

Around 1880 Peirce had discovered the functional completeness of the joint denial operator:

> Every logical notation hitherto proposed has an unnecessary number of signs. It is by means of this excess that the calculus is rendered easy to use and a symmetrical development of the subject is rendered possible; at the same time, the number of primary formulae is thus greatly multiplied, those signifying facts of logic being very few in comparison with those which merely define the notation. (W 4, p. 218, 1880–81)

But the algebra of joint denial is a *hapax* in Peirce's logical writings. Although it perfectly conforms to the ideal of an analytic algebra, this system was soon abandoned in favor

[15] In Peirce's notes for the entry on "Exact Logic" for Baldwin's *Dictionary* we read: "It is far simpler, without question, to admit disjunction and copulation, together with negation, as primitive relations. If the object were to produce a working calculus of logic, that should be done [...] The real purpose of logic is to analyze reasonings, explain them, to furnish general canons for application in difficult cases, and to guide the laying out of a general plan of procedure in reasoning. For that purpose, complete analysis is requisite; and a completely analytic statement must necessarily be very complicated" (MS 1147, pp. 13–14, c. 1901).

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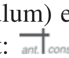
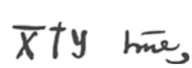
of inclusion. Since he proved that both joint denial and inclusion are minimally functionally complete (this latter when taken together with constant falsehood ⊥), that is, since one is as analytic as the other, the question may arise as to why Peirce settled on inclusion. The answer should at this point be clear, as inclusion mirrors inference while the joint denial operator does not: “of all the methods in which propositions may be analyzed and analyzed *correctly*, that one which uses the copula of inclusion alone corresponds to the theory of inference” (NEM 4, p. 174, 1898). If logic were a purely syntactic manipulation of signs, then any analytic relation could be taken as primitive as any other. Russell and Peano, for example, considered the choice of the primitive ideas of their logic as being to some extent arbitrary (Russell 1903, §31; Peano 1958, pp. 247, 302, 432). But for Peirce logic takes its reason for being from the nature of reasoning, and its signs should adhere to the representation of reasoning as much as it is allowed by the requirement of analyticity: if two notations are equivalently analytic, preference should be accorded to the one which more closely represents inference.

Peirce would continue using some copula of inclusion during the late 1880s and the beginning of the 1890s. Very remarkable is the so-called “sign of consequence”, invented circa 1886 (W5, pp. 341–343, 361–378) and used in the projected 1893 book *How to Reason: the Critick of Arguments* (MS 411, MS 559). The sign of consequence, , is formed by a horizontal line (the vinculum) extending over the antecedent and by a cross separating antecedent and consequent: 

It expresses the material conditional (W5, p. 341). From the point of view of the truth-functional calculus, the sign of consequence and the copula of inclusion are equivalent. (In the next section, however, we show that they are not equivalent under a somewhat different perspective.) The sign of consequence and its algebra, Peirce says, “completely describes the notation. It does not yield a convenient calculus, but it has the logical merit of doing everything that the Boolian algebra does without any superfluous sign” (“The Logic of Relatives”, 1886, W5, p. 373), that is, is a sole sufficient operator for the propositional calculus. Peirce also suggests that the sign of consequence can be “truncated” and read off as a disjunction:

> [W]e can cut the sign of consequence into two parts, the cross signifying ‘or,’ the vinculum ‘not’. Thus,
>
> 
>
> may be regarded as
>
> 
>
> meaning not X or Y is true. [. . .] this modification of our notation is so vastly more convenient than what we had before, that the student may well ask why I did not adopt it from the beginning. The answer is, that in thus breaking the sign of consequence and inconsequence we shutter all vestiges of the logical origin of the signs of aggregation and composition. Now, I consider the convenience of a logical algebra a very secondary consideration, since it is of no very great importance as a calculus, while it is very important as an instrument of logical analysis (“The Algebra of the Copula”, MS 411, pp. 232–233, 1893; formulas in Peirce’s hand).

The sign of consequence can be “truncated” into two Boolean parts, one the vinculum expressing negation, the other the cross expressing disjunction. The notational derivation

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from $\overline{x \prec} y$ to $\overline{x} \prec y$ shows how negation and disjunction can be derived from the conditional without any structural remodeling of the notation.

In one of his last logical papers, Peirce recalls that the development of Entitative Graphs in 1896 had started "from the conditional form" (MS 670, p. 11, 1911). In fact, in "The Logic of Relatives", where Entitative Graphs were first presented (Peirce 1897) he begins with the (quantificational) logic of relatives, without treating propositional logic as a separate part: the "algebra of the copula" section is conspicuously missing in that paper.[16] However, it should not come as a surprise that the conditional is again the primary conception also in Entitative Graphs: "it must be acknowledged that the illative relation (that expressed by 'therefore') is the most important of logical relations, the be-all and the end-all of the rest. It can be demonstrated that formal logic needs no other elementary logical relation than this" (Peirce 1897, p. 171). We return to Entitative Graphs in the next section.

Invented immediately after the system of Entitative Graphs, Existential Graphs too make the conditional form primary. It has sometimes been observed that in EGs Peirce abandons the conditional and resorts to conjunction and negation as the minimal set of connectives of the Alpha part. For example, Dipert wrote that

> the graphs make negation and conjunction primary, diminishing the importance of the conditional that had dominated Peirce's logic since 1885 and of some transitive and antisymmetric logical connective that had dominated his whole adult life (from 1867 through the 1890s). (Dipert 2006, p. 323n43)

This claim fails to do justice to Peirce's treatment of propositional logic in EGs. What in 1885 was the copula of inclusion and in 1893 the sign of consequence, in EGs becomes the scroll, composed of two closed lines one inside the other (Fig. 1), but often drawn with one continuous line (Fig. 2). The antecedent is placed in the outer compartment, the consequent in the inner compartment. Both Figs. 1 and 2 represent the material conditional, "If *A* then *B*":

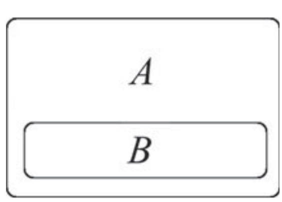

Fig. 1

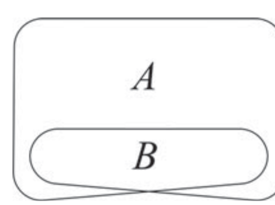

Fig. 2

As with the "truncation" of the sign of consequence into a disjunction (with the vinculum signifying negation), in EGs the scroll may be interpreted as a conjunction (with the

[16] The reason is that "The Logic of Relatives" (Peirce 1897) was intended as Peirce's second *Monist* review of Schröder's *Vorlesungen*, this one being especially concerned with the third volume devoted to the logic of relatives. Thus Peirce, wishing to include a presentation of his newly discovered Entitative Graphs in the review, felt himself forced to focus on the graphical treatment of quantification. He would however soon produce a "more formal statement" (MS L 77) of the theory (MSS 481–483; Pietarinen 2015c), including separate presentations of propositional and predicate logic. See Roberts 1973, pp. 25–27, for an explanation of the "Alpha" part of Entitative Graphs.

