OBSERVATIONAL SELECTION IN SPECTROSCOPIC BINARY ECCENTRICITIES

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ABSTRACT

The discovery-and-identification probability for different shaped orbits of spectroscopic binary stars is estimated. The eccentricity distribution observed in the sample of ~1000 binaries with known orbits and appearing as strongly peaked toward e=0 is corrected for observational selection effects. The resulting e-distribution seems to be flat for e in the range ~0.05-0.6 with some excess of circular (or almost circular) orbits and a deficiency of orbits with $e \gtrsim 0.6$.

1. INTRODUCTION

Statistical analysis of the data compiled in the "Seventh Catalogue of the Orbital Elements of Spectroscopic Binary Systems" (Batten et al. 1978, hereinafter referred to as the Catalogue) demonstrates the serious bias of all these data mostly by observational selection effects (Staniucha 1979). The observed distribution of orbital eccentricity for 978 binaries is strongly peaked toward e=0 (cf. Fig.4 in Staniucha 1979). Almost 28% of all systems have circular orbits and 55% have $e < 0.1$. A similar feature may be found probably for all close binaries, but the problem is whether it is real.

An attempt at extracting the real e-distribution from the observed one was made thirty years ago by Scott (1951). She analysed the selective identifiability of spectroscopic binaries and although she derived several very useful formulae, the rectification of the observed distribution did not give any convincing result.

The purpose of the present paper is to answer two questions: (1) How do the observational selection effects contribute to the observed eccentricity distribution? and then (2) What does the real distribution look like? From this view we estimate the probability

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of the spectroscopic detection-and-identification of orbits with different elements (Sec. 2). These estimates are then used (Sec. 3) to obtain the real eccentricity distribution from the observed one. Concluding remarks and discussion are given in Section 4.

2. OBSERVATIONAL SELECTION EFFECTS

For any observed spectroscopic binary system two conditions have to be fulfiled to place that system and its orbit in the Catalogue: (1) detection of its radial velocity variations, and (2) determination of the period for this variability and of the other orbital elements. Both steps may end with success only with a certain probability, the value of which depends on the parameters of the binary itself, on the available instruments, weather conditions and on the number of the observed spectra.

To estimate the probability, each step is accomplished with, we adopt (after Scott 1951) the following approach. First, we assume the moments of observations chosen at random in time and thus uniformly and independently distributed throughout the orbital period P. This assumption will be sufficiently good if we have to do with periods of reasonable length. We disregard therefore all difficulties with periods of a day, a month, a year etc., as well as with very long periods and even with the very short ones (comparable in length with the time of single observation). Second, we adopt the errors in radial velocity without regard to their origin as following a normal distribution with zero-mean and known standard deviation σ. Then, assuming the star as having variable radial velocity whenever

$$\frac{1}{\sigma^2} \sum_{j=1}^{n} \left(V_j - \bar{V}\right)^2 \geqslant \chi^2_\alpha \qquad (1)$$

(where $\bar{V}$ is the mean of the observed radial velocities, V_j, n is the number of observations and χ^2_α is the value of the classical χ^2 with the significance level α and n-1 degrees of freedom), we can obtain the desired formula for the probability of variable radial velocity detection in the form $\beta_1 = \beta_1(x, e, \omega, n)$ (Scott 1951 gives the formula as well as other details). Here $x = (\sigma/K)(1-e^2)^{-1/2}$, where K is the semi-amplitude of radial velocity and the second factor was introduced to release x from implicit dependence on eccentricity.

The problem of the probability, we accomplish the second step with, cannot be solved in an analytic way with mathematical strictness. Since we know quite many stars showing radial velocity variations, but having no derived orbit or even no period found for those variations (few examples are seen in the data of Abt and Levy 1976, 1978), we decided not to neglect this point and to give at least some, more or less rough estimate for that probability.

