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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Graham Leach-Krouse (Managing Editor), Albert Atserias, Mark van Atten, Clinton Conley, Johanna Franklin, Dugald Macpherson, Antonio Montalbán, Valeria de Paiva, Christian Retoré, Marion Scheepers, and Nam Trang. Authors and publishers are requested to send, for review, copies of books to *ASL, Department of Mathematics, University of Connecticut, 341 Mansfield Road, U-1009, Storrs, CT 06269-1009, USA.*

JOHNATHAN KIRBY. *An Invitation to Model Theory*. Cambridge University Press, Cambridge, UK, 2019, xiv + 182 pp.

The author himself describes this book as a companion reference for an undergraduate or master's-level course in model theory, which has evolved out of courses taught by himself in different institutions. Since there are several classical references in English aimed for an undergraduate or graduate course in model theory, such as Hodges's *Model Theory*, Cambridge University Press, Encyclopedia of Mathematics and its Applications 42, or Poizat's *A Course in Model Theory*, Springer, Universitext, among others, I would like to compare Kirby's volume to two popular books for a course in model theory, which are often referenced to even in research papers: one of them is Marker's *Model Theory: An Introduction*, Springer, Graduate Texts in Mathematics 217, and the second one, which is in my personal opinion more advanced, is Tent and Ziegler's *A Course in Model Theory*, Cambridge University Press, Lecture Notes in Logic 40, which contains some chapters which lie beyond the scope of an introductory course in model theory.

Kirby describes his book as an *invitation* and indeed, its content and the focus are different from those of the two aforementioned classical references. First of all, in contrast to the books of Marker and Tent-Ziegler, the chapters in Kirby's volume are surprisingly short and structured in such a way that every chapter could be indeed covered within the length of a 1-hour course (or in two sessions for some of the chapters). Second, whilst both Marker and Tent-Ziegler present Morley's theorem as a way to introduce the main features of stability (or in the case of Tent-Ziegler even simplicity), which is at the core of the so-called geometric model theory, Kirby decides to stay at a level more accessible for those with an undergraduate background. Some fundamental notions of geometric model theory, particularly o-minimality, appear in Chapter 22, but this is not meant as an attempt to present all of the relevant topics in this area, but rather to provide an accessible introduction, referencing instead to Marker's book or van den Dries's *Tame Topology and O-Minimal Structures*, London Mathematical Society, Lecture Note Series 248, for further reading. At the end of every chapter, there is a full set of exercises of various levels of difficulty for either individual work or possibly as a companion to the lecturers using this book as a manual. The selection of the exercises shows a sensible choice in order for the readers to get more familiar with the notions presented in the chapter, for most of the exercises are doable but not trivial.

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Some of the exercises (such as the proof of Schröder–Bernstein's Theorem in Chapter 10) are not immediate for the first-time reader, but I assume that they are included as a sort of complement or to be done during the lecture.

Regarding the use of this book as a manual for instructors, Kirby himself suggests in the preface different combinations of the chapters according to the total number of teaching hours as well as to the prerequisites of the targeted audience of students. Part I comprises Chapters 1–5 and introduces the basic syntactic and semantic notions of formulae and structures. Part II, comprising Chapters 6–11, focuses on theories. Whilst Kirby decides not to introduce formal deduction and thus it does not present a proof of Gödel's completeness theorem, it does present the compactness theorem, which is one of the main tools in model theory (and yet one of the results most difficult to fully grasp and internalise at the beginning). Chapter 11 contains an adaptation of Henkin's method (at a purely semantic level without formal deductions) to produce a canonical model whose universe consists of interpretations of the constants. This is crucial in order to obtain Löwenheim–Skolem Theorems (downwards and upwards) in Chapters 12 and 13, which belong to Part III (Chapters 12–16), in which more advanced notions such as elementary substructures and extensions as well as categoricity are introduced. Part IV (Chapters 17–22) presents quantifier elimination, one of the fundamental aspects of model theory concerning the study of definable sets. The notions presented in Parts III and IV reappear in Part V, in which complete types are introduced, and in particular the omitting types theorem is proved. However, Kirby does not assume a knowledge of topology for his audience, so the topological properties of the space of types are not explored in detail (in particular, the notion of an isolated type is given purely in terms of atomic or principal formulae). The last chapters of Part V concern the notions of prime models as well as saturated models. The construction of a countable saturated model is presented in Chapter 26 for countable complete small theories (or in Kirby's notation 0-stable). Part VI (Chapters 28–32) presents the theory of algebraically closed fields as an archetype for the comprehension of a tame mathematical structure (or a class thereof) from a model theoretic point of view, relating fundamental notions from algebra to their model-theoretic counterparts: Chevalley's theorem in Chapter 31 and quantifier elimination, or Hilbert's Nullstellensatz and model-completeness in Chapter 32.

In my personal opinion, this book is well suited for two different audiences: advanced researchers in mathematics who would like to get a first acquaintance with some of the notions and results in model theory without having to spend a considerable amount of time with some classical notions whose relevance may not be clear at the beginning. The second audience is students (or rather, those faculty members considering offering an introductory course in model theory) at universities which do not offer many courses in mathematical logic, but are interested in broadening their curricula with a first introductory course in model theory.

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Edwin Mares. ***The Logic of Entailment and its History***. Cambridge University Press, Cambridge, UK, 2024, xv + 264 pp.

At the heart of logic lies the conditional notion of *entailment*; certain arguments are logically good in virtue of having a conclusion which follows *from* the premises in some sense. From the very birth of the research program, relevantists have insisted upon a substantial notion of from-ness; a proposition may be true in virtue of its logical form, yet need not for that reason follow *from* any and every collection of premises. Edwin Mares' book *The History and Philosophy of Entailment* is a fresh and updated account of the philosophy of and the

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