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oval signifying negation); one may thus read Fig. 1 and Fig. 2 as $\neg(A \wedge \neg B)$. Although this reading becomes easier in more complicate cases, it would be a logical distortion to treat negation as primary and the conditional as derived. On the contrary, by taking the conditional and its corresponding notation as primary, Peirce derives the idea of the negation from it. To do this, to show that *if* the scroll signifies a conditional, *then* the single oval must signify negation, Peirce introduces the pseudograph as a constant falsehood ($\perp$). This may be represented as a single *empty*, or vacant, oval, as in Fig. 3. (Strictly speaking, the area of the oval of the pseudograph is not absolutely empty, for the sheet of assertion upon which the graphs are scribed represents all truths, and the pseudograph denies that.)

Fig. 3. The empty oval or pseudograph.

Since the pseudograph is a constantly false proposition or absurdity, this can also be taken to mean that every proposition is true: "Were every graph asserted to be true, there would be nothing that could be added to that assertion. Accordingly, our expression for it may very appropriately consist in completely filling up the area on which it is asserted. Such filling up of an area may be termed a *blot*" (MSS 455–456). If the blot and its boundary are regarded as irrelevant to the meaning of the graph, Fig. 4 may be taken to represent that from A the pseudograph follows, or that "not-A."

Fig. 4

As in 1885 "On the Algebra of Logic", Peirce proposes to derive negation from the conditional and constant falsehood. Now, since a blot may be made indefinitely small, this derivation, which is not a transformation according to the rules of inference, can also be represented as the following continuous transmutation:

Fig. 5

This mutation gives us what Peirce calls an *interpretational corollary* of the convention adopted (representing the conditional by the scroll): the single oval precisely denies its content.

> Since an obliterated area may be made indefinitely small, a single cut will have the effect of denying the entire graph in its area. For to say that if a given proposition is true, everything is true, is equivalent to denying that proposition. (CP 4.402, 1903)[17]

---

[17] At one point Peirce seems to disavow aspects of the above analysis, as the blot cannot be removed completely: "this error of assuming that, because the blackened Inner Close can be

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> Since a Conditional *de inesse* (unlike other conditionals) only asserts that either the antecedent is false or the consequent is true, it all but follows that if the latter alternative be suppressed by scribing nothing but the antecedent, which may be any proposition, in an oval, that antecedent is thereby denied. (CP 4.564, 1906)

It is important to notice that in this process–Peirce says–we "did not assume that any sign represented negation". Yet we have "proved that certain signs having certain significations, otherwise defined, must express negation. In other words, we have *virtually analyzed the concept of negation*" (MS 481, p. 8, 1896, our emphasis). The scroll analyzes negation and shows its logical origin: "I thus analyze the negation of *P* into a positing of *P* as a mere idea together with the assertion that falsity is sequent upon it" (MS 300, p. 46, 1908). The reverse is not true, however: one cannot define the conditional by negation without also employing either conjunction or disjunction. In EGs, the impossibility of deriving the conditional (the scroll with vacant areas) from negation (a single cut with a vacant area, or pseudograph) in the same way in which Peirce derives the latter from the former is reflected in the impossibility of introducing the cut with a vacant area (pseudograph) by scribing it on the sheet.

When each oval of the scroll is interpreted as expressing negation, the juxtaposition of two graphs on the Sheet of Assertion has to represent either conjunction or disjunction. This gives us the two, dual systems of Entitative and Existential Graphs. The full primitive sign is, both in Entitative and in Existential Graphs, the representation of the conditional. In Entitative Graphs, "*A* implies *B*" is represented as in Fig. 6, while in EGs, it is represented as Fig. 7:

Fig. 6

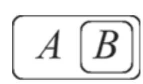

Fig. 7

As an abbreviation, we can interpret the oval as negation and then express the conditional in terms of negation and conjunction (as in EGs) or in terms of negation and disjunction (as in Entitative Graphs). In this way, it is possible to negate *a* without resorting to the more complex conditional form with the blot as in Fig. 4. But Peirce is clear that this is just a derived interpretation, an interpretational corollary (MS 450, 1903). Taking the idea of negation as primary is philosophically inaccurate:

> All my own writings upon formal logic have been based on the belief that the concept of Sequence, alike in reasonings and in judgments, whether

made indefinitely small, therefore it can be struck out entirely, like an infinitesimal. That led me to say that a Cut around a graph-instance has the effect of denying it. I retract: it only does so if the Cut encloses also a blot, however small, to represent iconically the blackened Inner Close. [. . .] a single Cut, enclosing only A and a blank, merely says: 'If A', or 'If A, then' and there stops. If what? you ask. It does not say. 'Then something follows', perhaps; but there is no assertion at all. This can be proved, too. For if we scribe on the Phemic Sheet the Graph expressing 'If A is true, Something is true', we shall have a Scroll with A alone in the Outer Close, and with nothing but a Blank in the Inner Close. Now this Blank is an Iterate of the Blank-instance that is always present on the Phemic Sheet; and this may, according to the rule, be deiterated by removing the Blank in the inner close. This will do, what the blot would not; namely, it will cause the collapse of the Inner Close, and thus leaves A in a single cut" (MS S-30, pp. 16–18, 1906). If this is right, then the oval that derivatively represents denial has an infinitesimally small, invisible blot residing on its boundary.

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> the latter be conditional or categorical, could in no wise be replaced by any composition of ideas [...] The simple Cut is a Scroll [...] Indeed, so far is the concept of Sequence from being a composite of two Negations, that, on the contrary, the concept of the Negation of any state of things, X, is, precisely, a composite of which one element is the concept of Sequence. Namely, it is the concept of a sequence from X of the essence of falsity. (MS 300, pp. 46–48, 1908)[18]

Negation is not a primitive idea; rather, it is a conception derived from that of implication. Therefore, the sign of negation ought to be considered as a complication or determination of a more primitive sign, the scroll. The analysis of negation in terms of the conditional is according to Peirce one of the few satisfactory proofs of the indecomposability or uncompoundness of a logical concept that can be obtained. He took that in logical analysis, it is "tolerably easy to demostrate compoundness, but next to impossible to make sure of elementality, or elementarity" (MS 300, p. 49, 1908). Once all other logical constants of the propositional calculus are defined in terms of the conditional relation, we have carried analysis to its extreme (truth-functional) limit: the conditional is logically unanalyzable, while other logical relations are analyzable through it.

§3. Combining Truth-function and Scope. Alpha is therefore as analytic as a notation for propositional logic can be, for it uses the least possible amount of logical connectives—in effect, just one. No system is more analytical than a single-sign notation. This was clearly perceived by Wittgenstein, who in the *Tractatus* pointed out that every proposition can be obtained from the elementary propositions by a recursive application of the N-operator (joint denial) to classes of propositions to obtain further propositions (*Tr.* 6.001). Russell and Whitehead used the Sheffer stroke in the second edition of *Principia Mathematica*, also persuaded by the Tractarian idea that austerity shows something essential about logic. Peirce had discovered the functional completeness of joint denial operator in 1880, but considered the other operator, material implication, to be superior because of its mirroring the very idea of inference itself.

