STELLAR EVOLUTION AND THE PLANETARY NEBULA FORMATION RATE

J.P. PHILLIPS
Physics Department, Queen Mary College,
Mile End Road, London, El 4NS,
England

1. INTRODUCTION

A knowledge of the formation rate of planetary nebulae is crucial to our understanding of a wide range of physical environments and processes, and as such has been fitfully investigated over the last forty or so years. The elemental composition of the interstellar medium, for instance, and in particular the seeding of the IS medium by CNO and s-process materials, turns out to be rather sensitively dependent upon the local rate of PN formation χ(PN) (Salpeter, 1978, Pottasch, 1984; Tinsley, 1978; and Wood et al, 1983). Similarly, and to varying degrees, the ionisation balance of the I.S. medium (cf. Pottasch, 1984; Salpeter, 1978), composition and number density of I.S. grains (Natta and Panagia, 1981), and ultimately the galactic star formation rate (cf. Miller and Scalo, 1979) can all (more or less directly) trace a dependency upon χ(PN). Finally, χ(PN) has proven historically important in establishing the evolutionary status of PN (cf. Abell and Goldreich, 1966), and is expected to be directly related to the rate of formation of white dwarfs, the death-rate of AGB-type stars (Mira variables, OH/IR sources), and the main-sequence turn-off rate in the mass range $2 \lesssim M/M_{\odot} \lesssim 10$, some 10^{10} years ago.

It is clear therefore that, be it ever so insecure, the value of χ(PN) has a bearing upon the broad range of important astrophysical questions. In the following, we shall briefly consider the procedures whereby χ(PN) is determined, together with the vagaries inherent in such estimates. We shall also review the full range of current estimates for χ(PN) together with χ_G(PN), the galactic formation rate, and suggest best current estimates for these parameters. Finally, we provide a brief survey of χ for various related stages of pre- and post-planetary evolution, and their relation to χ(PN).

2. DETERMINATION OF χ(PN): METHODS AND UNCERTAINTIES

The evaluation of χ(PN) is in principle straightforward, and for the optically thin regime of expansion may be determined through

S. Torres-Peimbert (ed.), Planetary Nebulae, 425–442.

general expressions of form

$$\chi(PN) \simeq 1.02.10^{-15} \left(\frac{\sigma_o(R)}{kpc^{-3}pc^{-1}} \right) \left(\frac{V_{exp}(R)}{km.sec^{-1}} \right) pc^{-3}yr^{-1} \qquad (1)$$

where $\sigma_o(R)$ is the local number density of PN per unit radius about R, and $V_{exp}(R)$ is the corresponding mean nebular expansion velocity. For a variation

$$\sigma(R, Z) = \sigma_o(R)\, e^{-kz}\, kpc^{-3}\, pc^{-1} \qquad (2)$$

with respect to height z above the galactic plane, then the local density may in turn be estimated through

$$\sigma_o(R) = N(D, R) \left| 2\pi \left(\frac{D^2}{k} - \frac{2}{k^3} + e^{-kD} \left(\frac{2D}{k^2} + \frac{2}{k^3} \right) \right) \right|$$

where N(D, R) represents the number of nebulae within unit radius of R, and within a given distance D of the sun. A summary of values $\chi(PN)$ determined through this, or similar procedures is outlined in table 1, together with optically thin PN number densities σ_o^*, galactic formation rates $\chi_G(PN)$, and total cumulative galactic number N_T.

Whilst the methodology is therefore simply stated, there are several uncertainties that require careful consideration:

(a) The value of D

It is clearly necessary, in determining $\sigma_o(R)$, to obtain a reasonably complete sample of nebulae for any particular radius R. This is increasingly difficult at large D, however, due to the effects of galactic IS absorption, which will preferentially obscure nebulae having large R (and low surface brightness). The consequences of this have been illustrated by Phillips (1984) and Daub (1982), from which it is apparent that N(D, R) is a very much steeper function of R when D is >1 kpc, than appears to be the case where D < 1 kpc.

Two primary methods have been outlined to overcome this difficulty. In the first case (cf. Cahn and Kaler (1971)), the variation of N(D, R) with R is determined for a variety of (decreasing) distances D, until the trend appears reasonably invariant, and consistent with current nebular expansion models (thus, Cahn and Kaler adopt the Seaton (1966) evolutionary model in explaining a deficit of sources at radii R $\lesssim$ 0.4 pc, whilst Phillips (1984) evaluates N(R) on the basis of an empirical function $V_{exp}(R)$. Both of these procedures have defects). Alternatively, $\sigma_o(R, D)$ may be evaluated for a variety of values R, and increasing D, until incomplete sampling causes $\sigma_o(R)$ to decrease. This

