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Construction of block designs and related results

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The work in this thesis centres mainly on the construction of various kinds of balanced designs. Chapter 1 contains some basic definitions and known results. In Chapter 2 the concept of a maximal arc in a finite projective plane is generalised to the case of an arbitrary balanced incomplete block design, and the "internal structure" of designs possessing maximal arcs is examined. Families of partially balanced designs are produced in some instances from designs with maximal arcs.

The remaining three chapters contain different construction methods for balanced designs. In Chapter 3 all non-isomorphic balanced incomplete block designs with parameters $(7, 21, 9, 3, 3)$ and $(9, 24, 8, 3, 2)$ are found, and also small 3-designs on eight elements with blocks of size four.

In 1893 Moore [1] discussed some designs of block size 3 in which elements could occur more than once in a block. In 1952 Tocher [2] defined a *balanced* n-ary *block design* to be a design on V elements, arranged in B blocks of size K, such that every element can occur $0, 1, 2, \ldots,$ or $n-1$ times in each block, and such that $\sum_{\substack{m=1 \\ i \neq j}}^{B} n_{im} n_{jm} = \Lambda$, a constant, where n_{im} is the number of times the ith element occurs in the mth block, $i = 1, \ldots, V$ and $m = 1, \ldots, B$. Moore's designs were not balanced in this sense. With Tocher's definition, a balanced incomplete block design is a balanced binary design. Chapter 4

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contains some properties of balanced n-ary designs, including a form of the Bruck-Ryser-Chowla theorem; also, existing balanced incomplete block designs are used to construct balanced n-ary designs. In Chapter 5, collections of cyclotomic classes in finite fields are used to produce n-ary supplementary difference sets, and the families of such sets that have constant block size generate balanced n-ary designs. All of these constructions have been adapted in some instances to yield balanced incomplete block designs.

References

[1] E. Hastings Moore, "Concerning triple systems", *Math. Ann.* 43 (1893), 271-285.

[2] K.D. Tocher, "The design and analysis of block experiments", *J. Roy. Statist. Soc. Ser. B* 14 (1952), 45-100 (including discussion).