If this were the whole story, however, Alpha would just be as analytical as the algebra of logic of 1885, because the scroll analyzes the logic of propositions exactly as the copula of inclusion does. Something is missing in our reconstruction. In fact, as we now proceed to show, with the graphs Peirce takes a step further in the analysis, which consists in the unification in one single sign of two distinct notational offices: truth-functionality and the indication of scope.

Such a step was first taken in 1886. We saw above that in 1886 Peirce substitutes the copula of inclusion with the sign of consequence. The decisive reason for this notational change was that the sign of consequence, unlike the copula of inclusion, also fulfills the office of parentheses. It thus does everything that the former notation does with a lesser amount of signs:

[18] Cf. "Before I had the concept of a cut, I had that of two cuts" (MS 650, p. 20, 1910); "Now the method of logical analysis of propositions that I recommend is that of the System of the Existential Graphs; and that system did, as a fact, arise, and could not have failed to arise, from the adoption of a suitable diagrammatic symbol for the relation between one supposition and another from which it follows; and Existential Graphs suffice for all the purposes of logical criticism and Critic" (MS S-30, p. 5, c.1906).

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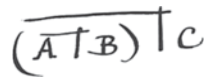
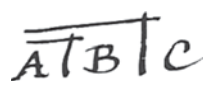
[A] further notational convention must be introduced. Using parentheses, just as they are used in algebra, as binding signs, we have to distinguish between



and

To do this, we have only to establish the convention that the vinculum, or horizontal line, which forms a part of the sign of consequence is *[*to*]* be extended over the whole antecedent, and all possible ambiguity is removed, without the use of parentheses. Thus, we write



and

(MS 559, p. 8, 1893, formulas in Peirce's hand)

In the 1885 notation that uses the copula of inclusion, in order to distinguish $A \prec (B \prec C)$ from $(A \prec B) \prec C$ we need to use parentheses or other conventions.[19] With the sign of consequence this is not necessary: the scope of vinculum denotes the antecedent in all cases.

To appreciate the significance of such differences, it is instructive to compare the sign of negation in the standard language (¬) and the sign of negation in Boolean algebra, namely the vinculum.

$$\overline{x+y} \qquad \bar{x}+\bar{y}$$

Fig. 8

$$\neg(x+y) \qquad \neg(x)+\neg(y)$$

Fig. 9

In Boolean notation (Fig. 8), the vinculum fulfills at once the offices that in the standard notation are obtained by the joint action of two different signs: the ¬ and the parentheses (Fig. 9).[20] In a similar way the sign of consequence fulfills the offices that in the 1885 notation were obtained by the joint action of two different signs, the copula $\prec$ and the

[19] Peirce considers inclusion as left-associative, and thus writes $x \prec y \prec z$ for $x \prec (y \prec z)$ (W5, p. 176, 1885).

[20] The first to achieve something similar may have been Descartes, who attached the vinculum to the radical sign (Cajori 1929, pp. 385-386), thus synthetizing two notational functions in one single sign. Leibniz, the Bernoullis and others used the *vinculum* to only express aggregation (Cajori 1929, pp. 386–390).

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parentheses.[21] This was the real novelty of the 1886-1893 sign, at once the sign of consequence and the sign of scope of the antecedent. None of the signs used in algebra, with the exception of the Boolean vinculum to express negation, has this double property.

Now, the sign of consequence, with its "double lecture" discussed in the preceding section, is *notationally* equivalent to the conditional form that we find in Entitative Graphs. In Entitative Graphs, the sign of consequence (Fig. 10) is replaced by the oval (Fig. 11):

Fig. 10

Fig. 11

The oval here fulfills the same offices as the sign of consequence. Both the sign of consequence and the oval express material implication by indicating the scope of the antecedent. Nests of ovals behave just as stacked *vincula*.[22] Both indicate the scope of the antecedent, merging in one single sign two distinct offices. When he presents Entitative Graphs in the 1897 *Monist* paper, Peirce insists that his main concern in devising the new notation was *the possibility of representing the scope of the antecedent in the conditional form*. That was his reason for passing from the sign of inclusion to the sign of consequence:

> Since, if the antecedent is compound, it is very important *to know just how much is included in the antecedent*, while it is a matter of comparative indifference how much is included in the consequent (though it is simply everything not in the antecedent), and since further (for the same reason) it is important *to know how many antecedents*, each after the first a part of another, contain a given relative or copula, I find it best to make the line which joins the antecedent and consequent encircle the whole of the former. (Peirce 1897, p. 174, our emphasis)

This is the form of Fig.11, which is read "If A, then B". Fig. 12 means "for any x, either it is not H or it is D," or "any H is D" (in contemporary notation, $\forall x[\mathrm{H}x \rightarrow \mathrm{D}x]$). Fig. 13 means "whoever loves only the virtuous is wise" (in contemporary notation, $\forall x \forall y$

Fig. 12

Fig. 13

$(\mathrm{L}xy \wedge \mathrm{V}y \rightarrow \mathrm{W}x)$). The oval performs the office of negation and parentheses, while the juxtaposition functions as disjunction. (A further convention prescribes taking any line of

[21] The editors of W5 state: "This may be thought of as a Peircean version of what is known as Polish notation. The location of the left tip of the 'streamer' makes it a prefixive (and parentheses-free) notation" (W5, pp. 459–460, n341.7). There are also interesting parallels with Frege's philosophy of notation, which we cannot address here for want of space. The main issue is that both Peirce with his 1886 sign of consequence and Frege with his 1879 *Begriffsschrift* managed to express the order of the operations in a second dimension. On Frege's two-dimensional notation see Macbeth 2006, pp. 45–56.

[22] The editors of W5 state: "The diagrammatic treatment of logic here is suggestive of Peirce's later graphical treatments but is more akin to his entitative than to his existential graphs. Note the emphasis on inclusion of antecedent in the consequent of a true hypothetical proposition" (W5, p. 458, n331.27-332.22)

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identity whose outermost part is unenclosed or evenly enclosed as a universally quantified variable.)

Since Peirce had been in possession of a graphical device for representing the logic of relatives over two dimensions ever since 1882 (cf. W4, pp. 391–399), why is it only in 1896 that he publishes the first version of his Graphs? The common answer is that in 1882 he lacked a devise to represent negation (cf. Roberts 1973, p. 20). But we believe it would be more appropriate to say that in 1882 he did not have *a sign fulfilling both the office of a logical connective and of a collectional sign*.[23] This was the great novelty introduced in 1886 with the sign of consequence (‾†), which does not simply represent a conditional form (as the copula of inclusion ⤙ does), but also represents the scope of the antecedent of that conditional. In this very important sense, the oval as a distinctive sign of Entitative and Existential Graphs evolved from the 1886-1893 sign of consequence. The sign of consequence is the missing link between the 1885 system with the copula of inclusion and the systems of graphs. Fig. 14 shows what the evolution from sign of consequence to Entitative Graphs would look like.[24]

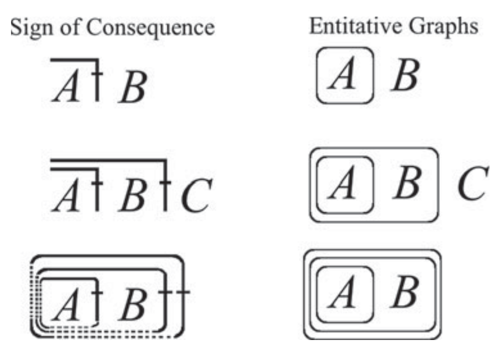

Fig. 14

The same is true of Existential Graphs. In the Alpha part of EGs, the scroll expresses a material conditional, and each of its ovals expresses negation. But the scroll (and *a fortiori* the ovals) indicates its own scope, so that no syntactic ambiguity is possible in Alpha. There is no way to scribe a graph in the Alpha system corresponding to A ⤙ B ⤙ C without thereby scribing either (A ⤙ B) ⤙ C (Fig. 15) or A ⤙ (B ⤙ C) (Fig. 16).