TABLE 1

PLANETARY NEBULAE FORMATION RATES

Reference	χ(PN) 10^{-12}pc^{-3}. yr^{-1}	N_T x10^4	$^+\chi_G$(PN) yr^{-1}	σ_o kpc^{-3}	k kpc^{-1}	PN Lifetime yrs	Distance Scale
Paranego (1946)	-	.6-1.0	0.3-0.5	-	5.08	-	Own
Vorontsov and	-	10	5	-	-	-	Own
Velyaminov (1950)	-	-	-	-	-	-	-
Minkowski (1956)	-	6	3	-	-	-	Own
O'Dell (1962)	0.4	4.8	2.4	-	-	-	Own
Abell & Goldreich (1966)		0.5-5.0				$2.0.10^4$	Own
Seaton (1966)	0.1	-	-	-	-	$1.5.10^4$	Own
Cahn (1968)	1.4-0.5	-	-	12	-	-	Seaton (1966)
O'Dell (1968)	.39-.46	-	-	14	-	$3.5.10^4$	Own
Perek (1968)	-	-	-	30	-	-	Various
Seaton (1968)	2.0	-	-	-	-	-	Own
Gurzadyan (1969)	-	-	1-10	-	-	-	-
Cahn & Kaler (1971)	3.2	29-43	42	40-54	11.1	$1.6.10^4$	Seaton (1968)
O'Dell (1971)	0.4			14			
Osterbrock (1973)	2.5	-	-	-	-	-	Seaton (1968)
Osterbrock (1974)	3.1	-	-	-	8.0	-	-
Alloin et al (1976)	.63-3.1	1→2.25	0.5-1.1	-	6.25	-	Cudworth (1974) /Seaton (1968)
Cahn & Wyatt (1976)	5.1±1	3.8±1.2	1.9	80±15	8.70	$1.6.10^4$	Seaton (1968)
Smith (1976)	11±3	-	-	150	10.0	3.10^4	Seaton (1968)
Weidemann (1977)	1.8-2.6	-	-	-	-	$1.6.10^4$	Own
Cahn & Wyatt (1978)	-	1.35-4.5	.3-.7	-	8.70	-	Cudworth (1974)
Acker (1978)	3.0	2.5	1.25	48	5.0	$1.6.10^4$	Own
Tinsley (1978)	1.2-4.8	-	-	-	8.0*	-	Cudworth /Seaton
Jacoby (1980)	-	1±.4	0.5	-	-	-	-
Maciel (1981)	2.0	2.1	1.0	41	6.94	2.10^4	Own
Mallik (1982)	2.4±.2	2.8	1.4	44±4	-	-	Own
Daub (1982)†1	5±2	1.4	0.7	55	8.0	$4.8.10^3$	Own
Isaacman (1983)	2.9	2.1	1.0	-	-	-	-
Phillips (1984)	4.4±1.5	-	-	-	8.0	-	Daub (1982)
Pottasch (1984)	1.5	0.2-0.5	0.1-0.3	50	4.0	-	-
Amnuel et al (1984)	4.6±2.2	4.0	1-2	117±49	7.69	$1.6.10^4$	Own
Ishida & Weinberger (1987)	8.0	14	7.0	326	10.0	4.10^4	Various
Phillips (1987)†2	2.39±.32	2.96	1.31±.18	90±6	6.7		Own

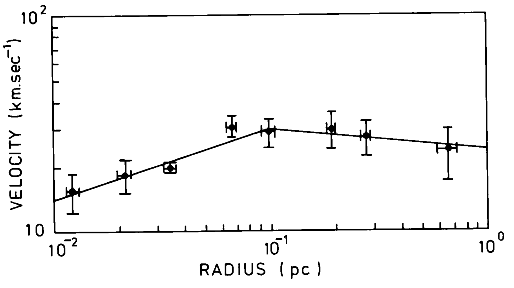
* Assumed value of k $^+\chi_G$ determined from $N_T/(2.10^4$ years), unless otherwise specified. †1 σ_o* determined for optically thin *and* thick nebulae; †2 This review.

procedure has been discussed in some detail by Daub (1982), and it is clear that D, in consequence, may become a variable function of R. As a result of the application of this procedure, however, Daub (1982) finds a severe increase in $\sigma_o(R, D)$ for radii R > .24 pc; a result which is not readily explicable in terms of most current models of nebular expansion or, indeed, of observed velocity trends (see Section 2e).

Finally, several investigators have adopted the simpler recourse of taking a low limiting distance D < 1 kpc (cf. Ishida and Weinberger (1987), who chose D = 0.5 kpc), or have attempted to correct for the effects of IS absorption using plausible models of the galactic dust distribution (cf. Smith, 1976).

(b) Distribution with height z above the Galactic Plane

The adoption of an exponential law for σ_o (expression 2) is in many ways no more than an analytical convenience; it is clear that such a trend can only very approximately represent the true z-distribution of PN. Nevertheless, various investigators have shown that such an expression represents a reasonable working assumption (cf. Daub, 1982), and one that is unlikely to lead to gross errors in χ(PN). The value of the scaling factor k, on the other hand, has varied widely over the years, and such uncertainties feed directly through to χ - although most estimates appear to cluster about $\sim$ 7-8 kpc^{-1}.



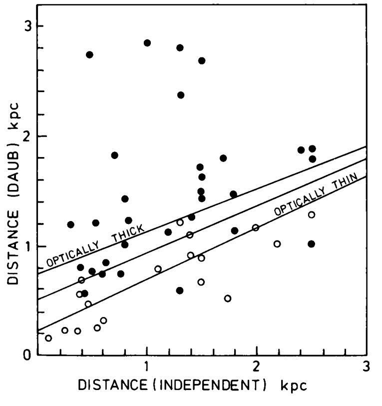


Figure 1. Variation of mean nebular expansion velocity with radius (see text for details). The solid lines indicate the trends for δ = 0.3 (R < .1 pc) and δ = -0.1.

Figure 2. Comparison of Daub (1982) distances with values based upon model independant measures, where the upper line represents a least-squares regression analysis for optically thick sources (● ; correlation coefficient $r_c \sim$ .37), and the lower line corresponds to the optically thin results (o : $r_c \sim$ .76). The least-squares fit to the entire data set is indicated by the central line.

