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ESSENTIAL NORM OF EXTENDED CESÀRO OPERATORS FROM ONE BERGMAN SPACE TO ANOTHER

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Abstract

Let $A^p(\varphi)$ be the pth Bergman space consisting of all holomorphic functions f on the unit ball B of $\mathbb{C}^n$ for which $\|f\|_{p,\varphi}^p = \int_B |f(z)|^p \varphi(z)\, dA(z) < +\infty$, where φ is a given normal weight. Let T_g be the extended Cesàro operator with holomorphic symbol g. The essential norm of T_g as an operator from $A^p(\varphi)$ to $A^q(\varphi)$ is denoted by $\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)}$. In this paper it is proved that, for $p \le q$,

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)} \simeq \limsup_{|z|\to 1} |\Re g(z)| \Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}$$

with $1/k = (1/p) - (1/q)$, where $\Re g(z)$ is the radial derivative of g; and for $p > q$,

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)} = \lim_{r\to 1} \int_{|z|\ge r} |g(z) - g(0)|^s \varphi(z)\, dA(z)$$

with $1/s = (1/q) - (1/p)$.

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1. Introduction

Let B be the unit ball of $\mathbb{C}^n$; if $n = 1$ then the unit disc is also denoted by D. Let dA be the Lebesgue volume measure on B and let $d\sigma$ be the normalised surface measure on ∂B. Write $\beta(\cdot,\cdot)$ for the Bergman distance on B. Given $z \in B$ and $r > 0$, the Bergman ball with centre z and radius r is $E(z, r) = \{w \in B : \beta(z, w) < r\}$. Let $H(B)$ be the family of all holomorphic functions on B. A positive continuous function φ on $[0, 1)$ is called normal if there are two constants $b > a > -1$ such that

$$\frac{\varphi(r)}{(1-r)^a} \downarrow 0, \qquad \frac{\varphi(r)}{(1-r)^b} \uparrow \infty \tag{1.1}$$

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as $r \to 1^-$. If φ is normal, then we extend it to B by $\varphi(z) = \varphi(|z|)$. For $0 < p < \infty$, the weighted Bergman space $A^p(\varphi)$ consists of all functions $f \in H(B)$ for which

$$\|f\|_{p,\varphi}^p = \int_B |f(z)|^p \varphi(z)\, dA(z) < +\infty.$$

For $g \in H(B)$, with symbol g, the extended Cesàro operator T_g is defined on $H(B)$ as

$$T_g(f)(z) = \int_0^1 f(tz)\Re g(tz)\frac{dt}{t}, \qquad f \in H(B), z \in B,$$

where $\Re g(z) = \sum_{j=1}^n z_j(\partial g/\partial z_j)$ is the radial derivative of g, as in [Ru80].

Let X and Y be two Banach (or Fréchet) spaces, and let T be a linear operator from X to Y with the operator norm $\|T\|_{X\to Y}$. Let K be the set of all compact linear operators from X to Y. The essential norm of T, denoted by $\|T\|_{e,X\to Y}$, is defined as

$$\|T\|_{e,X\to Y} = \inf_{Q\in K} \|T - Q\|_{X\to Y}.$$

The operator T_g in one variable was studied in [AC01, AS95, AS97]. In the higher-dimensional case, it was first studied in [Hu03, Hu04], where the boundedness and compactness on Bergman spaces (or mixed norm spaces) were completely characterised. Recently, in [HT10], Schatten(–Herz) class extended Cesàro operators on $A^2(\varphi)$ were considered. The purpose of this note is to study the essential norm for T_g as an operator from $A^p(\varphi)$ to $A^q(\varphi)$ for all possible $0 < p, q < \infty$. Some of our results in the one-variable case with $p \le q$ were obtained in [Ra07].

In what follows, we use C to denote a positive constant whose value may change from line to line but does not depend on the functions in $H(B)$. The expression '$A \simeq B$' means there exists some C such that $C^{-1}A \le B \le CA$.

2. Main theorem

Given $g \in H(B)$, write $M_\infty(g, r) = \sup_{|z|=r} |g(z)|$. It is well known that $M_\infty(g, r)$ is increasing with r. In the proof of our main theorem, we need the following lemma, which is of independent interest.