Fig. 15

Fig. 16

23 Having a sign that fulfills both functions (truth-function and collectional function) allows the Beta graphs to represent *dependent quantification*. The problem with Peirce's 1882 graphical experiments with the logic of relatives (W4, pp. 392–396) is that he cannot express dependent quantification without considerably complicating the notation, while in the 1896 Graphs dependent quantification is perfectly represented by means of ovals and lines only. On this point, see Pietarinen 2015a.

24 The three formulas expressed with the sign of consequence (on the left-hand side) and in Entitative Graphs (on the right-hand side), correspond to the following standard formulas: "$A \rightarrow B$", "$(A \rightarrow B) \rightarrow C$", "$\neg\neg(A \rightarrow B)$".

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In the *Minute Logic* of 1902 Peirce explains that the ovals indeed combine a number of offices: "even when there are no lines of identity, they fulfill three distinct offices, and [...] in introducing these lines we have imposed upon them two more". They "fulfill all five with success" (MS 430, p. 53, 1902). The first function is a *truth-function*: the ovals mean negation. Second, they serve as collectional signs, and thus have a *collectional function*. Third, they can express all modes of logical combination when juxtaposition upon the Sheet of Assertion is taken to mean logical conjunction. The third office is a consequence of the first two. In the Beta part, they add two further offices. The fourth "is to indicate the order of succession of the identifications", namely dependent quantification (see footnote 23). The fifth, which he does not mention in this manuscript, is to cut continuous segments out of the lines of identity[25] in order to represent nonidentities between the individuals denoted by extremities of the line (MS 430, pp. 53–63, 1902; see Pietarinen 2015a).

The invention of such a multitask oval was an important achievement, showing a basic analytical truth: unlike what is the case with superfluous logical connectives which can be dispensed with if we seek a functionally analytical notation, *the signs of scope are not dispensable*. Quite the contrary, Peirce regarded such "collectional signs" as the most important signs of algebra:

> Treatises on algebra do not dwell upon the subject of the enclosure: they do not need to do so. They use it incessantly, however, and it merits the attention of the logician as the very type of an efficient algebraical tool,—the embodiment in purest form of the quintessential characteristic of mathematical thought, which consists in making individual objects out of relations. (MS 430, pp. 71–72, 1902)
>
> [A]long with the sign of negation we require one of those Collectional Signs,—"Klämmern", Schröder calls them,—such as in algebra are the parentheses, brackets, braces, the vinculum, period. The functional signs when more than single letters are attached to them belong to this class of Collectional Signs, which class is the most important,—it would, indeed, be strictly true to say they are the only indispensable,—signs of algebra. But the whole of the strict truth is, in this case, not important. What *is* important is to understand that the essential power of algebra is due exclusively to collectional signs. (MS 670, pp. 13–14, 1911)

It is strictly true that the collectional signs are the sole indispensable signs of algebra. For we can imagine an algebra in which the only signs are the parentheses. Alpha is such an algebra:

> But while the syntax of existential graphs thus needs both a sign of negation and an endless series of collectional signs, there is no reason why a single sign should not perfectly fulfill both these purposes. (Ibid.)

[25] A line of identity is defined by Peirce as "a Graph any replica of which, also called a line of identity, is a heavy line with two ends and without other topical singularity (such as a point of branching or a node), not in contact with any other sign except at its extremities. Otherwise, its shape and length are matters of indifference. All lines of identity are replicas of the same graph" (CP 4.416). A line asserts the numerical identity of the individuals denoted by its extremities. By being attached to Beta spots, lines represent predication. On the lines and more generally on Beta see Roberts 1973, pp. 47–63 and Pietarinen 2011.

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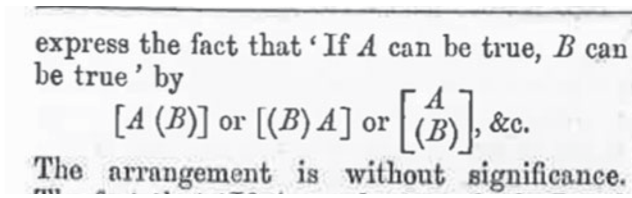
Alpha forces us to the correct analysis of propositional logic by pushing to the extreme the requirement of analyticity. Signs of logical connectives are superfluous if we already have a powerful collectional sign. In a sense, the step that Peirce took with the Graphs was that of *getting rid of signs of connectives*. In the entry "Symbolic logic" written for Baldwin's *Dictionary* (MS 1147) Peirce presents the graphs using parentheses and brackets instead of the ovals (Fig. 17 and 18). The shape of the signs is immaterial (Fig. 19):

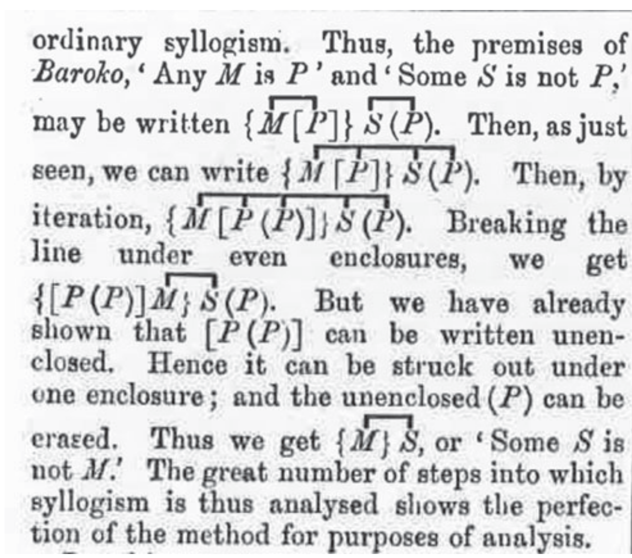


express the fact that 'If A can be true, B can be true' by

$$[A\,(B)] \text{ or } [(B)\,A] \text{ or } \begin{bmatrix} A \\ (B) \end{bmatrix}, \text{\&c.}$$

The arrangement is without significance.

Fig. 17. Alpha formulas in Baldwin's *Dictionary*.



ordinary syllogism. Thus, the premises of *Baroko*, 'Any M is P' and 'Some S is not P,' may be written $\{M[P]\}\ S\,(P)$. Then, as just seen, we can write $\{M[P]\}\ S\,(P)$. Then, by iteration, $\{M[P\,(P)]\}\ S\,(P)$. Breaking the line under even enclosures, we get $\{[P\,(P)]\,M\}\ S\,(P)$. But we have already shown that $[P\,(P)]$ can be written unenclosed. Hence it can be struck out under one enclosure; and the unenclosed (P) can be erased. Thus we get $\{M\}\ S$, or 'Some S is not M.' The great number of steps into which syllogism is thus analysed shows the perfection of the method for purposes of analysis.