In this regard, the values K $\lesssim$ 5 kpc^{-1} adopted by Acker (1978) and Pottasch (1984) appear aberrant, and are unlikely to be consistent with the PN progenitor mass distribution (see later).

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(c) The Distance Scale

Earlier estimates of χ depended upon a variety of distance estimates, ranging from the early Shklovsky (1956) scale through to the analyses of O'Dell (1962), Seaton (1966, 1968) and Cudworth (1974). All of these predicated upon some assumed invariant nebular property (usually ionised mass) and they have all, for this reason, been subsequently found wanting. More recently, Pottasch (1980) has noted that if such assumptions are side-stepped, and distances based on extinction trends and the like are employed, then a steeply varying trend of nebular mass with radius is deduced for $R \lesssim 0.1$ pc. This analysis has subsequently been expanded and generalised by Daub (1982) and Maciel and Pottasch (1980), and forms the basis of what must be regarded as the more reliable, recent estimates of $\chi(PN)$.

Having noted this, however, the current reliability of such scales is by no means so great as to leave us sanguine, and Ishida and Weinberger (1987) and Mallik (1985) have recently emphasized the extreme sensitivity of $\chi(PN)$ to any such uncertainties - for a distance scaling parameter κ, then in general $\chi(PN) \propto \kappa^4$.

Given the range of scales espoused by Phillips and Pottasch (1984) and Cudworth (1974), say, then it is clear that $\chi(PN)$ may potentially vary by a factor ~ 10.

The degree of uncertainty is further illustrated in figure 2, where we compare the Daub distances with a range of 'model independent' values based upon extinction trends, HI absorption measurements, kinematic parallax, and central star gravities (cf. Pottasch (1983, 1984) and Gathier et al (1986 a, b); see also the summaries of Sabbadin (1986) and Phillips (1984)). Apart from the obviously poor correlation between the respective distance scales (the correlation coefficient r_c is ~ 0.6) and the disparate trends between optically thick and thin nebulae, the scaling factor $\kappa \sim \langle d \text{ (independent)} \rangle / \langle d \text{ (Daub)} \rangle \sim .84$ would imply a reduction of the Daub/Phillips formation rates by a factor ~ 2.0, to of order $\chi(PN) \sim (2.2 \rightarrow 2.5).10^{12}$ pc^{-3} yr^{-1}, together with a revised scale height $k \sim 6.7$ kpc^{-1}.

(d) Influence of Progenitor Mass

The rate of PN formation is usually calculated on the assumption that central star masses are similar for all PN - with the consequence that the rate of PN evolution is also invariant. In fact, this is far from being the case, and it is clear that whilst central stars having masses $M_{NPN} \sim .55$ $M_\odot$ evolve rather slowly within the HR plane, a relatively small change in M_{NPN} to $\sim .6$ $M_\odot$, say, is likely to result in (i) a 20-fold secular increase in evolution within the $L-T_{eff}$ plane (cf. Schonberner, 1981, 1983); (ii) a grossly enhanced envelope mass (cf. Weidemann, 1987; Weidemann and Koester, 1983); and (iii) an appreciable alteration in envelope C, He, and N abundances, reflecting the influence of convective-zone dredging of CNO-process elements (Renzini and Voli, 1981). Higher mass (population I) sources appear also to be characterised by distinctive morphologies and high expansion velocities (cf. Peimbert and Torres-Peimbert, 1982), and it is clear that the straightforward application of expression (1) must be viewed with caution. Similarly, it may be noted that the differing progenitor masses would imply appreciably differing scale heights $z_o (\equiv k^{-1})$, and the dependency of velocity dispersion and nebula type upon z_o is by now reasonably well established (cf. Cudworth, 1974; Amnuel et al, 1984). A significant dispersion of this kind is also to be found in the progenitor OH (IRAS, Mira, and OH/IR) sources (cf. Herman and Habing, 1985), and any accurate assessment of σ_o should therefore fully take account of this hierarchy of central star masses. Having said this, however, the available information does not encourage us to believe that such a detailed analysis is yet possible. Similarly, the sharply peaked nature of the central star mass-distribution (cf. Schonberner, 1981) implies that errors arising from a single mass analysis are likely to be low.

(e) Expansion Velocities

The value of $V_{exp}(R)$ is usually taken to be constant, and of order 20 $km.sec^{-1}$ in the optically thin regime ($R \lesssim .1$ pc). Larger values have on occasion been employed, however, and these account for at least some of the variation in χ(PN) to be noted in table 1.

It is clear, from this, that although χ(PN) may depend rather sensitively upon V_{exp}, most previous investigations have been happy to adopt a rather hand waving estimate of this parameter. Phillips (1984) has investigated this question in a little more detail, by establishing an empirical relation of form $V(R) = V_o R^{\delta}$, although even for this case the relation is strongly biassed towards optically thick nebulae, for which dR/dt may take a value appreciably at variance with V_{exp}, and of order $\sim V_I$ (the velocity of the ionisation front). This disparity may also account for the apparent inability of such velocity trends to replicate the sharp rise in N(R) ($\propto \chi$/dR/dt)) for low values of R (see later).

For the purposes of this review we have updated this analysis to include a range of new and improved expansion velocities due, primarily, to Sabbadin et al (1983, 1984, 1985 and 1986) and Sabbadin (1984 a, b; 1986), together with earlier data referenced in Phillips (1984). Combining these with the model independent distance estimates referenced in Section 2c, we then obtain results which are closely similar to those of the earlier analysis, implying $\delta \sim 0.3$ for $R \lesssim 1$ pc (see figure 1). In the R > 0.1 pc optically thin regime, on the other hand, there is evidence for a peaking of velocities close to $V_{exp} \sim 30$ $km.sec^{-1}$, followed by a more-or-less gradual decline to higher radii.

