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ARTICLE

Unconventional policy instruments in the New Keynesian model

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Abstract

This paper analyzes the use of unconventional policy instruments in New Keynesian setups in which the "divine coincidence" breaks down. The paper discusses the role of a second instrument that, in addition to the effect of conventional interest rate policy, may enter the Phillips curve, the investment–saving (IS) curve, and the welfare function, thus influencing inflation and output. The paper presents theoretical results on equilibrium determinacy, the inflation bias, the stabilization bias, and the optimal central banker's preferences when both instruments are available. We show that the use of an unconventional instrument reduces the zone of equilibrium indeterminacy and may reduce the volatility of the economy. However, in some circumstances, committing to not use the second instrument may be welfare-improving (a result akin to Rogoff (*Journal of International Economics* 18(3-4), 199–217, 1985) example of counter-productive coordination). We also show that the optimal central banker should be both aggressive against inflation and interventionist in using the unconventional policy instrument, and we analyze the optimal central banker's preferences when social preferences would yield equilibrium indeterminacy.

Keywords: Unconventional monetary policy; equilibrium indeterminacy; stabilization bias; inflationary bias

1. Introduction

Since the 2008 global financial crisis, central banks around the world have been forced to rethink their monetary and financial stability frameworks. Concerns about financial stability, the risk of deflation, and the risk of widespread corporate bankruptcies and low growth due to the COVID-19 pandemic led central banks to use a variety of policy instruments, from credit policy and guarantees to quantitative easing, macro-prudential tools, and foreign exchange intervention (the latter, especially in emerging markets). As a result, old questions about the appropriate objectives of monetary policy and the instruments that should be in the central bank's toolkit have re-emerged. These questions had seemed settled by the success that inflation-targeting central banks enjoyed during the so-called "Great Moderation." For instance, in his volume *Interest and Prices*, Woodford (2003) argues that central banks should *only* target the inflation of the basket of goods whose prices are updated the least frequently, because volatility in these prices is what distorts most relative prices.[1] However, the COVID-19 pandemic highlighted the value of credit policies, while the global financial crisis put the spotlight on frictions in financial intermediation. The literature has thus investigated the benefits of policy regimes different from strict inflation

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targeting, starting from flexibilizing [e.g., Woodford (2012)] to more radical rethinking [Giavazzi and Giovannini (2010)] of inflation targeting.

Our contribution is to present a New Keynesian framework with multiple instruments and to derive results that are applicable to a wide variety of models. We study the role of a second policy instrument in the general class of three-equation New Keynesian models, assuming that the second instrument may enter the Phillips curve, the investment–saving (IS) curve, and the welfare function, thus influencing inflation and output in addition to the effect of conventional interest rate policy. In the simplest New Keynesian model, the policy interest rate is sufficient to achieve economic stability because the inflation target and output at its first best level coincide—what is often called *divine coincidence* [Blanchard and Gali (2007)]—in which case a second policy instrument is not needed. Optimal monetary policy simply consists of indexing the real interest rate on the natural rate of interest.[2] But when additional elements are added to the model, this "divine coincidence" often breaks down and the conduct of monetary policy becomes more challenging [Zanetti (2006)], opening a role for second instruments. Such elements could be reduced-form, exogenous, cost-push shocks, which lead to trade-offs between reducing output volatility and inflation volatility [Taylor (1979); Clarida et al. (1999)]. Or there could be frictions beyond the nominal rigidities already included in the New Keynesian model. In models with real wage rigidities, stabilizing inflation and the output gap is not optimal [Blanchard and Gali (2007)]. In models where interest rates affect marginal costs or the timing assumption for the use of money is changed, standard policy rules may lead to indeterminacy [Surico (2008); Carlstrom and Fuerst (2001)], or monetary policy may be inefficient [the output gap and inflation both fluctuate following productivity or demand shocks; see Ravenna and Walsh (2006)]. Or there could be limits to the efficacy of standard interest rate policy—for instance, because of the zero-lower bound [Eggertsson and Woodford (2003); Debortoli et al. (2020); Ikeda et al. (2023)], because of risk premia in international capital markets [Farhi and Werning (2014)], or because of disruptions in the process of financial intermediation [Curdia and Woodford (2010)]. In each of these circumstances, it is natural to ask how a secondary, unconventional, policy instrument can alleviate the challenges faced by policymakers. Different instruments have been discussed, depending on the source of the friction: fiscal policy can support monetary policy if it is constrained [Correia et al. (2013)]; quantitative easing can help reduce credit spreads that hamper financial intermediation [Curdia and Woodford (2011)]; macroprudential policy can help resolve financial instability or aggregate demand externalities [e.g., De Paoli and Paustian (2013); Farhi and Werning (2013)]; capital controls can lean against volatile capital flows when there are shocks to risk premia [Farhi and Werning (2014)]. This literature has also touched upon the capacity of monetary policy alone to do the job [Woodford (2012)] and the need for coordination of the different policy instruments [Svensson (2014)].