Fig. 18. Beta formulas in Baldwin's *Dictionary*.

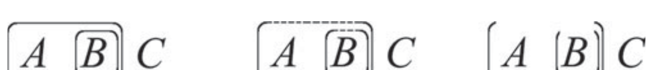


Fig. 19

Peirce even suggests that the *primary* office of ovals should be that of collectional signs, and *secondarily* that of signs of negation:

> The first office which the ovals fulfil is that of negation. [. . .] The second office of the ovals is that of associating the conjunctions of terms. [. . .] This is the office of parentheses in algebra. [. . .] The ovals are able to combine these offices because the last does not refer to single terms; so that we only have to use the ovals so as rightly to associate the elementary parts of the assertion we wish to express; and then, if any such parts

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> have the wrong *quality* (which is the technical term for the distinction between affirmative and negative), it only needs to have an oval drawn around it so as to enclose nothing else. (MS 430, pp. 54–56, 1902)

To appreciate this insight, it may help to compare Peirce's position to Wittgenstein's. In the *Tractatus*, Wittgenstein claimed that his *Grundgedanke* was that "the 'logical constants'do not represent. That the logic of the facts cannot be represented" (*Tr.* 4.0312); "there are no such things as'logical objects' or 'logical constants' (in the sense of Frege and Russell)" (*Tr.* 5.4). One argument in support of the *Grundgedanke* is that the behaviour of logical constants is akin to that of signs of punctuation:

> The apparently unimportant fact that the apparent relations like $\vee$ and $\supset$ need brackets—unlike real relations is of great importance. The use of brackets with these apparent primitive signs shows that these are not the real primitive signs; and nobody of course would believe that the brackets have meaning by themselves. (*Tr.* 5.461)
>
> Logical operation signs are punctuations [*Die logischen Operationszeichen sind Interpunktionen*] (*Tr.* 5.4611)

Cheung (1999) explains *Tr.* 5.4611 by considering the case of a propositional calculus with one single logical connective, the Sheffer stroke. If we write (p q) for p | q (neither p nor q), then the formula $p \rightarrow (q \vee r)$ may be written as follows:

(((pp)((qr)(qr)))((pp)((qr)(qr)))).

The idea is that in any context in which there is but one logical operation, the sign of the operation can be dispensed with as long as the order and scope of its application is clearly indicated. For example, in a system of arithmetic with only addition, the sign of addition "+" can be dispensed with; 7 + (4 + 6) would be written as (7 (4 6)), and the rule of associativity of addition as (x (y z)) = ((x y) z).[26] Such a notational device is applicable whenever we have a logical system with a single operation: in any such system, the sign of the truth-function only needs to occur as a sign of scope. Since in the *Tractatus* all other connectives are definable in terms of the N-operator, all signs of logical constants are, at bottom, *Interpunktionen*.

Milne (2013) has argued that in fact Wittgenstein goes too far when he asserts that signs for logical operations are punctuation marks. There is a difference of *function* here: while logical constants determine truth-conditions, the signs of scope determine *what it is* whose truth-conditions are to be determined.

> While neither punctuation marks nor logical constants stand for anything, both contribute to the sense expressed. In this they are alike. But *how* they contribute differs, and it is for this reason that it is misleading—I'm tempted to say, it is just plain wrong—to say that signs for logical operations *are* punctuation marks. (Milne 2013, p. 123)

[26] Cf. Milne 2013, pp. 121-122n127. Of course in this example the juxtaposition of numbers is the "implicit" symbol of addition, just as in EGs the juxtaposition of graphs is the "implicit" symbol of logical conjunction. In the notation based on the representation of p | q as (p q), juxtaposition means joint denial. In both cases, the parentheses delimit scope of the operation whose symbol is only implicit.

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This is a typical problem in the philosophy of notation. Let us put it in Peircean terms. Logical constants and punctuation marks have different notational functions: the former determines truth-conditions (i.e., is a truth-function), and the latter determines that whose truth-conditions are determined (i.e., is a collectional function). Yet it is possible to devise a notation in which one sign fulfills both functions, thus showing that "there is no reason why a single sign should not perfectly fulfill both these purposes" (MS 670, pp. 13–14, 1911). In such a notation as well as in whatever notation follows the same principle, the logical constants *are* punctuation marks that bear truth-functional meaning. *Tr.* 5.4611 by no means suggests regarding the logical constants as "void" signs of punctuation: rather, it suggests regarding the signs of punctuation as "filled" with truth-functional meaning. If we interpret *Tr.* 5.4611 as asserting that logical constants *can be expressed in the notation* by punctuation marks (as Cheung 1999 does), then Peirce's Alpha graphs may be said to constitute a convincing *example* of *Tr.* 5.4611.

It might however be argued that according to Peirce's explication of analysis in terms of homogeneous parts, those symbols that combine truth-function and collectional function (such as Peirce's sign of consequence and scroll) are not fully analytic, as they combine or synthetize, rather than analyse, heterogeneous elements. But to say that the sign of consequence or the scroll is not analytic because it does not separate truth-function and collectional function is to *presuppose* that these functions are in fact different heterogeneous elements that the notation has to keep apart. We might call this presupposition the *Atomistic Postulate* concerning notations.

However, on Peircean principles, the results of the analysis cannot be presupposed at the outset, because "to say that an analysis is logically correct only means that it will be so represented in such a system that is, as this will then be, stripped of all superfluities" (MS 296, p. 9, 1908). A system is more analytic than another if *ceteris paribus*, that is, provided that the former can represent everything that the latter represents, the former does it with fewer signs and conventions. It is one thing to recognize that notations exist in which the truth-function and the collectional function are represented by distinct symbols; quite another to "reify" the two functions and maintain that an analytic notation should keep them apart. The idea that the two functions are to be separately represented in the notation is a presupposition that nothing in the analysis can warrant, and which on the contrary directly derives from one's habitus of thinking in standard, linear notation, in which the signs of the operations and the signs of their scope are sharply distinct. But that truth-function and collectional function would be heterogeneous elements that ought to be separated in the notation is a presupposition that nothing in the analysis can warrant. On the contrary, the fact—exemplified well by Alpha—that those two functions can be expressed in the analysis by the same sign reveals that *they should not have been divorced in the first place*. We take this to be the real "notational" impact of Wittgenstein's remark that "[w]hen we have rightly introduced the logical signs, the sense of all their combination has been already introduced with them [. . .] We should then already have introduced the effect of all possible combinations of brackets" (*Tr.* 5.46). Of course some standard notations such as four operators plus parentheses or some other such conventions may be useful for some purposes and philosophical for some other reasons. But there must be some element of truth in the idea of a notation without parentheses that Peirce and Wittgenstein (and for instance Łukasiewicz when inventing the Polish notation) cultivated: the mere fact that it is possible to express the whole propositional calculus with one single symbol reveals essential truths about logic itself. Peirce's genius allowed him to see what a parentheses-free notation like in the Alpha teaches about logic: that truth-function and collectional function are *different* functions only in a notation that *represents* them as different. But

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since a notation is constructible in which no such difference is represented, this not only *does*, but *must* suffice for logic's analytic purposes.