A mean for the optically thin nebulae in figure 1 yields the expansion velocity $\bar{V}_{exp} \sim 26.1 \pm 2.9$ km. sec^{-1}, whilst a more balanced weighting with respect to the complete sample of nebulae in figure 3 would imply $\bar{V}_{exp}$ (0.1 < R < 0.6 pc) 26.0 $km.sec^{-1}$ for optically thin sources, and $\bar{\bar{V}}_{exp} \sim 24.5$ $km.sec^{-1}$ for all nebulae having R < 0.6 pc.

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(f) Sample Completeness

Finally, recent investigations of large ($R \gtrsim .5$ pc), previously unrecognised, and/or high galactic latitude nebulae are increasing the corpus of known PN close to the sun; examples of such surveys include the recent analysis by Ishida and Weinberger (1987), and the careful discussion of Pottasch (1984). Nevertheless, perhaps of order ∿ 25% of low surface brightness nebulae possess neither independantly estimated distances, nor integral Hβ fluxes with which to evaluate Shklovsky distances, and this results in a rather sharp cut-off in $\sigma(R)$ for $R > 0.6$ pc (see later).

Finally, we may remark that question of sample completeness also arises for low values of R, where nebulae are likely to be unresolved, and possibly mis-identified or unrecognised. Where a mean value $\sigma_0(R) \approx \int_{\Delta R}\sigma_0(R)\, dR/\Delta R$ is determined for the full range of nebulae (as in, say, Daub (1982) or Ishida and Weinberger (1987)) then it is evident that $\chi(PN)$ may be correspondingly underestimated. However, the trends for local compact nebulae (cf. figure 3) suggest, on the contrary, that the values of σ_0 ($R \lesssim .1$ pc) are significantly in excess of the mean, and any further enhancement would compound an already puzzling trend.

Given all these caveats and uncertainties, it is perhaps not surprising to find that estimates of $\chi(PN)$ have varied appreciably over the years, with more recent estimates extending through the range ∿ $(1\rightarrow4).10^{-12}$ pc^{-3} yr^{-1}, implying a mean ∿ $3.4.10^{-12}$ pc^{-3} yr^{-1}. Ishida and Weinberg (1987) have recently attempted to buck this trend, however, and follow Smith (1976) in deriving a high value $\chi(PN)$ ∿ 8.10^{-12} pc^{-3} yr^{-1}; an estimate which, if true, would be not a little perplexing, implying a ratio $\chi(WD/\chi(PN) \lesssim 0.25$. A re-analysis of the Ishida and Weinberger data base, however, suggests that for $R \lesssim 0.4$ pc and $k = 10$ kpc^{-1}, then $\sigma_0{}^*$ takes a value ∿ 83. kpc^{-3}, and $\chi(PN)$ is of order ∿ $4.3.10^{-12}$ pc^{-3} yr^{-1}. Similar values are also obtained for $R < 0.6$ pc. We have not allowed for a N-S disparity in PN numbers for this more limited range of radii, a feature which increases the $R < 0.8$ pc sample analysed by Ishida and Weinberger by a factor 1.4. Similarly, we express some surprise at the apparent disparity in volume number densities between samples having $D \leq 1$ kpc (see later), and $D \leq 0.5$ kpc; a difference which is unlikely to be entirely due to extinction.

Taken as a whole, therefore, it is apparent that whilst such high values of $\chi(PN)$ can probably be discounted, the derived range in $\chi(PN)$ remains uncomfortably broad - and somewhat at variance with the optimistic prognoses of Weidemann (1977, 1978). Nevertheless if a mean of certain of the more recent and realistic estimates of $\chi(PN)$ are selected, then we find a value ∿ $(2.60 \pm .24).10^{-12}$ pc^{-3} yr^{-1} which is tolerably consistent with earlier values of $\chi(WD)$ (see Section 4a). Alternatively, model independant distances and radii (see section 2(c) and (e)) have now been determined for the larger fraction of nebulae having $D < 1$ kpc; and where these are not available (in general for the larger ($R \gtrsim .5$ pc) optically thin sources) then Shklovsky distances may be derived. The results, in the form of the trend of $\sigma_0(R)$ with R are illustrated in figure 3, where sampling for $R > .5$ pc is clearly increasingly incomplete (a full description of this analysis will be provided in Phillips (1987)). The comparative function $^T\sigma_0(R) \propto \chi(PN)/V_{exp}(R)$, based upon the velocity trends in figure 1 and scaled to match the observed integral of sources for $R \lesssim 0.6$ pc, appears to reproduce the observed trend for optically thin sources tolerably well. At shorter radii, on the other hand, it is clear that there is a strong peaking in $\sigma_0(R)$ for $R \lesssim 0.05$ pc, a feature which it is impossible to replicate given the observed velocity trends.