Suppose, a star's radial velocities are equal to $V(t)$ and $V(t^*)$ at times t and t^* respectively. The difference between them will be observable only if

$$|V(t) - V(t^*)| \geqslant V_{lim}, \tag{2}$$

where V_{lim} is the minimum change in radial velocity still detectable, i.e. it has practically the same meaning as the above introduced σ, thus we are using both parameters equivalently throughout the paper. The longer is the fraction of the orbital period when Eq.(2) is fulfiled (with the value of t^* fixed and t running from zero to P), the larger is the probability we will be able to obtain a clear velocity curve and then good estimates of all orbital elements. Defining therefore

$$\beta^*(t^*) = \int_{|V(t)-V(t^*)| \geqslant V_{lim}} dt \cdot \left(\int_0^P dt \right)^{-1} \tag{3}$$

and then integrating β^* over t^* from 0 to P we obtain an estimation for the probability of the orbital elements determination:

$$\beta_2(x, e, \omega) = \frac{1}{P} \int_0^P \beta^*(t^*)\, dt^*, \tag{4}$$

where the notation is the same as previously used one. Also the other expressions were tested for the function β^* (e.g. dt in Eq. 3 was weighted by the relative radial velocity changes) but since all of them show practically the same behaviour and give the same final result, we chose the simpl est form of Eq.(3). Few examples of the β_2-function behaviour are shown in Fig. 1. For reasons of symmetry only the values of ω from the first quadrant were presented. The dependence on x though not given is also very strong: β_2 steeply decreases with increasing x (e.g. β_2 = 1.0, 0.5, 0.25 and 0.13 for x = 0.0, 0.5, 1.0 and 1.5 respectively when e=0.8 and $\omega=45^\circ$).

We should notice however that β_2 gives in fact the probability of the determination of period, since in a predominant majority of cases we can always estimate all the other orbital parameters once we know the value of period. Therefore the evaluation of the accuracy, the elements are determined with, becomes essential. A good measure for this accuracy is given by

$$\beta_3 = \beta_3 (K', e', \omega' / K, e, \omega), \tag{5}$$

which is the probability density of the estimates K', e', ω' of elements with their true values K, e, ω. Analytic treatment of β_3 meets many difficulties, however some numerical results in that subject were already obtained. Namely, Scott (1951), using computer

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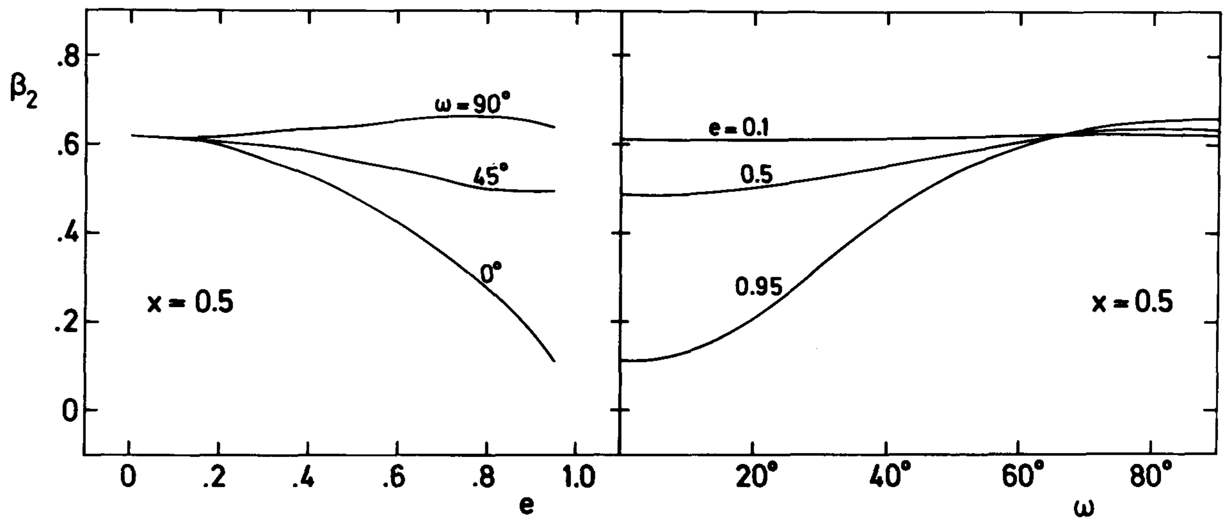