§4. The Fallacy of Multiple Readings. The functional minimality of the scroll and its fulfilling at the same time the office of a collectional sign give us a better idea of what Peirce meant when claiming that EGs are as analytical as a notation can be. Alpha is itself a direct effect of the pursuit of analysis that motivated all of his studies upon formal logic. Strictly speaking, the decisive step towards a diagrammatic representation of the logic of propositions was taken already in 1886 and consisted in reunifying in one single sign (the sign of consequence) truth-functional and collectional meaning.

We saw above that in Alpha graphs it is impossible to write $A \prec B \prec C$ without thereby scribing either $(A \prec B) \prec C$ (Fig. 15) or $A \prec (B \prec C)$ (Fig. 16). This is a consequence of the notational reunification of truth-function and scope and a rejection of the Atomistic Postulate. Unlike what may happen in algebraic notations that have to introduce some further stipulative conventions, it is structurally impossible for an Alpha graph to represent nonequivalent propositions, such as in Figs. 15 and 16. Interestingly, an apparently similar distinction has been proposed by Shin (2002). Shin claims that what distinguishes Alpha (and EGs in general) from symbolic notation is that Alpha graphs can have multiple *equivalent* interpretations or "readings." The aim of the present section is to expose the confusion that lies at the bottom of this view.

According to Shin, the literature on Peirce's logic has "taken for granted that Peirce's Existential Graphs are diagrammatic and it is a different type of representation from his symbolic logical system" (Shin 2011, p. 334). Shin wants to replace such a take-for-granted distinction with a criterion that distinguishes diagrammatic from symbolic notations. We agree with her caution that, at this stage of our knowledge of how diagrams work, "it is desirable to search for linguistic and diagrammatic elements of a system, rather than to come up with necessary and sufficient conditions either for linguistic or diagrammatic systems in general" (Shin 2011, p. 334). So what Shin is in search of is an element that distinguishes EGs from those logical notations commonly termed symbolic. Such an element is, according to Shin, that an Alpha graph can have multiple equivalent readings while a formula of an ordinary symbolic language cannot. This element is taken to constitute the main difference between Alpha and symbolic languages for propositional logic.

> I demonstrate how differently a meaningful unit of each system can be read off. In order to prevent ambiguity, the semantic interpretation of a symbolic sentence requires its unique readability, and hence no possibility of multiple readings. On the other hand, in the case of EG, multiple readings do not generate ambiguity. (2002, p. 4)

The following is the proposed Multiple Reading Algorithm for Alpha graphs (Shin 2002, p. 74):

Multiple-Reading Algorithm Let X and Y be Alpha graphs

1. If X is an empty space, its translation is $\top$.
2. If X is a sentence letter, its translation is X.
3. If a translation of X is α, then a translation of [X] is $\neg\alpha$.
4. If a translation of X is α and a translation of Y is β, then
   (a) a translation of XY is $(\alpha \wedge \beta)$,
   (b) a translation od [XY] is $(\neg\alpha \vee \neg\beta)$,

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(c) a translation of [X[Y]] (i.e., scroll with X in the outside cut and Y in the inner cut) is $(\alpha \rightarrow \beta)$, and
(d) a translation of [[X][Y]] is $(\alpha \vee \beta)$.

Fig. 20

Take the Alpha graph in Fig. 20. According to Shin, it may be "carved up" in different ways (Fig. 21), corresponding to the steps 4(a)–(d) of the Multiple-Readings Algorithm:

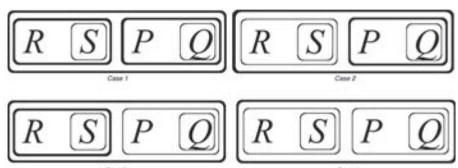

Fig. 21

For example, the following equivalent readings of the Alpha graph in Fig. 20 may be obtained that correspond to cases 1 and 2 in Fig. 21:

20a) [[X] [Y]], with X = (R ∧ ¬ S) and Y = (P ∧ ¬ Q) (Case 1): (R ∧ ¬ S) ∨ (P ∧ ¬ Q)
20b) [X [Y]], with X = ¬(R ∧ ¬S) and Y = (P ∧ ¬Q) (Case 2): ¬(R ∧ ¬S) → (P ∧ ¬ Q)

According to Shin, ordinary symbolic languages cannot have such multiple readings. As a simpler example, take the Alpha graph in Fig. 22 and the sentences (22a)–(22c).

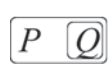

Fig. 22

22a) ¬ P ∨ Q 22b) P → Q 22c) ¬ (P ∧ ¬ Q)

According to the Multiple-Readings Algorithm, Fig. 22 can be "read off" indifferently as (22a)–(22c): 22a is obtained by clause 4b preceded by clause 3; 22b by clause 4c; 22c by 4a preceded by two applications of clause 3. One and the same diagram can be read off in these different ways, while the contrary is not true: (22a)–(22c) are *different* formulas expressing the same fact (the same truth-table). Being different, they require proofs of such logical equivalence. No such proof is needed in EGs, simply because there are no such *different* expressions: there is only one graph in Fig. 22.

There are two assumptions operative here.

Condition 1: "Reading off" an Alpha graph corresponds to translating it into a formula in ordinary symbolic notation.

According to this condition, EGs are the object or source-language and the symbolic language is the target-language into which the first is translated.

Condition 2: In order to generate multiple readings the target-language must have *a richer logical vocabulary* than the source-language.

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For had the target-language only conjunction (∧) and negation (¬), being thus a conjunctive-negative fragment of the full language of propositional logic, then (22a)–(22c) above would be indistinguishable from one another (cf. Fig. 23 and Fig. 24).

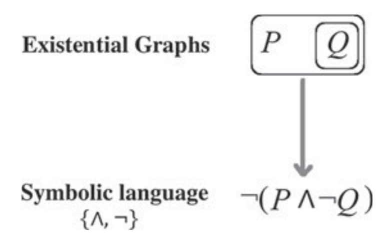

Fig. 23. Single reading of an Alpha graph in a Symbolic Language with{∧, ¬ }.

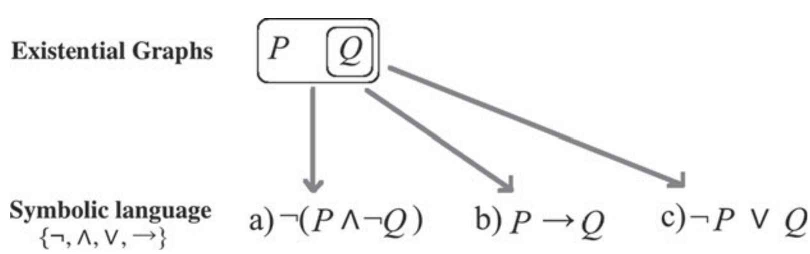

Fig. 24. Multiple readings of an Alpha graph in a Symbolic Language with {¬ , ∧, ∨, →}.