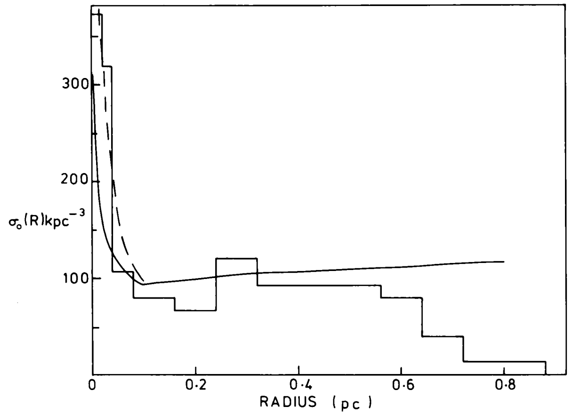

Figure 3. Variation of $\sigma_o(R)$ for sources within D = 1 kpc of the sun. The solid curve (fitted to the R < 0.6 pc results) represents the trend expected for the velocities in figure 1, whilst the dashed curve indicates the possible consequence of ionisation fronts.

In the optically thick expansion regime, on the other hand, it is likely that $\sigma_o(R) \propto \chi(PN)/(dR/dt) \propto V_I^{-1}$, where V_I is the velocity of the ionisation front. If for illustrative purposes we therefore assume the secular relations

$$V_G = V_{GO}\,(t/t_o)^{\alpha} \qquad V_I = V_{IO}\,(t/t_o)^{\beta}$$

where V_G is the gas expansion velocity ($\equiv V_{exp}$), then we determine

$$R_I = \frac{V_{IO}\,t_o}{\beta + 1}\left(\frac{V_G}{V_{GO}}\right)^{\frac{\beta + 1}{\alpha}}$$

$$\sigma_o(R) = \frac{\chi(PN)}{V_{IO}} \left(\frac{(\beta + 1)\, R_I}{V_{IO}\, t_o} \right)^{\frac{-\beta}{\beta + 1}}$$

Fitting these functions to the observed trends in figures 1 and 3 then yields the solutions α = 2.75, β = 7.33, and the trends in $\sigma(R)$ indicated by the dashed curve in figure 3.

It is clear, in short, that for such an hypothesis to reproduce the observed trends in $\sigma_o(R)$ requires an extremely rapid variation in V_I, presumably preceded by a more benign phase of I-front development (unless local densities are to become extraordinarily high). Alternatively, of course, it is possible that our zeal in identifying compact nebulae has resulted in a spate of mis-identifications, or that the distances to these sources have for some reason been grossly undervalued.

In any case it is apparent that by taking the optically thin results alone ($0.1 \leq R \leq 0.6$ pc), and adopting a value $\bar{V}_{exp} \sim 26$ km. sec^{-1} (Section 2e), then we find a reasonably invariant $\sigma_o(R) \simeq 89.6 \pm 6.4$ kpc^{-3} pc^{-1}, $\sigma_o^* \simeq 44.8$ kpc^{-3}, and a formation rate $\chi(PN) \simeq (2.39 \pm 0.32)\ 10^{-12}$ pc^{-3} yr^{-1}.

3. GALACTIC FORMATION RATE

The surface density of PN throughout the galaxy is frequently characterised by a function

$$\mu(R_G) = \mu_o\, e^{-k_G R_G}$$

where R_G is the galactocentric radius and μ_o the central density of PN; a trend which also appears to be representative of kinematic, density, and surface brightness trends in other galaxies (cf. de Vaucouleurs, 1959; Freeman, 1979; Toomre, 1972, and Cruz-Gonzalez, 1974). By scaling this function through comparison with $\mu(R_O)$(the local surface density) it is then possible to obtain an estimate for N_T, the total number of nebulae in our galaxy (cf. Cahn and Kaler, 1971). Earlier attempts to estimate k_G utilised the observed galactic radial decrement in σ_o near the sun, yielding values $k_G \sim 0.9$ kpc^{-1} which would have implied phenomenal galactic centre densities, and total numbers $N_T \sim 2.9 \rightarrow 4.3 . 10^5$ - and in turn suggesting, for a typical expansion period $\sim 2.10^4$ years, a formation rate $\chi(PN) \sim 20$ PN.year^{-1}. More recent analyses have resulted in a down grading in k_G to values ~ 0.3 kpc^{-1}, however, more clearly relating to the distribution of F-M type stars and PN number densities between R_G = 5 and 10 kpc (cf. Alloin et al, 1976; Cahn and Wyatt, 1978; and Amnuel et al, 1984), and this has resulted in a commensurate reduction in N_T to $\sim 10^4$. Other procedures employed to estimate N_T include (a) estimates of the local mass specific number density κ_m in the range $1.1 \rightarrow 2.5\ 10^{-7}\ M_\odot^{-1}$, a value which is presumed to apply galaxy-wide. By taking an overall galactic mass $M_G \sim 1.3 - 1.5.10^{11}\ M_\odot$, therefore, it follows that N_T must reside in the range $(1.3 \rightarrow 3.8).10^4$. There are several problems with such an analysis, the more serious of which concerns the presumed invariance of κ_m - such a value will certainly not apply in the galactic halo, for instance. Nevertheless, values of κ_m determined for the galactic centre PN (Isaacman, 1981), and planetary nebulae in the (presumably similar) galaxy M31 (Jacoby, 1980), appear to confirm the widespread viability of such procedures.

(b) Jacoby (1980) has investigated the luminosity function of PN in the Magellanic clouds, and concludes that the trends are well represented by a simple model of nebular expansion due to Henize and Westerlund (1963). Under these circumstances, he argues that luminosity functions are likely to be similar in other galaxies - the effects of differing progenitor mass distribution, of chemical composition and nebular expansion velocities being merely to 'smear' these trends. A rather more important uncertainty may however arise from his extrapolation to fainter nebulae which, although uncertain, roughly doubles the presumed number of PN.