In many of these papers, despite the diversity of circumstances considered, the formal models often boil down to an extended New Keynesian model, where the linearized expected IS curve and Phillips curve are affected by the "friction," by the new instrument, and where (the quadratic approximation of) the welfare function directly includes the unconventional policy instrument, typically penalizing its use. That is the general problem we study. Our objective is to provide a unified framework to draw results on the use of additional policy instruments. We show that additional policy instruments can be useful in ruling out equilibrium indeterminacy and in reducing welfare losses after exogenous shocks or in the presence of a distorted steady state, although under some circumstances, committing to *not* use the unconventional instrument may be welfare-improving. We also establish that the inflationary bias and the stabilization bias are mitigated if the central bank aggressively uses the secondary instrument. Finally, we characterize the optimal preferences for the central bank governor in cases where societal preferences would result in indeterminacy.

Section 2 presents the analytical framework, which is a general linear New Keynesian model with two policy instruments, and discusses how it relates to different strands of the literature.

Section 3 analyzes equilibrium determinacy and characterizes the stabilization bias and the inflationary bias. Section 4 discusses the optimal preferences (over inflation and over the use of the second instrument) of the central bank to mitigate the inflationary bias and the stabilization bias, given the weights in the social welfare function. Section 5 concludes by discussing some of the policy implications of our analysis.

2. Analytical framework

2.1. The extended New Keynesian framework

We want to analyze the optimal use of a secondary policy instrument, denoted θ, in a framework that comprises a Phillips curve, an expected IS curve, and a quadratic loss function. Since optimal policy under commitment is not time-consistent, we consider a purely discretionary framework in which expected values of future variables are taken as given and analyze the ways in which a central bank can reinforce its credibility.

2.1.1. The dynamic equations

A fairly general model is one in which the New Keynesian Phillips curve (NKPC) and the IS curve take the following forms:

$$\pi_{H,t} = \Phi(\pi^e_{H,t+1}, y_t, y^e_{t+1}, i_t, \theta_t, \theta^e_{t+1}, u_t), \quad y_t = \Psi(y^e_{t+1}, \pi_{H,t}, \pi^e_{H,t+1}, i_t, \theta_t, \theta^e_{t+1}, v_t)$$

where π_H is domestic inflation, y is the output gap, i is the policy interest rate, θ is the unconventional policy instrument, u and v are exogenous shocks, and Φ and Ψ are linear functions. This formulation is more general than the standard New Keynesian model. In particular, in that model, the NKPC and IS curves take the form:

$$\pi_t = \beta\pi^e_{H,t+1} + \lambda(\sigma + (\phi + \alpha))/(1 - \alpha)y, \quad y_t = y^e_{t+1} - \frac{1}{\sigma}(i_t - \pi^e_{H,t} - \bar{r})$$