Shin is aware of this latter condition, as she tells that "EG's Alpha system has fewer syntactic devices than propositional languages, but without suffering from the inconvenience of a symbolic system with only two connectives" (Shin 2002, p. 97; cf. 2011, p. 340).

Therefore, the only possible comparison is between Alpha and a *given* symbolic language Ls as expressive as Alpha (say, with ∧ and ¬). A comparison cannot be done otherwise, because the comparison has anyway to be made with *some* specific language Ls. From such a comparison it should result that there is something in Alpha not present in Ls. Accepting this method, as we should if we want to differentiate Alpha from symbolic notations, the problem can then be reformulated according to Conditions 1 and 2. The difference between the language of Alpha graphs and a given symbolic language Ls with the set of connectives {∧, ¬ } is that Alpha graphs have multiple readings in another target-language Lt with at least {¬ , ∧, ∨, →}, while Ls has no such multiple readings.[27]

But now, if Conditions 1 and 2 are enough to be able to speak of "multiple readings" in a target language Lt of a formula of the source language Ls, then also symbolic formulas can have multiple readings. To see this, we may provide an algorithm to translate a sentence of Ls with {¬ , ∧} into a sentence of Lt with {¬ , ∧, ∨, →} as follows:

[27] Given an Ls with the set of connectives {∧, ¬ } one may recursively generate indefinitely many "multiple equivalent readings" of one single formula: $\neg P = \neg\neg\neg P = \neg\neg\neg\neg\neg P$ and so on. One can likewise recursively produce "multiple equivalent readings" of one single Alpha graph by adding double cuts. However, such "multiple readings" are made *within* one given language, while Shin's multiple readings are *translations* between *languages*. Shin's point concerns such *external*, not *internal*, multiple readings.

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3. If a translation of X is α, then a translation of $\neg$ X is $\neg\,\alpha$.
4. If a translation of X is α and a translation of T is β, then
   (a) a translation of X $\wedge$ Y is $(\alpha \wedge \beta)$,
   (b) a translation of $\neg$ (X $\wedge$ Y) is $(\neg\,\alpha \vee \neg\,\beta)$,
   (c) a translation of $\neg$ (X $\wedge\, \neg$ Y) is $(\alpha \rightarrow \beta)$,
   (d) a translation of $\neg$ ($\neg$ X $\wedge\, \neg$ Y) is $(\alpha \vee \beta)$.

That is, in order to translate (1s) of Ls we can take different paths.

(1s) $\neg\,[(\neg\, A \wedge \neg\, B) \wedge \neg\,(C \wedge \neg\, D)]$.

We can for example "read off" the whole formula according to clause 4(c), with X = $(\neg\, A \wedge \neg\, B)$ and Y = $(C \wedge \neg\, D)$, as

(1t) $(\neg\, A \wedge \neg\, B) \rightarrow (C \wedge \neg\, D)$.

Likewise, we can "read off" the whole formula according to clause 4(b), with X = $(\neg\, A \wedge \neg\, B)$ and Y = $\neg\,(C \wedge \neg\, D)$, as

(2t) $\neg\,(A \wedge \neg\, B) \vee \neg\,[\neg\,(C \wedge \neg\, D)]$.

There is nothing surprising here. (1t) and (2t) are multiple readings in Lt of (1s) in Ls. To return to our simple example, the sentence "$P \rightarrow Q$" of Ls with $\{\rightarrow\}$ can be multiply read either as "$\neg\, P \vee Q$", "$P\rightarrow Q$", or "$\neg\,(P\wedge\neg\, Q)$" in Lt with $\{\neg, \wedge, \vee, \rightarrow\}$, as shown in Fig. 25:

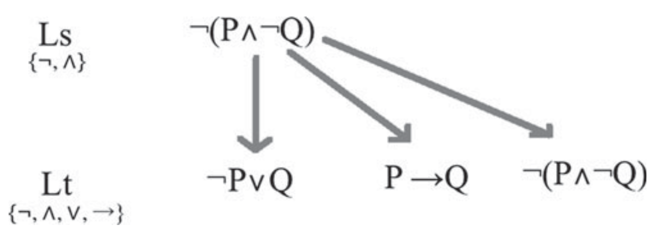

Fig. 25. Multiple readings of a formula of Ls $\{\neg, \wedge\}$ in Lt $\{\neg, \wedge, \vee, \rightarrow\}$.

Why should not these be multiple readings in the sense in which Alpha is claimed to have multiple readings? Shin argues that in the case of symbolic languages, we do not "read off" a formula in multiple ways. Rather, we *extend* the vocabulary of our language in order to provide multiple readings of its formulas. With Alpha, on the contrary, no extension of the vocabulary is needed in order to have multiple readings.

But if we assume that any language is specified by its logical and nonlogical vocabularies, then any addition to the vocabulary of a given language necessarily produces a *different* language.[28] Strictly speaking, no language can extend its own notation without producing a *different* (albeit not necessarily more expressive) language. But then, on Shin's account, when a language Ls has multiple readings in another language Lt, but Lt is, *or may be considered as*, an extension (again, not necessarily a proper or conservative extension) of Ls, then the phenomenon in question is not one of multiple readings.

The idea of extension implies the idea of a *family* of languages, some more extended than others. In taking Lt as an extension of Ls, as Shin does, one can only mean that we are taking them to belong to the same genus of languages, of which some are extensions

[28] Cf. e.g. Church 1956, p. 48, p. 48n111.

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of others in having some additional logical or nonlogical constants in their vocabulary. For what other reason could there be to distinguish Alpha's multiple readings in Lt from Ls' multiple readings in Lt, if not that this latter is performed within a *unique language* (say L, to which Ls and Lt belong)? Shin's illusion is created by the fact that the object-language Ls and the target language Lt, although two distinct languages, are both *symbolic*. But it is one thing to say that two languages are *both symbolic*, and quite another to say that they are the *same* language.

There is thus a third condition tacitly operative in Shin's multiple-readings argument:

> ***Condition 3***: A formula can have multiple readings in a symbolic target-language only if it is not itself symbolic.

Condition 3 is a definitory condition, and is implicitly assumed in Shin's argument. It merely states that when we multiply translate a formula of a source language into a target language, and both languages belong to the same family (both are symbolic, both are diagrammatic), this is *not called* multiple reading. Condition 3, being definitory, is virtually harmless. The problem is that it renders Shin's argument circular. For Shin's aim was to find a feature that would prove that Alpha is not symbolic, and in general one that distinguishes nonsymbolic from symbolic notations. She thinks she has found in that the former may have multiple readings while the latter cannot. Unfortunately, the notion of multiple readings is itself defined in terms of such distinction: when a symbolic language has multiple translations in another, possibly more expressive symbolic language, we do not call these translations "multiple readings". As it is apparent, there is a *circulus in definiendo* here: the differentiation between the graphical and the symbolic is exactly what the possibility of multiple readings is supposed to provide. But this very distinction is employed to define multiple readings.

In sum, Shin's argument is that:

1) What distinguishes Alpha from symbolic notations is that Alpha is capable of multiple readings while symbolic notations are not.
2) A formula of a given language Ls has multiple readings in a symbolic target-language Lt (richer in its connectives) if it has multiple equivalent translations in Lt without Lt being, or being capable of being, an extension of Ls.