A specific mass number density for M31 ($\sim 1.1.10^{-7}\ M_\odot^{-1}$) is thereby found which appears closely similar to that of our own galaxy - although Jacoby prefers to use the M31 luminosity specific density $(6.1 \pm 2.2).10^{-7}$ PN $L_\odot^{-1}$ to obtain a final estimate $N_T \sim (10 \pm 4).10^3$ PN.

Other, and in certain cases simpler procedures have also been employed; Cahn and Wyatt (1976), for instance, adopt the local column density of PN as typical for the entire galactic plane, and integrate over the disc area to obtain $N_T \sim 1.3.10^4$ - a procedure which, with all its faults, at least lacks the contrivance of certain more model-intensive estimates.

Despite the adoption of these varying procedures, and the wide range of σ_o^* upon which they are based, the values of N_T thus obtained (see table 1) are for the most part tolerably consistent - a mean of values over the past decade yields $N_T \sim (2.1 \pm 0.4).10^4$ PN. For the purposes of this review, these estimates have been subsequently divided by a typical expansion period 2.10^4 years to yield the galactic formation rate χ_G(PN), implying a characteristic value $\sim 1.0 \pm 0.2$ PN. year^{-1}. This may be compared with the values $N_T \sim 2.47.10^4$ (for $R < 0.6$ pc), and $\chi_G(PN) \sim 1.31 \pm .18\ yr^{-1}$ determined from a local formation rate $\chi(PN) \sim 2.39.10^{-12}\ pc^{-3}\ yr^{-1}$ (section 3), and assuming a galactic mass $M_G \sim 1.4.10^{11}\ M_\odot$, and $\mu_m \sim 75\ M_\odot\ pc^{-2}$ (Schmidt, 1963).

Finally, and for the future, it is plain that the vast amount of FIR data collected through IRAS is likely to yield yet further estimates of χ_G. Specifically, it is clear that PN occupy a rather closely defined zone within FIR colour-colour plots, and the distribution of such sources within the galactic plane should allow a similar investigation to that of the GC radio sources (cf. Isaacman, 1983). Further details of such an analysis are provided in Phillips (1987).

4. THE PLANETARY NEBULA FORMATION RATE AND STELLAR EVOLUTION

The broad progress of stellar evolution, the behaviour of stars along the AGB branch, and the general development of these stars through the subsequent white dwarf and PN phases of evolution are by now tolerably well understood. Nevertheless, this understanding is far from complete, and several phases of both AGB and RGB evolution remain obscure and ill-defined. In this respect, values of χ for the Mira stars, the OH/IR sources and the white dwarfs not only act as a constraint upon our estimates of χ(PN), but also enable us to test the viability (or otherwise) of current models of stellar evolution - the proposed rates of evolution through the HR diagram, the assumed relation between PN, progenitors, and post-PN phases, and (through comparison with main-sequence turnoff) the variation of the star-formation rate over the last 10^{10} years, together with the presumed PN progenitor mass range. In the following, we briefly review our current understanding of these issues.

(a) White Dwarf Formation Rate

A summary of values for the white dwarf formation rate is provided in table 2, whence it is seen that prior to 1986, a consensus had been reached whereby χ(WD) varied between $1.4.10^{-12}$ pc^{-3} yr^{-1} and $\sim 2.6.10^{-12}$ pc^{-3} yr^{-1}, suggesting a plausible average $\sim 2.10^{-12}$ pc^{-3} yr^{-1}. Given that all PN ultimately evolve into white dwarfs, it was apparent (within errors) that χ(PN) would be of order similar to χ(WD). Several recent developments in our understanding of stellar evolution, PN central star (M_{NPN}) and white dwarf (M_{WD}) mass distributions, and the value of χ(WD), however, have somewhat muddied these waters. In the first place, estimates of M_{NPN} based on the location of PN with respect to evolutionary tracks (cf. Schonberner and Weidemann, 1983) suggest a trailing-off in numbers to higher masses, but an extraordinarily sharp cut-off at $M_{NPN} \simeq 0.55\ M_\odot$. This compares with a distribution in M_{WD} which appears to be symmetrical, and peaked at $M_{WD} \sim 0.55\ M_\odot$.

A description of the underlying reasons for this trend has been outlined by Drilling Schonberner (1985), whereby it is clear that $\sim .2$-3% of stars evolving off the main sequence, and ending their lives as white dwarfs, have masses $\lesssim .53\ M_\odot$ which may imply direct evolution from the horizontal branch to the white dwarf phase - the intermediate AGB-PN stage is altogether avoided (cf. Schonberner and Drilling, 1984). For another and more important fraction of stars having masses $M \sim .55\ M_\odot$, constituting perhaps 5-70% of white dwarf progenitors, the evolution from the AGB to the white dwarf sequence is extremely slow, and PN shells have generally dissipated before the central star temperatures have appreciably increased, and shell ionisation takes place. For these cases, therefore, we observe no associated PN shell. Finally, the remaining 30-95% of higher mass stars evolve at a reasonably rapid rate, and proceed to the observed PN.