Lemma 2.1. *Let ψ be a positive continuous function on the interval $[0, 1)$ with $0 < \limsup_{r\to 1} \psi(r) \le \infty$. Then there is some constant C such that, for all $g \in H(B)$,*

$$\sup_{z\in B} |g(z)|\psi(|z|) \le C \limsup_{|z|\to 1} |g(z)|\psi(|z|). \tag{2.1}$$

Proof. First we prove that there exist a constant C and a sequence $\{r_j\}$, $r_j \to 1$ as $j \to \infty$, such that

$$\sup_{0\le\rho<r_j} \psi(\rho) \le C\psi(r_j). \tag{2.2}$$

In fact, if $0 < \limsup_{r\to 1} \psi(r) < \infty$, then we can pick some sequence $\{r_j\}$ such that $r_j \to 1$ as $j \to \infty$ and $\psi(r_j) \ge \frac{1}{2} \limsup_{r\to 1} \psi(r)$. Hence

$$\sup_{0\le\rho<r_j} \psi(\rho) \le \sup_{0\le\rho<1} \psi(\rho) \le \frac{2 \sup_{0\le\rho<1} \psi(\rho)}{\limsup_{r\to 1} \psi(r)}\psi(r_j) = C\psi(r_j). \tag{2.3}$$

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If $\limsup_{r\to 1}\psi(r)=\infty$, then we can take some $r_j\to 1$ so that

$$\sup_{0\le\rho\le r_j}\psi(\rho)=\psi(r_j). \tag{2.4}$$

Otherwise, we would have some r_0 such that, for all $r\in[r_0,1)$,

$$\sup_{0\le\rho\le r}\psi(\rho)>\psi(r).$$

Then $\sup_{0\le\rho\le r}\psi(\rho)$ cannot be achieved at any point in $[r_0,r]$. Hence $\limsup_{r\to 1}\psi(r)\le\sup_{0\le\rho\le r_0}\psi(\rho)$, a contradiction. From (2.3) and (2.4), (2.2) follows.

For $g\in H(B)$, we claim that there is some $\eta=\eta(g)\in(0,1)$ such that

$$M_\infty(g,r)\psi(r)\le 2\limsup_{r\to 1}M_\infty(g,r)\psi(r) \tag{2.5}$$

for all $\eta\le r<1$. In fact, if $\lim_{r\to 1}M_\infty(g,r)\psi(r)=0$, then $\lim_{r\to 1}M_\infty(g,r)=0$ by the hypothesis $\limsup_{r\to 1}\psi(r)>0$. This means that g is identically zero. Hence (2.5) is valid for all $\eta\in[0,1)$. If $\lim_{r\to 1}M_\infty(g,r)\psi(r)>0$, the estimate (2.5) is valid for all η sufficiently near 1 by the definition of lim sup.

Now, for any $g\in H(B)$, fix some r_j satisfying (2.2) such that $r_j\in[\eta(g),1)$. Then, by (2.5),

$$\begin{aligned}\sup_{0\le r<1}M_\infty(g,r)\psi(r)&\le\sup_{0\le r\le r_j}M_\infty(g,r)\psi(r)+\sup_{r_j\le r<1}M_\infty(g,r)\psi(r)\\&\le M_\infty(g,r_j)\sup_{0\le r\le r_j}\psi(r)+2\limsup_{r\to 1}M_\infty(g,r)\psi(r)\\&\le CM_\infty(g,r_j)\psi(r_j)+2\limsup_{r\to 1}M_\infty(g,r)\psi(r)\\&\le C\limsup_{r\to 1}M_\infty(g,r)\psi(r),\end{aligned}$$

where the constant C is independent of $g\in H(B)$. The estimate (2.1) follows. □

Lemma 2.2. *Suppose that $g\in H(B)$. Then, for $0<p\le q<\infty$,*

$$\|T_g\|_{A^p(\varphi)\to A^q(\varphi)}\simeq\sup_{z\in B}|\Re g(z)|\bigg(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\bigg)^{1/k}$$

with $1/k=(1/p)-(1/q)$; and, for $0<q<p<\infty$,

$$\|T_g\|_{A^p(\varphi)\to A^q(\varphi)}\simeq\|g-g(0)\|_{s,\varphi}$$

with $1/s=(1/q)-(1/p)$.