Figure 1. Dependence of the probability β_2 on the eccentricity e and the longitude of periastron ω for x=0.5.

generated observations, estimated the β_3-function for the case of $K/\sigma=2$, i.e. $x = 0.5/(1-e^2)^{1/2}$. Although the estimation is a little rough (the elements were determined by a visual comparison of the data with a grid of synthetic velocity curves), it shows (cf. Fig. 9 in her paper) that even in such extreme case - where the semi-amplitude K is only twice the standard error σ - the circular orbits might be distorted in very small percentage of binaries only (the probability density highly peaks at e=0 and e'=0). As we are going to the higher eccentricities the probability density function becomes wider and flatter, but since at the same time the x value increases, the probability, that we detect such a binary and then we determine its elements, very fast becomes small enough to allow us to argue that the neglection of all effects connected with β_3 will not enter any serious bias into the present investigation. We do not have perhaps to add, that the derivation of the formula for β_3 might be of great importance in the rectification of distributions for other orbital elements (like e.g. ω).

With the β_1, β_2 and β_3 functions already defined we can answer the first question of Introduction: the influence of the observational selection effects on the distribution of the elements of spectroscopic binaries can be represented by the formula:

$$\mathcal{O}(K', e', \omega') = \frac{\iiint \beta_1 \beta_2 \beta_3 \mathcal{R}(K, e, \omega) \, dK \, de \, d\omega}{\iiint \beta_1 \beta_2 \, \mathcal{R}(K, e, \omega) \, dK \, de \, d\omega}, \quad (6)$$

where the observed and the real distributions are denoted by $\mathcal{O}$ and $\mathcal{R}$ respectively and the range of integration is over the extreme limits of each variable. Moreover, if we put

$$\beta_3 = \delta(K' - K) \cdot \delta(e' - e) \cdot \delta(\omega' - \omega), \quad (7)$$

we obtain

$$\mathcal{O}(K, e, \omega) = \beta_1 \cdot \beta_2 \cdot \mathcal{R}(K, e, \omega), \quad (8)$$

where the constant factor was omitted.

3. INCOMPLETENESS CALCULATIONS

To apply the procedure from the previous section to the correction of the observed e-distribution for the observarional selection effects we need, apart from e and ω listed in the Catalogue, the values of V_{lim} ($\equiv \sigma$) for every system in our sample. Abt and Levy (1976, 1978) assumed fixed values of V_{lim} for stars with similar spectral types. In general, V_{lim} should be a function of the star's temperature, with a minimum for middle spectral types, being a compromise between broad lines in hot stars and the predominance of many

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metallic lines in cold ones. The good approximation for the V_{lim} values can be given by the minimum semi-amplitudes of radial velocity, K_{min} found for different spectral type ranges in the Catalogue. Fig. 2 presents log K_{min}-spectral type relation for single (SB1) and double line (SB2) systems treated separately. As we might expect, K_{min} values