where $0 < \beta < 1$ is the utility's discount rate, $\sigma > 0$ is the inverse of the risk aversion coefficient (and is equal to the intertemporal elasticity of substitution), $\phi > 0$ is the inverse of the Frisch labor supply elasticity, $0 < 1 - \alpha < 1$ is the labor share of income, and $\bar{r}$ is the natural rate of interest rate [Gali (2008)]. In this formulation, there is no additional instrument ($\partial\Phi/\partial\theta_t = \partial\Phi/\partial\theta^e_{t+1} = \partial\Psi/\partial\theta_t = \partial\Psi/\partial\theta^e_{t+1} = 0$) and the interest rate does not enter the NKPC ($\partial\Phi/\partial i_t = 0$). Independently of the specific calibration of the parameters $\alpha, \beta, \phi, \sigma$ in the standard New Keynesian model, determinacy is guaranteed because $\beta < 1$ [Gali (2008)], a case our Proposition 1 will encompass.

Moving away from the standard New Keynesian model can change coefficients, add instruments, and thus can affect essential results in the monetary policy literature, including equilibrium determinacy. Going back to the general formulation represented by Φ and Ψ, when substituting for the interest rate in the Phillips curve,[3] we can summarize the model's dynamics by the law of motion:[4]

$$\pi_{H,t} = k_\pi\pi^e_{H,t+1} + k_y y_t + k_{y^e} y^e_{t+1} + k_\theta\theta_t + k_{\theta^e}\theta^e_{t+1} + u_t \tag{1}$$

Let us describe briefly how some of the aforementioned secondary instruments can affect economic dynamics according to the existing literature.

- In Farhi and Werning (2014), capital controls introduce a wedge between domestic and foreign consumption levels and thus impact domestic output and domestic consumption asymmetrically in an open economy framework. Capital controls then enter the IS curve since they affect consumption choices. In addition, since an increase in domestic consumption increases the real wage at which domestic households supply labor, capital controls also affect firms' marginal costs and thus enter the Phillips curve. Similar equations are obtained when models assume central banks can use FX intervention to alter capital flow patterns [Cavallino (2019); Alla et al. (2020)]. We present Farhi and Werning (2014)'s model of capital controls in more detail in Section 2.3 to explain how the parameters of the law of motion are affected by the secondary instrument.

- Alla (2015) analyzes the optimal VAT and labor tax (fiscal devaluation) paths following a variety of exogenous macroeconomics shocks. The VAT affects domestic consumption and thus both the inflation rate and output dynamics. The labor tax, paid by firms, on top of wages, affects the firms' marginal cost and thus enters the Phillips curve linearly, in a way that could be described as an "endogenous cost-push shock."
- Woodford (2012), allowing for heterogeneous households whose marginal utilities of income and savings differ, models the difference between these two marginal utilities as an endogenous state variable representing a financial friction. This variable measures the distortion in the allocation of expenditure due to credit frictions and is positively related to leverage and output (in a nonlinear way). This variable impacts the IS curve, since a worsening of the financial friction affects aggregate demand. It also enters the Phillips curve since changes in this financial friction shift the relationship between aggregate real expenditure and the marginal utility of income.

2.1.2. The objective function

The objective is to minimize the welfare loss function:

$$\sum_{t=0}^{\infty} \beta^t \left[\alpha_\pi \pi_{H,t}^2 + y_t^2 + \alpha_\theta \theta_t^2\right]$$

where α_π and α_θ are the weights on inflation and on the unconventional instrument, respectively, with the weight on the output gap normalized to unity. The first two terms are standard in the New Keynesian framework and stand for the distortionary costs due to variations in inflation and output. Since the secondary instrument affects allocations, its distortive effect must also be costly (from a welfare point of view) in the quadratic approximation of the social welfare function; else, an extreme use of this instrument would not be costly from a welfare point of view, a situation that is unlikely for a tool that affects macroeconomic dynamics. Since we work in a discretionary policy framework, the central banker's problem boils down to minimizing the current term of the above expression:[5]

$$\alpha_\pi \pi_{H,t}^2 + y_t^2 + \alpha_\theta \theta_t^2 \tag{2}$$

2.2. The rationale for unconventional policy instruments

2.2.1. Breaking the divine coincidence

Since our purpose is to analyze the relevance and implications of additional policy instruments, we consider two cases where monetary policy alone cannot perfectly stabilize the economy.