See [Hu04, Theorem 5]. Things to pay attention to are that, as pointed out in [Hu04, Remark 2], normality here is the same as that defined by conditions (P_1) and (P_2) in [AS97, Hu04] in the sense that they induce the same pth Bergman space with equivalent norms. Also, we have $\varphi^*\simeq\varphi$, where $\varphi^*(r)=(1/(1-r))\int_e^{(1+r)/2}\varphi(t)\,dt$, as in [Hu04].

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THEOREM 2.3. *Let $g \in H(B)$. Then, for $0 < p \le q < \infty$,*

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)} \simeq \limsup_{|z|\to 1} |\Re g(z)|\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k} \tag{2.6}$$

with $1/k = (1/p) - (1/q)$; and, for $p > q$,

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)} = \lim_{|z|\to 1} \int_{|z|\ge r} |g(z) - g(0)|^s \varphi(z)\, dA(z) \tag{2.7}$$

with $1/s = (1/q) - (1/p)$.

PROOF. We suppose first that $0 < p \le q < \infty$. Given $\zeta \in B$, let the function f_ζ be

$$f_\zeta(z) = \Big(\frac{(1-|\zeta|^2)^\beta}{\varphi(\zeta)(1-\langle z,\zeta\rangle)^{n+1+\beta}}\Big)^{1/p},$$

where $\beta > b$ is fixed with b as in (1.1). As indicated in [Hu04, proof of Theorem 2],

$$\|f_\zeta\|_{p,\varphi} \le C \quad \text{and} \quad f_\zeta(\zeta) = \frac{1}{(\varphi(\zeta)(1-|\zeta|^2)^{n+1})^{1/p}}.$$

Further, it is easy to check that $f_\zeta(z)$ goes to 0 uniformly on any compact subset of B as $|\zeta| \to 1$. Hence, for each $Q \in K$,

$$\lim_{|\zeta|\to\infty} \|Qf_\zeta\|_{q,\varphi} = 0.$$

Let $\zeta_j \in B$ be such that

$$\lim_{j\to\infty} |\Re g(\zeta_j)|\Big(\frac{(1-|\zeta_j|^2)^{k-(n+1)}}{\varphi(\zeta_j)}\Big)^{1/k} = \limsup_{|z|\to 1} |\Re g(z)|\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}.$$

Notice that $\Re(T_g f) = f\Re g$. Then, for $Q \in K$, by [Hu04, Theorem 1],

$$\begin{aligned}
\|T_g - Q\|_{A^p(\varphi)\to A^q(\varphi)} &\ge C \limsup_{j\to\infty} \|(T_g - Q)f_{\zeta_j}\|_{q,\varphi} \\
&\ge C\Big(\limsup_{j\to\infty} \|T_g f_{\zeta_j}\|_{q,\varphi} - \lim_{j\to\infty} \|Qf_{\zeta_j}\|_{q,\varphi}\Big) \\
&= C \limsup_{j\to\infty} \|T_g f_{\zeta_j}\|_{q,\varphi} \\
&\simeq C \limsup_{j\to\infty} \|\Re(T_g f_{\zeta_j})(z)(1-|z|^2)\|_{q,\varphi} \\
&= C \limsup_{j\to\infty}\Big(\int_B |f_{\zeta_j}(z)\Re g(z)(1-|z|^2)|^q \varphi(z)\, dA(z)\Big)^{1/q} \\
&\ge C \limsup_{j\to\infty}\Big(\int_{E(\zeta_j r)} |f_{\zeta_j}(z)\Re g(z)(1-|z|^2)|^q \varphi(z)\, dA(z)\Big)^{1/q} \\
&\ge C \limsup_{j\to\infty}(|f_{\zeta_j}(\zeta_j)\Re g(\zeta_j)|^q (1-|\zeta_j|^2)^{q+(n+1)} \varphi(\zeta_j))^{1/q} \\
&= C \limsup_{j\to\infty} |\Re g(\zeta_j)|\Big(\frac{(1-|\zeta_j|^2)^{k-(n+1)}}{\varphi(\zeta_j)}\Big)^{1/k}.
\end{aligned}$$