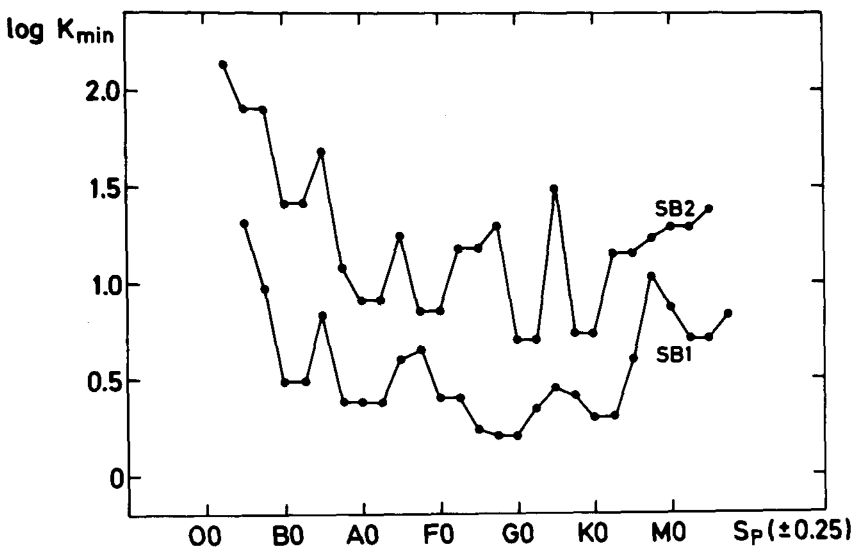

Figure 2. Common logarithm of the minimum semi-amplitude of radial velocity as a function of the spectral type of primary. Spectral type bins overlap each other by one-half.

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are in the average about eight times larger for SB2 systems than for SB1s, since the detection of two separate lines needs relatively larger values of K compared to the single line changing its position. Very similar results have been recently found by Kraitcheva et al. (1979), though they used the mass of primary component instead of spectral type as a parameter.

Two curves giving lower boundaries were fitted to the points in Fig. 2 and the values of V_{lim} and x were computed for almost all (i.e. 933) binaries in the Catalogue. Only systems with unknown spectral types or those with Wolf-Rayet star or subdwarf as primary were omitted. Also all binaries originally observed as visual ones were rejected, since the probability of their detection is slightly different (this was confirmed by our calculations). Probabilities $\beta_1(x, e, \omega, n)$ and $\beta_2(x, e, \omega)$ were then determined with the number of observed spectra, n, usually not given in catalogues, assumed to be uniformely distributed between 2 and 15 (several runs have been made to avoid some casuality which might arise from the random sampling of n).

The correction for the incompleteness due to observational selection was made for each 0.1-wide eccentricity bin separately. Circular orbits were also taken as a separate group. The real number of systems in each bin, N_r was obtained by simple summation:

$$N_r = \sum_{j=1}^{N} \frac{1}{\beta_1 \cdot \beta_2}, \qquad (9)$$

where N is the observed number of binaries in that e-bin. The errors, δN_r were computed from:

$$(\delta N_r)^2 = \sum_{j=1}^{N} \frac{\left(\frac{\delta\beta_1}{\beta_1}\right)^2 + \left(\frac{\delta\beta_2}{\beta_2}\right)^2}{(\beta_1\,\beta_2)^2} + (N_r\,\delta N/N)^2, \qquad (10)$$

where δN was taken according to the Poisson distribution as square root of N. Different values were tried for $\delta\beta_1$ and $\delta\beta_2$ which are not known, but since the sensitivity of the resulting δN_r-error was rather weak to both quantities they were finally fixed and both taken equal to 0.05.

The eccentricity distribution corrected for the observational selection effects can be characterised as follows:
(1) Large fraction (about one fourth) of all systems has circular orbits. No significant difference in that group of binaries was therefore noticed.
(2) The distribution for eccentric orbits seems to be uniform for e in the range ~0.05-0.6 (see Fig. 3). Some excess of very small eccentricities as well as a clear deficiency of highly eccentric