The first case, which we refer to as *cost-push shocks*, represents any model element that leads to additive factors in the Phillips curve or the IS curve and breaks the divine coincidence. Since monetary policy is then insufficient to ensure perfect stabilization, the secondary instrument can help smooth economic fluctuations. These exogenous shocks have appeared in recent models, for instance: risks to financial intermediation in the version of the model of Curdia and Woodford (2016) where leverage is exogenous [see Woodford (2012)], risks premia in models of capital flows that model the investors preferences for the government bonds of different countries [Farhi and

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Werning (2014)]. Because they break the "divine coincidence," they are important for much of our analysis. Exogenous shocks that affect the welfare criterion (by moving the stabilization targets) have similar implications and can also be analyzed within our framework.

We also introduce *financial frictions* in a reduced form, as any model component that implies that the (domestic or foreign) interest rate enters the Phillips curve. This is a fairly general definition of a financial friction, since balance sheet valuation, costs of working capital, etc., can all be related to the interest rate. The cost channel of monetary policy is operative when firms' marginal costs are affected directly by the interest rate and is well documented in empirical studies. For instance, Tillman (2008) shows that the cost channel adds significantly to the explanation of inflation dynamics. Such a financial friction can lead to monetary policy indeterminacy [Surico (2008)], and it reduces the efficacy of monetary policy as a stabilization tool [Ravenna and Walsh (2006)].

2.2.2. Forward-looking determinants of inflation

The presence of such a financial friction is not, however, required for our analysis. What matters is the presence of the expected terms y^e_{t+1} and θ^e_{t+1} in the Phillips curve.[6] There is no empirical consensus on the role of expectations in the NKPC [Mavroeidis et al. (2014)], but this has not prevented the macroeconomic literature from restricting itself to future inflation and future output gap as the sole forward-looking determinants of inflation. This assumption can lead to strong policy prescriptions. For instance, Clarida et al. (1999)'s argument in favor of a conservative central banker is based on the assumption that only inflation expectations influence price-setting behaviors entering the Phillips curve. We investigate in this paper how other instruments, whose expected values could also affect current inflation, should be used by central banks.

2.3. An example

At this stage, it may help intuition to provide an example of such an extended New Keynesian model, the model by Farhi and Werning (2014) who study capital controls as an instrument of monetary policy. Farhi and Werning (2014) start from a traditional open economy New Keynesian model [as in Gali (2008)] and introduce risk premium shocks Ψ_t in the Uncovered Interest Parity (UIP) condition. These shocks capture time-varying risks premia that foreign investors require for investing in country Home.[7] In this setup, the central bank may be interested in using capital controls, represented by a time-varying tax-equivalent τ_t applied to capital inflows' returns (or a subsidy applied to capital outflows' yields). Assuming the foreign country, labeled with superscript $*$, is homogenous, is not affected by risk premium shocks, and does not use capital controls, the UIP is

$$1 + i_t = \Psi_t \left(1 + \tau_t\right) \left(1 + i^*_t\right) \frac{E_t + 1}{E_t}$$

where E_t is the Home exchange rate.[8] Under these assumptions, the solution of the consumer's optimization problem leads to a wedge Θ_t between consumption of the Home customers and consumption of the foreign customers (also called a wedge in the Backus–Smith condition):

$$C_t = \Theta_t C^*_t Q_t^{\frac{1}{\sigma}} \quad \text{with} \quad \frac{\Theta^\sigma_t}{\Theta^\sigma_{t+1}} = \Psi_t \left(1 + \tau_t\right)$$

where Q is the real exchange rate. The wedge Θ_t is a function of both the risk premium shock and capital controls.