TABLE 2

White Dwarf Formation Rates

Reference	χ_{WD} 10^{-12} pc^{-3} yr^{-1}
Weidemann (1968b)	2.0
Weidemann (1977)	1.4
Weidemann (1978)	2.0
Liebert (1980)	1.4
Wesemael (1981)	1.4
Guseinov et al (1983)	2.5
Fleming et al (1986)	.49 - .75

What does all this entail for χ(WD), and its relation to χ(PN)? Superficially, at least, if most of the white dwarfs having mass > .55 $M_\odot$ are assumed to derive from a PN progenitor, then we might expect $\chi(PN) \sim 0.6\ \chi(WD)$, and given a value $\chi(WD) \sim 2.10^{-12}\ pc^{-3} yr^{-1}$, then a rate $\chi(PN) \sim 1.2.10^{-12}\ pc^{-3}\ yr^{-1}$ would be anticipated. The situation has recently, however, become even more severe, with Fleming et al (1986) re-analysing the data base of Green (1980), and spectrally re-classifying a large fraction of the previously identified white dwarf population to lower gravities. The result is an apparent lowering of χ(WD) to $\sim 4.5 - 7.5.10^{-13}\ pc^{-3} yr^{-1}$ which, if confirmed, would imply $\chi(PN) \sim (2.7 - 4.5).10^{-13} pc^{-3} yr^{-1}$. Such a range of values would, surprisingly, be more consistent with earlier estimates of χ(PN) (e.g. O'Dell (1968); Seaton (1966), and Cahn (1968)), and disagree with practically the entire corpus of more recent values. If we take these recent estimates to be the more reliable, therefore, it is clear that we are confronted with an interesting paradox - the rate of PN production is approximately 4→7 times greater than would be predicted from χ(WD).

In this respect, we note that the procedures for evaluating χ(WD) (which depend upon a cooling rate defined from stellar theory) are almost certainly reasonably accurate - more so, indeed, than those employed in the determination of χ(PN). Under these circumstances, one might be inclined to argue that the white dwarf statistics are at fault - perhaps many white dwarfs, for instance, represent hitherto undetected binary companions (cf. Grauer and Bond, 1983; Fleming et al, 1986). Under these circumstances, however, and given an incidence of spectroscopic binaries in PN no greater than ∿ 10-15% (see Bond, these proceedings), we would require between 65 and 80% of PN nuclei to contain longer period binaries. Alternatively, Kudritzki and Barlow (these proceedings) have argued in favour of the Cudworth (1974) scale of PN distances, and Weidemann (these proceedings) suggests a value ∿ 1.3 κ (Cahn and Kaler). Either of these would result in a sharp reduction in χ(PN) to of order$(0.6 \rightarrow 1.2).10^{-12}\ pc^{-3}\ yr^{-1}$ - closer, certainly, to χ(WD), although more disparate with χ(Mira) and, on a galactic scale, χ_G(OH/IR). Similarly, such re-scaling would imply appreciable errors in the M.I. scales adopted here (section 2c), a result which would be difficult to readily comprehend.

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We should finally remark that the scale height z_o of white dwarfs appears to lie somewhere in the range 250 and 300 pc (Fleming et al (1986), Downes (1984) and Ishida et al (1982)), and is similar to that of Mira variables ($\sim$ 300 - 400 pc according to Wood and Cahn (1977) and Wyatt and Cahn (1983)). The disparity with z_o(PN) $\sim$ 150 pc is readily understood if only the more massive stars proceed through a PN phase of evolution, and indeed similar arguments have been used to argue for a progenitor mass range < 2.5 $M_\odot$ (Mallik, 1985). Subsequent dynamical interactions with the galactic disc may also operate to increase z_o(WD) even further (cf. Wielen, 1977; Koester and Wiedemann, 1980). The more massive Miras are also expected to translate through the OH/IR phase prior to becoming PN (cf. Habing 1987; Kwok 1987), and it is therefore gratifying to note that the mean height $|z\ (OH/IR)|$ $\sim$ 56-194 pc is indeed similar to that of PN (Herman and Habing, 1985; the higher values corresponding to a ZAMS mass $\sim$ 1.6 $M_\odot$, and shell expansion velocities < 15 km.sec^{-1}, whilst the lowest mean height appears consistent with M(ZAMS) $\sim$ 8 $M_\odot$, and V_e > 28 km.sec^{-1}). The distribution of z_o with stellar type is therefore consistent with only a fractional post-AGB evolution through the PN phase (ignoring, of course, those that subsequently chose to become supernovae, in the progenitor range M $\gtrsim$ 10 $M_\odot$), and confirms the expected low value of χ(PN) with respect to χ(WD).

(b) Rate of Evolution of OH/IR stars

The evolution of Mira variables along the AGB is ultimately believed to result in a change in pulsation mode, from first overtone to fundamental. Whilst the origins and consequences of this behaviour are far from being well understood, it is believed that the resulting, pulsationally driven mass loss leads to mass-loss rates $\sim 10^{-4}$ $M_\odot$.yr^{-1} over a period of between < 10^3 and (for low mass sources) 10^5 years, giving rise to an extensive, opaque dusty thermosphere. Recent analysis of the statistics of such sources by, in particular, Hermann and Habing (1985), implies a galaxy wide formation rate χ_G(OH/IR) $\sim$ 0.9 yr^{-1}. Given that the larger part of these sources are expected to become PN, it is clear that χ_G(OH/IR) is in tolerable agreement with the best current estimate of χ_G(PN) (see section 3).