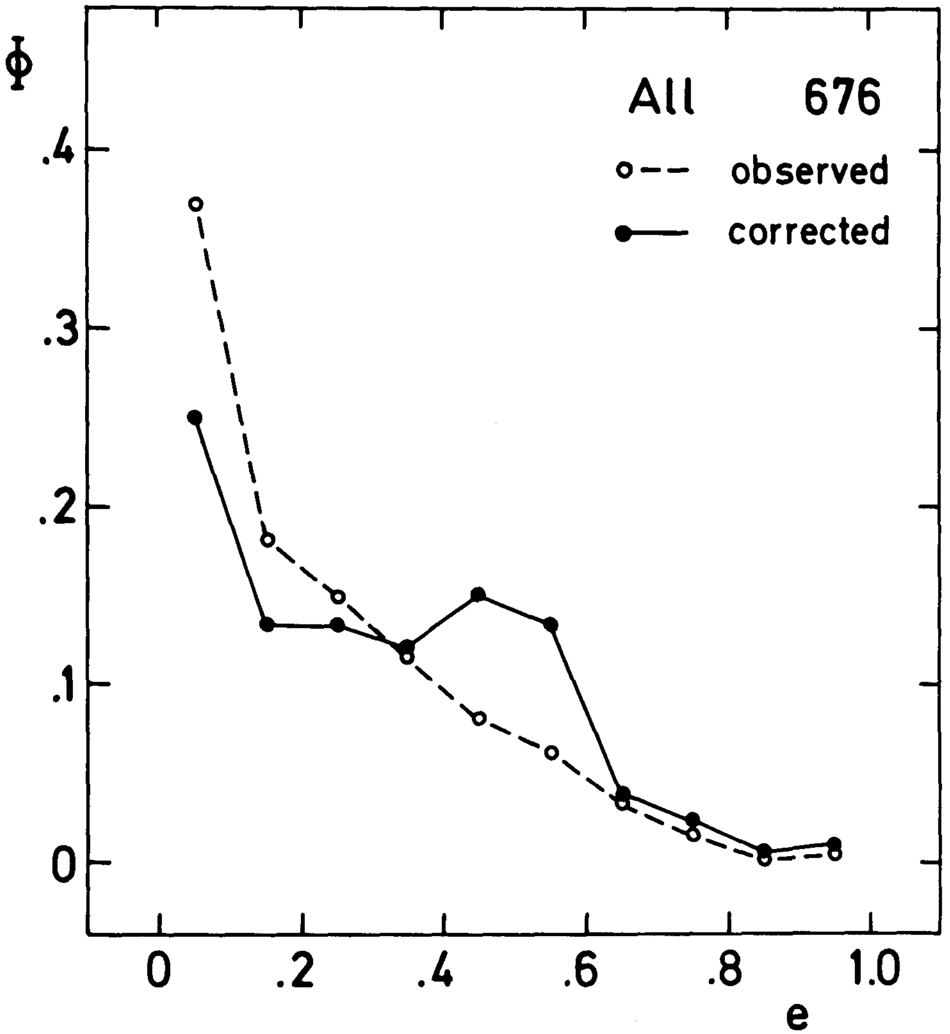

Figure 3. Observed and corrected eccentricity distributions for all non-circular orbits. The number of systems in the observed sample is given in the upper right corner.

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orbits is seen. Although in Fig. 3 the error bars were not plotted to avoid ambiguity, we should stress the statistical significance of both jumps: at $e \cong 0.6$ and e close to zero, the latter being even more pronounced when the circular orbits are added to the sample. The flat part, on the other hand, fits well within the one sigma-wide band marked by the data points.
(3) If the samples of SB1 and SB2 systems are treated separately the e-distribution in each sample remains almost the same as for all systems together (cf. Fig. 4).

Some additional attention may be called to the systems with high quality orbits, i.e. those with quality classes a and b according to Batten et al. (1978). The corrected e-distribution in this group is intermediate between the two distributions presented in Fig. 3, but since these are the systems having (to some extent by assumption) the highest probabilities of detection and identification, their usefulness in this case is restricted to supporting our results for the total sample.

4. CONCLUDING REMARKS

The results from our rectification procedure show that the certain number of binaries (at least 25%) have circular or almost circular orbits, while in the group of remaining systems the eccentricity is distributed more or less uniformly, with the exception of very eccentric orbits ($e \gtrsim 0.6$) which are evidently unfrequent. Every trial to explain such result must be of necessity only speculative, since our knowledge on the mechanisms of binary star creation and/or evolution is still very uncertain.

If we assume after Popova et al. (1981, this meeting) that most of SB2s are "systems prior to mass exchange" (what probably makes the period-eccentricity correlation existence due to observational selection effects) we arrive at the conclusion that the eccentricity distribution for SB2s, shown in Fig. 4, is (1) very close to the primordial one, or (2) defined mainly by the dynamical evolution effects (of what sort?), or (3) both. To clarify this point some support from a theory is needed.