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Under these hypotheses, Farhi and Werning (2014) show that the model collapses to the following objective function, linearized Philips curve and IS curve:[9]

$$\min_{\{i_t,\pi_{H,t},y_t,\theta_t\}} \alpha_\pi \pi_{H,t}^2 + y_t^2 + \alpha_\theta \theta_t^2 \tag{3}$$

$$\pi_{H,t} = \beta \pi_{H,t+1}^e + \kappa_y y_t + \kappa_\theta^\pi \theta_t + \kappa_{\theta^e}^\pi \theta_{t+1}^e + \kappa_f i_t + u_t \tag{4}$$

$$y_t = y_{t+1}^e - (i_t - \pi_{H,t+1}^e - \rho) + \kappa_\theta^y \theta_t + \kappa_{\theta^e}^y \theta_{t+1}^e$$

As common in open economy New Keynesian models, the welfare cost of inflation α_π is independent of the economy's openness α. However, the cost of using capital controls α_θ goes to zero in the closed economy limit and to infinity in the fully open economy scenario.[10] Indeed, the more open the economy, the more costly are the distortions associated with the trade balance [$nx_t = -\alpha_\theta \theta$ in Farhi and Werning (2014)]. Similarly, the impact of capital controls on inflation is also higher, the more open the economy: $\kappa_\theta = \lambda\alpha$, where λ only depends on the utility function discount rate and on the frequency of price updating.

Finally, it may be useful to present a micro-foundation of the cost channel of monetary policy and to show how it affects parameters in the key law of motion presented in equation (1). In Ravenna and Walsh (2006), the cost channel of monetary policy stems for a financial friction: firms must borrow an amount $W_t N_t$ from financial intermediaries, at the gross nominal interest rate R_t, to finance wages. The real marginal cost is the same for all firms and equal to $MC_t = (R_t w_t)/A_t$, where A_t is labor productivity and w_t is the real wage. Ravenna and Walsh (2006) then show that in the log-linearized setup with sticky prices, the real marginal cost depends on the nominal interest rate given the presence of a cost channel of monetary policy so that the term $\kappa_f i = \frac{\kappa_y}{1+\phi} i$ is included additively in the Philipps. Adding this cost channel to equation (4) yields the extended Philips curve:

$$\pi_{H,t} = \beta \pi_{H,t+1}^e + \kappa_y y_t + \kappa_\theta^\pi \theta_t + \kappa_{\theta^e}^\pi \theta_{t+1}^e + \kappa_f i_t + u_t \tag{5}$$

Finally, substituting for the interest rate in the Phillips curve by using the IS curve leads to the dynamic equation:

$$\pi_{H,t} = (\beta + \kappa_f)\pi_{H,t+1}^e + (\kappa_y - \kappa_f) y_t + \kappa_f y_{t+1}^e + (\kappa_\theta^\pi + \kappa_f \kappa_\theta^y)\theta_t + (\kappa_{\theta e}^\pi + \kappa_f \kappa_{\theta^e}^\pi)\theta_{t+1}^e + u_t$$

3. The need for unconventional policy instruments

3.1. Equilibrium determinacy

We first analyze the conditions under which equilibrium determinacy is guaranteed under discretionary policy. To do so, we solve the maximization problem and substitute optimal policies in equation (1) to assess the dynamics of π_H. The first-order conditions for y_t and θ_t are, respectively:

$$y_t = -\alpha_\pi k_y \pi_{H,t} \tag{6}$$

$$\theta_t = -\frac{\alpha_\pi}{\alpha_\theta} k_\theta \pi_{H,t} \tag{7}$$

Domestic inflation thus obeys the following law of motion:

$$\pi_{H,t} = \frac{k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta}\right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta}\right)} \pi_{H,t+1} \tag{8}$$

Equation (6) shows that the optimal policy is to choose a positive level of inflation together with a negative output gap (or a negative level of inflation with a positive output gap)—otherwise, if the output gap and the inflation were positive, the central bank could reduce both by increasing the interest rate. In other words, the central bank "leans against the wind," engineering a contraction if inflation is excessive. Similarly, the unconventional instrument is used to moderate inflation. We also use equation (8) to determine the conditions for equilibrium determinacy.