(c) Estimates of χ(Mira)

Wood and Cahn (1977) have determined a value χ(Mira) of order 3.10^{-13} pc^{-3} yr^{-1}, using a scale height z_o(Mira) of 314 pc, and implying a local space density $\sim$ 245 kpc^{-3}. In the case of these sources, the rate of evolution is by no means so well established, and Willson (1981) for instance takes account of mass loss to evaluate a very much higher figure $\sim 3.10^{-12}$ pc^{-3} yr^{-1}. Given that only $\sim$ 60% of Miras are expected to result in PN, then very approximately we might expect χ(PN) $\sim$ 0.6 χ(Mira) $\sim 1.8.10^{-12}$ pc^{-3}, in tolerable conformity with our earlier derived values. Note also that Wyatt and

Cahn (1983) find a local density ∿ 200 kpc^{-3}, whilst Oort van Tulden (1942) determine a value ∿ 10^2 kpc^{-3} - estimates which may imply somewhat lower values of χ(Mira).

(d) Main-Sequence Turn-off Rate

Finally, we note that several attempts have been made to relate the rate of main-sequence turn-off some $5\text{-}10.10^9$ years ago with the current density of degenerate and PN central stars. Koester and Wiedemann (1980) for instance find it possible to match degenerate star number densities and mass distributions providing the SFR varies slowly ($\propto e^{-t/5.10^9}$ yr, for instance), and either a Salpeter (1955) or Larson and Tinsley (1978) IMF is adopted, whilst Miller and Scalo (1979) also require (from a less detailed comparison) that the SFR varies only slowly, and by less than a factor ∿ 2 over the last 10^{10} years. Cahn and Wyatt (1976) deduce a PN formation rate ∿ $2.3.10^{-12}$ pc^{-3} yr^{-1} which is largely independent of the assumed galactic age, or indeed

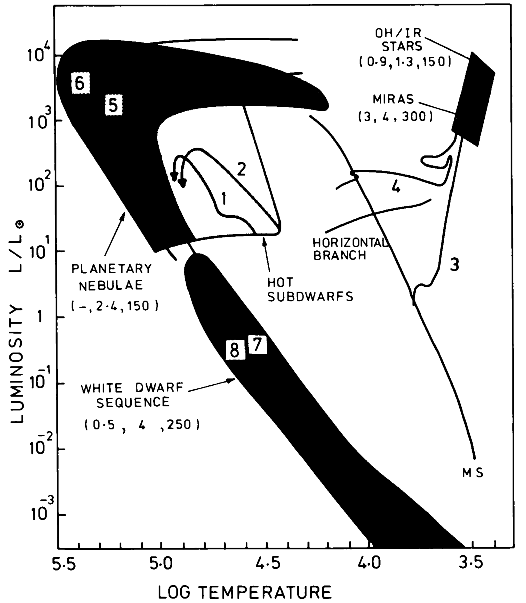

Figure 4. Schematic diagram illustrating the primary regimes occupied by planetary nebulae, hot sub-dwarfs, Miras, and OH/IR stars. For comparison, we also show evolutionary tracks for 0.5 $M_\odot$ stars away from the EHB (1, 2), 1.1 $M_\odot$ and 3 $M_\odot$ main sequence stars (3, 4), 0.6 $M_\odot$ and 0.8 $M_\odot$ central stars (5, 6), and 0.6 $M_\odot$ and 1 $M_\odot$ white dwarfs (7, 8). The codes (χ, χ_P, z_o) for Miras, PN, and white dwarfs refer respectively to the observed formation rate (x 10^{12}), the predicted formation rate for $\chi(PN) = 2.4.10^{-12}$ pc^{-3} yr^{-1}, and the typical mean scale height. The codes for OH/IR stars correspond to galactic formation rates $\chi_G(PN.yr^{-1})$ and are based on a rate $\chi_G(PN) = 1.3$ yr^{-1}.

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of the fractional gas content, although uncertainties in the adopted correlation between M_{BOL} and stellar mass, for instance, may lead to factors of two uncertainty in χ(PN). If the relevant input parameters of Abell and Goldreich (1966) are adopted, for instance, then a PN formation rate $1\text{-}2.10^{-12}$ pc^{-3} yr^{-1} would be implied. Similar results are also deduced by Tinsley, who predicts χ(WD) $\sim 2.10^{-12}$ pc^{-3} yr^{-1}.

Finally, we note that by using an improved understanding of the mass range M_{NPN}, an initial/final mass relation as given by Weidemann and Koester (1983), and assuming a secularly invariant SFR, Mallik (1985) determines a formation rate $\sim 5.10^{-13}$ pc^{-3} yr^{-1} - similar, indeed, to the values quoted by Alloin et al (1976), O'Dell (1962, 1968), and Cahn (1968) - although less than our best current estimate.

In summary, it is clear that the overall picture of PN as emerging from the higher mass AGB sequence via a high mass-loss OH/IR phase is reasonably well attested through the respective main-sequence turn-off rate, Mira death rate, and OH/IR formation rate (see figure 4 for a summary of these parameters). The errors (and uncertainties) in all of these estimates are not such as to create an overwhelming confidence, but appear nevertheless to confirm a value close to χ(PN) $\sim 2.4.10^{-12}$ pc^{-3} yr^{-1}. The relation of χ(PN) to χ(WD) is somewhat more problematic. Whilst earlier values of χ(WD) appeared to relate quite well to χ(PN), the realisation that only $\sim$ 60% of white dwarfs pass through an observable PN phase of evolution makes the agreement less secure. Similarly, the recent estimates of χ(WD) by Fleming et al (1986) appear consistent with only the lowest estimates of χ(PN), and may require a complete re-evaluation of the M.I. distance scale, or alternatively a large ($\sim$ 80-90%) population of PN central star binaries.

Clearly, therefore, there is room here for a considerable improvement in either χ(PN), χ(WD), or perhaps both, unless current models of stellar evolution are grossly in error.

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