If the orbital distribution in phase space is a function of binary energy only, then the expected eccentricity distribution is $\Phi(e) = 2e$ (Ambartsumian 1937). Either numerical or theoretical investigations show that it occurs especially in star clusters which are relaxed (see e.g. Heggie 1975). But it has no place for the field stars (which make about 96% of all Catalogue entries), since any significant encounter between them is almost impossible. On the other hand, the escape rates of binaries in clusters as well as the distribution of eccentricity among escapers are indeed unknown. Binaries which result from the dynamical decay of small (five bodies) unstable stellar systems were investigated by Harrington (1976). He found the

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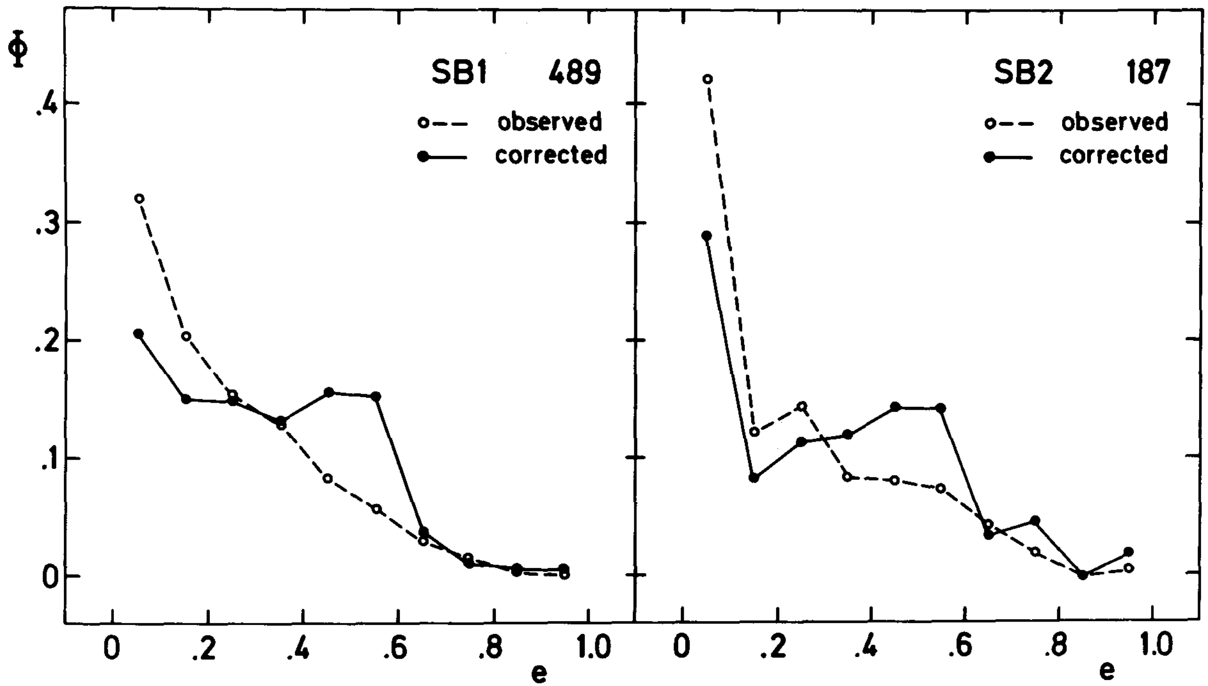

Figure 4. The same as in Fig. 3 but for SB1s (left panel) and SB2s (right panel) separately.

resulting eccentricity distribution very close to the linear Ambartsumian function, but a somewhat flatter at high ($\gtrsim 0.6$) eccentricities. Also a fission of single rotating protostar was simulated (Lucy 1977), but no information on the resulting eccentricities was obtained.

Therefore, as one can see , no prediction to the eccentricity distribution for field (close) binaries can be given. Also the knowledge of mechanisms, the dynamics of binary stars is governed by, is rather poor. Further mainly theoretical developement in those subjects might put a new light on the real e-distribution presently obtained.

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