Proposition 1 ***[Equilibrium determinacy under discretionary policy].***

Equilibrium determinacy is ensured when the Blanchard–Kahn condition is satisfied, that is, when[11]

$$\alpha_\pi > \frac{k_\pi - 1}{k_y(k_y + k_{y^e}) + \frac{k_\theta(k_\theta + k_\theta^e)}{\alpha_\theta}} \tag{9}$$

Proof. The proof simply consists in applying the Blanchard–Kahn condition to equation (8), that is, verifying that:

$$\left| \frac{k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \right| < 1$$

□

What are the conditions under which the model leads to indeterminacy? In the standard New Keynesian model, $k_{y^e} = k_\theta = k_\theta^e = 0$ and $k_\pi = \beta < 1$. This implies that the denominator of the right-hand side of (9) is positive, that its numerator is negative, and, since $\alpha_\pi > 0$, the Blanchard–Kahn condition is satisfied.[12] Equilibrium determinacy is thus guaranteed. In the more general model, however, there are parametrizations for which the Blanchard–Kahn condition could be violated. An important situation where this could happen is when the financial friction is sufficiently large [e.g., $k_f > 1 - \beta$ in equation (4)], in which case the numerator in (9) becomes positive.

To understand the role of the second instrument in ensuring determinacy, it is useful to first understand the determinacy condition when the second instrument is not used. This is found by adding a constraint $\theta_t = 0$ to the minimization problem (2), or, alternatively, by assuming that the cost of using the secondary instrument is infinite, that is, $\alpha_\theta \to +\infty$ (so that $\theta \to 0$). The determinacy condition is then:

$$\alpha_\pi > \frac{k_\pi - 1}{k_y(k_y + k_{y^e})}$$

Denoting by X_y the recession engineered by the central bank when inflation is 1% [i.e., $X_y = \alpha_\pi k_y > 0$, from equation (6)], the first condition for equilibrium determinacy is

$$(k_y + k_{y^e})X_y > k_\pi - 1$$

Intuitively, determinacy requires that the central bank's optimal decision is to engineer recessions such that the total impact on today's inflation $1 + (k_y + k_{y^e})X_y$ is greater than the impact of expected inflation on today's inflation, k_π: this ensures that current inflation is low, ruling out multiple equilibria. However, with a financial friction, the decision to increase the interest rate also affects the marginal cost in the Phillips curve (k_π increases; this is the cost channel of monetary policy). The recession must thus be deeper, or the sensitivity of inflation to the output gap higher, to ensure marginal costs are sufficiently reduced. If the weight on inflation in the loss function is too low, the recession engineered by the central bank may be insufficient to offset the impact of the financial friction on inflation. Current inflation may then be too high, resulting in multiple equilibria.

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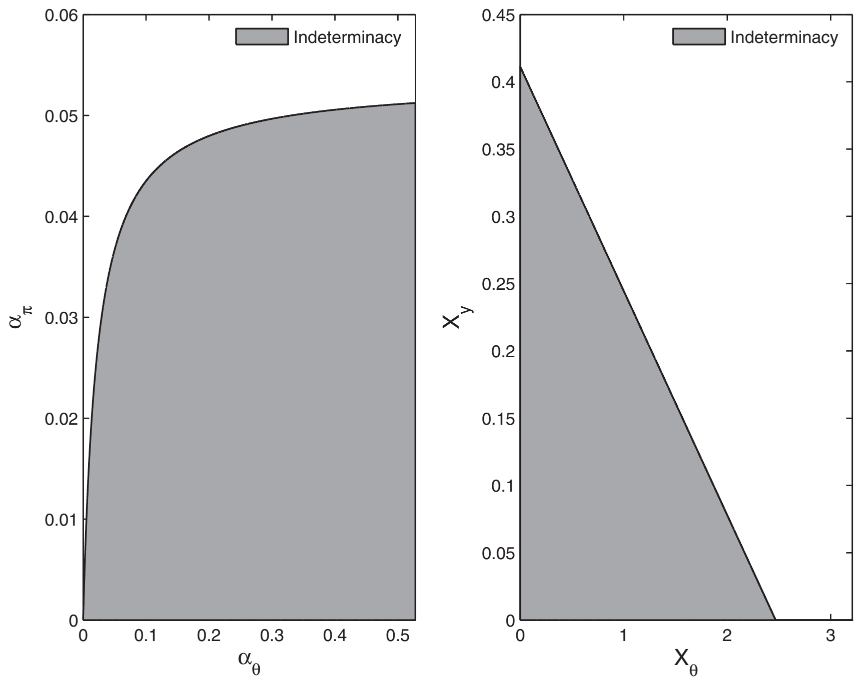