This argument is circular. The refutation is: When Lt is an extension of Ls, Lt, and Ls belong to the same family of languages. Since (2) states that Lt is symbolic, Ls must also be symbolic. But then, according to Shin, a formula can have multiple readings only if it is *not* symbolic, for no symbolic formula can have multiple readings. Therefore, an Alpha graph is nonsymbolic because it can have multiple symbolic readings; but it can have multiple symbolic readings only because it is nonsymbolic. Alpha graphs are differentiated from symbolic notations by saying that they have a character that only nonsymbolic notations have.

Shin in fact does not explain *why* symbolic notations cannot have what she calls multiple readings. What she says is that they do have multiple *nonequivalent* readings, and then suggests that this is the reason why they cannot have multiple *equivalent* readings. But this amounts to confounding the two:

> [A] symbolic system is very careful to prevent multiple readings of a formula, since it would yield ambiguity [...] To secure unique readability, parentheses or prefix notations have been adopted so that one and only one way of parcelling up a sentence is available. (Shin 2002, p. 79)

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> I demonstrate a fundamentally different way that sentences and graphs are read off. In the case of sentences, unique readability should be observed to prevent ambiguity in a system, but graphs can be read off in many different ways without causing ambiguity. (2011, p. 335)

This way of differentiating between sentential and graphical assertions is exceedingly odd. Both assertions—that in symbolic languages unique readability should be observed to prevent ambiguity in the system, and that Peirce's graphs can have multiple readings without causing ambiguities—are certainly true. But on the one hand, also the formulas of a symbolic language may have multiple readings in another language, as shown in Fig. 25 above. On the other hand, Shin's use of "ambiguity" is itself ambiguous: by proposing the parallelism between unique readability of symbolic languages and multiple readability of diagrammatic ones, she conflates two senses of unique readability. A sentence in symbolic language may be *syntactically ambiguous*, and in order to prevent such ambiguity parentheses and other conventions are commonly introduced. However, Shin nowhere explains how the requirement of syntactical unique readability of symbolic sentences could be the *cause* of their not allowing multiple readings. Rather, the parallel suggested between unique-readability of sentences and multiple-readability of graphs gives the impression that, for want of a parallel multiple-reading phenomenon in symbolic language, no better solution has been found than the idea that both sentences and graphs have multiple readings, but while the multiple readings of graphs are all logically equivalent, the multiple readings of sentences are not. This is an unfortunate way of putting the matter, for if a phenomenon (multiple readings) gives two distinct results (equivalent and nonequivalent sentences), then it is more prudent to suspect that *we are in the presence of two distinct phenomena*. It would be far better to say, in the first place, that EGs do not have the sort of syntactical ambiguity that sentences of symbolic language can have (that is, nonequivalent multiple readings). *This* is the discrepancy to be explained. That EGs also have multiple *equivalent* readings is a distinct claim, and Shin does nothing to unpack her misleading suggestion that unique readability of symbolic sentences is the *cause* of their having only nonequivalent multiple readings.

We saw above that the real reason why Alpha graphs cannot express a syntactically ambiguous formula is that the Alpha graphs consist of signs that fulfil at once the office of truth-functional operators and the office of scope indicators. It is impossible in Alpha graphs to scribe a graph without indicating its compositional allocation. The interesting discrepancy is therefore between a notation in which it is possible to leave scope of a logical constant unexpressed and a notation in which this is not possible. This, if any, is a notational difference between Alpha and a corresponding symbolic notation.

In the terms of Shimojima's conceptual framework (Shimojima 1996a, 1996b), the convention of combining negation and scope indicator in one single sign functions as a *structural constraint* that excludes a false or ambiguous analysis of a given sentence. Such a structural constraint provides EGs with a special variety of *content specificity* (see Shimojima 1996a, Chapter 3): they cannot represent certain information (truth-functional operation) without adding certain other information (scope of the operation). The symbolic notation, on the contrary, has no such structural constraint and therefore is not provided with such a peculiar variety of content specificity. A symbolic notation can represent certain information (truth-functional operation) without adding certain other information (its scope). The absence of such structural constraint is what makes it possible to have syntactically ambiguous sentences in symbolic notation. The presence of such structural

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constraint in EGs, in contrast, renders the system nonambiguous; or perhaps better put, syntactic ambiguity ceases to be an applicable property.[29]

However, a charitable reading of Shin's proposal may be that while the formulas of a symbolic language Ls with $\{\neg, \wedge\}$ can be multiply read in a richer symbolic language Lt with $\{\neg, \wedge, \vee, \rightarrow\}$ which is a proper extension of the former, graphs in the language of Alpha graphs cannot be multiply read in a richer Alpha language, because *there is no such a thing as a richer Alpha language*. In other words, Shin's argument might be taken to imply that *the language of the Alpha system is not extendible* (that is, it cannot have more connectives than it actually has), without increasing its expressivity. This might well be true, and one might try to connect this fact with the discrepancy indicated above along the following lines: since *all* of Alpha's truth-functional signs also express scope of the truth-function, the system is in this sense *sui generis* or saturated and no further extension of it is thereby possible. In this perspective, only those systems in which the truth-functions and the collectional functions would be notationally separated would be capable of proper extension of their vocabulary. In our terms: only incompletely or nonnotationally analytic systems are extendible. The discrepancy that while symbolic notations are extendible Alpha graphs are not, would then only be the *superficial effect of those deeper structural constraints*.

Shin however does not suggest anything like this. She *assumes* the nonextendibility of Alpha without further notice and infers from that assumption that nonextendible languages have multiple readings in some more extended languages. But lest the nonextendibility of Alpha is explained in terms of some yet more fundamental feature, nothing but the notational synthesis of truth function and collectional function differentiates Alpha from any other symbolic language (like Ls above) that is *de facto* as rich as Alpha in terms of its logical vocabulary.

§5. Conclusion. We explained why Peirce considered the Alpha part of the theory of Existential Graphs to meet the requirements of the most perfect notation for the analysis of the propositional calculus. We have discussed Peirce's idea that the relation of illation is the primitive relation of logic and we have shown that this idea constitutes the fundamental motive of Peirce's philosophy of notation, both algebraic and graphical. We explained how in his algebras and graphs Peirce arrived at a unifying notation for logical constants that represents both truth-function and scope, thus employing the least amount of logical vocabulary. We also showed that Shin's claim that the possibility of multiple readings is what makes Alpha a nonsymbolic notation is viciously circular. What differentiates Alpha from typical symbolic and equivalently expressive notations is that Alpha is constituted by a minimal notation obtained by a notational reunification of the truth-function and collectional function.

[29] Shimojima further explains that a structural constraint may be "nomic" or "stipulative": "We call a structural constraint 'purely nomic' if it holds on a set of representations without needing any stipulation on our part. We call a constraint 'purely stipulative' if it holds on a set of representations purely in virtue of the syntactic stipulations we make for the representations" (1996a, p. 64). The structural constraint that obliges us to express scope together with operation is plainly a "stipulative" constraint, for it depends on the conventions adopted in our syntax, not on the properties of space (Pietarinen & Bellucci 2015a).

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