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By the definition of essential norm and the estimate above, we know that

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)} \geq C \limsup_{|z|\to 1} |\Re g(z)|\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}. \tag{2.8}$$

We now prove the reverse inequality. This will be split into two cases. First, let

$$\limsup_{r\to 1}\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k} = 0. \tag{2.9}$$

We may suppose that $g \in H(B)$ satisfies

$$\limsup_{|z|\to 1} |\Re g(z)|\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k} < \infty.$$

By (1.1), there is some positive constant α such that

$$\sup_{z\in B} |\Re g(z)|(1-|z|^2)^{\alpha} < \infty.$$

Hence [Zh05, Theorem 2.7] tells us that

$$\Re g(z) = c_\alpha \int_B \frac{\Re g(w)}{(1-\langle z, w\rangle)^{n+1+\alpha}}(1-|w|^2)^{\alpha}\, dA(w), \tag{2.10}$$

with c_α a fixed constant depending on n and α. For $0 < \rho < 1$, define G_ρ by

$$G_\rho(z) = c_\alpha \int_B \frac{\Re g(w)}{(1-\langle z, w\rangle)^{n+1+\alpha}}\chi_\rho(w)(1-|w|^2)^{\alpha}\, dA(w), \tag{2.11}$$

where

$$\chi_\rho(w) = \begin{cases} 1 & \text{if } |w| \leq \rho, \\ 0 & \text{if } \rho < |w| < 1. \end{cases}$$

It is trivial to verify that $G_\rho(z)$ is holomorphic on the closed unit ball $\overline{B}$, and also $G_\rho(0) = 0$ since $\Re g(0) = 0$. Set $g_\rho(z) = \int_0^1 (G_\rho(tz)/t)\, dt$; then g_ρ is also holomorphic on $\overline{B}$, and

$$\Re g_\rho(z) = G_\rho(z). \tag{2.12}$$

Hence, using (2.9),

$$\lim_{|z|\to 1} |\Re g_\rho(z)|\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k} = 0.$$

Theorem 6 in [Hu04] tells us that T_{g_ρ} is compact from $A^p(\varphi)$ to $A^q(\varphi)$. Therefore, by Lemma 2.2 and (2.10), (2.11), (2.12),

$$\begin{aligned}\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)} &\leq \|T_g - T_{g_\rho}\| \\ &\leq C \sup_{z\in B} |\Re g(z) - \Re g_\rho(z)|\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}\end{aligned}$$

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$$= C\sup_{z\in B}\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}\Big|\int_{|w|\ge\rho}\frac{\Re g(w)}{(1-\langle z,w\rangle)^{n+1+\alpha}}(1-|w|^2)^{\alpha}\,dA(w)\Big|$$
$$\le C\sup_{|w|\ge\rho}|\Re g(w)|\Big(\frac{(1-|w|^2)^{k-(n+1)}}{\varphi(w)}\Big)^{1/k}\sup_{z\in B}\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}$$
$$\times\int_B\Big(\frac{\varphi(z)}{(1-|w|^2)^{k-(n+1)}}\Big)^{1/k}\frac{(1-|w|^2)^{\alpha}}{|1-\langle z,w\rangle|^{n+1+\alpha}}\,dA(w).$$

Using the approach in [Hu03, proof of Lemma 2],

$$\int_0^1\frac{(1-|t|^2)^{\alpha-1+((n+1)/k)}}{(1-t|z|)^{1+\alpha}}(\varphi(t))^{1/k}\,dt\le C\Big(\frac{\varphi(z)}{(1-|z|^2)^{k-(n+1)}}\Big)^{1/k}.$$