Figure 1. Optimal policy determinacy condition.

It is worth noting that with the cost channel modeled by Ravenna and Walsh (2006), $k_\pi = \beta + \frac{k_y}{\sigma}$, and thus the condition (9) indeed imposes a constraint on α_π (since k_π is likely to be larger than 1). As a result, if the weight on inflation is not high enough in the central bank's objective function, equilibrium determinacy is not guaranteed in the presence of a cost channel of monetary policy.

We now reintroduce the second instrument and define $X_\theta = \frac{\alpha_\pi}{\alpha_\theta} k_\theta > 0$ as the marginal increase in the optimal use of the unconventional instrument for a decrease in the level of inflation. The determinacy condition becomes[13]

$$(k_y + k_{y^e})X_y + (k_\theta + k_\theta^e)X_\theta > k_\pi - 1 \tag{10}$$

The rationale is as before. The optimal use of the new instrument (and its use in period $t+1$) can mitigate current inflation, the more so if the effect of the instrument on today's inflation is high (i.e., k_θ and k_θ^e are high) and if the central bank uses this instrument aggressively (if X_θ is high). Figure 1 shows the zone of indeterminacy provided by conditions (9) and (10). When the use of the unconventional policy instrument comes at no cost ($\alpha_\theta = 0$, see left-hand chart), or when the new instrument has a strong effect on inflation (X_θ is high, see right-hand chart) the risk of indeterminacy is eliminated, even if the central bank is not very willing to engineer recessions. The downward sloping frontier in the right-hand chart clarifies the trade-off: for given impacts of the interest rate and conventional policy instruments, the central bank must either be willing to engineer large recessions or to be more activist with the second instrument.

3.2. Optimal stabilization policy following real shocks

In this section, we analyze the complementarity of policy tools by focusing on optimal stabilization policy after exogenous shocks. We assume that the model parameters are such that equilibrium

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determinacy is guaranteed. We thus focus on how the unconventional instrument is used in presence of a cost-push shock. Our objective is to find theoretical results, which is why we consider cost-push shocks that enter the Phillips curve linearly; this allows us to obtain closed-form solutions. These cost-push shocks, which are common in the literature, can also capture financial disruption, as in, for example, Curdia and Woodford (2010).

Proposition 2 ***[Optimal policy following cost-push shocks].***
Following a cost-push shock with autoregressive process $u_t = \rho_u^t u_0$, the optimal paths of inflation, output, and of the unconventional instruments are

$$\pi_{H,t} = \frac{1}{D(\rho_u)} u_0 \rho_u^t, \quad y_t = -X_y \pi_{H,t}, \quad \theta_t = -X_\theta \left[\rho_u^t - \frac{1-\beta}{1-\beta\rho} \right] u_0 \tag{11}$$

where[14]

$$D(\rho_u) = 1 - \rho_u k_\pi + \alpha_\pi \left[k_y (k_y + \rho_u k_{y^e}) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\alpha_\theta} \right]$$

Proof. The proof consists of iterating forward equation (8):

$$\pi_{H,t} = \sum_{i=0}^{\infty} \left(\frac{k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \right)^i \frac{1}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} u_0 \rho_u^{t+i}$$

□

Using the unconventional instrument enables the central bank to stabilize inflation and output more efficiently. The impact of the unconventional instrument is captured by the term $\frac{\alpha_\pi}{\alpha_\theta} k_\theta (k_\theta + \rho_u k_\theta^e)$ in $D(\rho_u)$. This formula is intuitive: the stabilization power of the second instrument is increasing in its current impact on inflation (coming from both current and expected actions) and is decreasing in the cost of using it.

