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Introduction

Ich behaupte aber, dass in jeder besonderen Naturlehre nur so viel eigentliche Wissenschaft angetroffen werden könne, als darin Mathematik anzutreffen ist.
[But I maintain that in every special natural doctrine only so much science proper is to be met with as mathematics.]

Immanuel Kant, 1786

Eratosthenes of Cyrene, who lived basically in the third century BC, was one of those first mathematicians whose knowledge and abilities at these early stages of human civilization were remarkable. Besides his method for seeking prime numbers, he particularly also contributed to the measurement of the Earth by, for example, determining its circumference. In this respect, he might have been the first geomathematician, or at least one of the first. Many more followed him, and definitely Carl Friedrich Gauß must be mentioned here, as he can be seen as the greatest genius in mathematical history. His works and their influence are widespread in mathematics, and they are also of essential importance in various applications, in particular and (in the author's possibly biased point of view) first of all in Earth sciences, especially geomagnetics and potential theory. The awareness, which reaches back to classical antiquity, that mathematics is the foremostly required skill and toolbox for understanding the objects and processes that surround us has been preserved up to the present. It has nicely and more generally been put in a nutshell by the preceding quotation, which is from Kant (1786); for the English translation, see Kant (1883). Over the centuries, Earth sciences and mathematics have both advanced. While the achievements at the time of Eratosthenes and his fellows are nowadays parts of the curricula at schools, many modern challenges in geosciences are equally challenges to twenty-first century mathematics.

The importance of mathematics for the understanding of the entire phenomenologies which are associated to the Earth was recognized and highlighted by the initiative Mathematics of Planet Earth (MPE), which was launched by UNESCO in 2013. Since then, the interest in geomathematics has grown extensively and many publications have occurred in the wake of MPE. Certainly, the Earth is as complex as it is versatile. Therefore, one single book cannot cover the whole mathematics which models processes occurring in the Earth, at its surface, and in the atmosphere. This new book that you are currently reading concentrates on specific topics: gravitation, magnetics, and seismology, though also these fields could easily fill books of the same size on their own. Moreover, this book is not only devoted to the modelling of these physical areas, but also to the mathematical foundations which

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are necessary for understanding and solving the occurring challenges. This comprises, in particular, a selection of basis function systems, including an algorithm for best basis choice, and the theory of inverse problems and their regularization.

Given the limitations that naturally occur when an author tries to squeeze these topics into approximately 500 pages, one needs to concentrate on certain essentials. A particular focus was put on the interconnections. There are mathematical tools which are essential in more than one of the three considered Earth sciences. Nevertheless, each chapter can be read on its own, but cross references are set where theorems and concepts are needed which have been derived in other contexts. Looking back at approximately a quarter of a century of being a scientist, the author has noticed that often the solution of one's currently urgent problems is located beyond one's own nose. Moreover, for opening the right drawer, one needs to know what is inside each drawer – otherwise the search can become very time consuming. Therefore, gravity experts, geomagnetics scientists, and seismologists are particularly encouraged to look not only into those chapters which have headings that are familiar in their own disciplines. Another focus was set on clear and rigorous deductions. For example, in the chapter on gravitation, we start with nothing more than Sir Isaac Newton's law of gravitation, which leads us to an integral. Then we observe the properties of this integral and their consequences. This brings us automatically, for instance, to the modelling with spherical harmonics and radial basis functions. Many tools and common facts, which are often accepted without scrutinizing them, occur here as logical consequences of elementary starting points in the modelling.

This book addresses several groups of readers: mathematicians who are interested in the theoretical foundations of certain areas of Earth sciences but also those who need tools for solving applied problems with mathematical means; geoscientists who look for a mathematical reference which explains why common physical realities are actually mathematical facts; and also those who look for new inspirations and alternative approaches for solving various topical problems in their fields. The monograph has been written for students as well as researchers who will both find their own particular benefit from it. It was also very important for the author to produce a book where proofs and other derivations are comprehensible to a wide audience. Numerous graphical illustrations support the understanding of complicated mathematical concepts. Moreover, for finding important keywords more easily, many of them have been set in boldface.

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