Therefore, by [Ru80, Proposition 1.4.10],

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)}$$
$$\le C\sup_{|w|\ge\rho}|\Re g(w)|\Big(\frac{(1-|w|^2)^{k-(n+1)}}{\varphi(w)}\Big)^{1/k}$$
$$\times\sup_{z\in B}\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}\int_0^1 dt\int_{\partial B}\frac{(1-|t|^2)^{\alpha-1+((n+1)/k)}}{|1-t\langle z,\zeta\rangle|^{n+1+\alpha}}(\varphi(t))^{1/k}\,d\sigma(\zeta)$$
$$\le C\sup_{|w|\ge\rho}|\Re g(w)|\Big(\frac{(1-|w|^2)^{k-(n+1)}}{\varphi(w)}\Big)^{1/k}$$
$$\times\sup_{z\in B}\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}\int_0^1\frac{(1-|t|^2)^{\alpha-1+((n+1)/k)}}{(1-t|z|)^{1+\alpha}}(\varphi(t))^{1/k}\,dt$$
$$\le C\sup_{|w|\ge\rho}|\Re g(w)|\Big(\frac{(1-|w|^2)^{k-(n+1)}}{\varphi(w)}\Big)^{1/k}.$$

This implies that

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)}\le C\limsup_{|z|\to1}|\Re g(z)|\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}. \tag{2.13}$$

For the case

$$\limsup_{r\to1}\Big(\frac{(1-|z|^2)^{k-(n+1)}}{\varphi(z)}\Big)^{1/k}\ne0,$$

by Lemmas 2.1 and 2.2 we have

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)}\le\|T_g\|$$
$$\simeq\sup_{0\le r<1}M_\infty(\Re g,r)\Big(\frac{(1-r^2)^{k-(n+1)}}{\varphi(r)}\Big)^{1/k}$$
$$\le C\limsup_{r\to1}M_\infty(\Re g,r)\Big(\frac{(1-r^2)^{k-(n+1)}}{\varphi(r)}\Big)^{1/k}. \tag{2.14}$$

The estimates (2.6) come from (2.8) and (2.13), (2.14).

We now suppose that $0 < q < p < \infty$. Let $s > 0$ be such that $1/s = (1/q) - (1/p)$. If $\|g - g(0)\|_{s,\varphi} < \infty$, by [Hu04, Theorem 6] T_g is itself compact from $A^p(\varphi)$ to $A^q(\varphi)$. Notice that $\|g - g(0)\|_{s,\varphi} < \infty$ implies that

$$\lim_{r\to 1} \int_{|z|\geq r} |g(z) - g(0)|^s \varphi(z)\, dA(z) = 0.$$

Hence

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)} = 0 = \lim_{r\to 1} \int_{|z|\geq r} |g(z) - g(0)|^s \varphi(z)\, dA(z). \tag{2.15}$$

On the other hand, if $\|g - g(0)\|_{s,\varphi} = \infty$, then, for each $r \in [0, 1)$,

$$\int_{|z|\geq r} |g(z) - g(0)|^s \varphi(z)\, dA(z) = \infty.$$

Theorem 5 in [Hu04] tells us that T_g is not bounded from $A^p(\varphi)$ to $A^q(\varphi)$. Hence, for each compact operator Q, $\|T_g - Q\|_{A^p(\varphi)\to A^q(\varphi)} = \infty$. Therefore,

$$\|T_g\|_{e,A^p(\varphi)\to A^q(\varphi)} = \infty = \lim_{r\to 1} \int_{|z|\geq r} |g(z) - g(0)|^s \varphi(z)\, dA(z). \tag{2.16}$$

The estimate (2.7) follows from (2.15) and (2.16). The proof is complete. □

Remark 2.4. The case in which

$$\limsup_{r\to 1} \left(\frac{(1 - |z|^2)^{k-(n+1)}}{\varphi(z)} \right)^{1/k} \neq 0$$

may happen for a suitable pair p, q with $p < q$ even for the simplest weight $\varphi \equiv 1$. To see this, for each $p \in (0, n + 1)$ and q sufficiently large, since $1/k = (1/p) - (1/q)$, observe that $k - (n + 1) < 0$; then

$$\limsup_{r\to 1} \left(\frac{(1 - |z|^2)^{k-(n+1)}}{\varphi(z)} \right)^{1/k} = \lim_{r\to 1} ((1 - |z|^2)^{k-(n+1)})^{1/k} = \infty.$$

Remark 2.5. Of course, in our Theorem 2.3, when $p = q$ the expression

$$\left(\frac{(1 - |z|^2)^{k-(n+1)}}{\varphi(z)} \right)^{1/k}$$

should be read as $1 - |z|^2$.

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