As long as this term is positive, the impact of the cost-push shock on the economy is minimized thanks to the availability of the unconventional policy instrument.[15] This is notably the case in Farhi and Werning (2014), in which $k_\theta^e = 0$. As a result, Farhi and Werning (2014) find that using capital controls helps stabilizing the economy.

However, if the impact of the expected use of the instrument more than offsets the impact of its current use (i.e., $k_\theta (k_\theta + \rho_u k_\theta^e) < 0$), then it is preferable to commit to *not* use the secondary instrument. In that case, the availability of the secondary instrument makes the economy more volatile, and a commitment to not use that instrument may be welfare-improving. This result is akin to that of Rogoff (1985a), who argues that international monetary policy coordination could affect inflation expectations and worsen the trade-off faced by central banks.[16]

4. Central banker's preferences

We analyzed above optimal policy assuming the central banker's and the social preferences coincide. However, the central bank's inability to commit to future policies restricts the space of feasible allocations, reduces its ability to stabilize the economy, and worsens social welfare. Kydland and Prescott (1977) and Barro and Gordon (1983) first showed how discretionary policy could lead to inefficient levels of inflation when the central bank targets a positive output gap (the inflationary bias). If the central bank cannot commit to future policies, it should thus target inflation more aggressively and tolerate a larger output gap in the current period in order to reduce inflation expectations, thus improving the trade-off characterized by the forward-looking

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Phillips curve [Rogoff (1985b)]. Clarida et al. (1999) extended this result by showing that even when the output objectives are realistic and the steady state is efficient, the central bank could improve its short-run trade-offs by assigning to inflation a higher cost than the true social cost (the *stabilization bias*).

We investigate in this section which central banker's preferences (with respect to the weights on inflation and on the unconventional policy instrument in the loss function) minimize the welfare losses due to the stabilization bias and to the inflationary bias. Although alternative design strategies for central banks have been proposed [in particular in Walsh (1995) and in Svensson (1997)], we focus on preference weights for simplicity. We first explore the stabilization bias and then present similar results for the inflationary bias—which may be seen as a particular case featuring a permanent shock.

4.1. The stabilization bias

If the weight that the central banker assigns to inflation is $\tilde{\alpha}_\pi$ and the weight on the unconventional instrument is $\tilde{\alpha}_\theta$, the central banker's objective is (using Proposition 2):

$$W\left(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\right) = \frac{\tilde{\alpha}_\pi + \tilde{\alpha}_\pi^2 k_y^2 + \alpha_\theta \frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2} k_\theta^2 \frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}}{\tilde{D}(\rho_u, \tilde{\alpha}_\pi, \tilde{\alpha}_\theta)^2} \frac{u_0^2}{1-\beta\rho_u^2}$$

where

$$\tilde{D}(\rho_u, \tilde{\alpha}_\pi, \tilde{\alpha}_\theta) = 1 - \rho_u k_\pi + \tilde{\alpha}_\pi \left[k_y(k_y + \rho_u k_y^e) + \frac{k_\theta(k_\theta + \rho_u k_\theta^e)}{\tilde{\alpha}_\theta} \right]$$

The central banker who should be appointed is the one whose preferences are

$$\left\{\tilde{\alpha}_\pi^{opt}, \tilde{\alpha}_\theta^{opt}\right\} = \text{argmin}\ W\left(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\right) \tag{12}$$

under the constraint that her preferences lead to equilibrium determinacy, that is,

$$\tilde{\alpha}_\pi^{opt} > \frac{k_\pi - 1}{k_y(k_y + k_{y^e}) + \frac{k_\theta(k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta^{opt}}}$$

Proposition 3 presents the solution assuming that social preferences remain in the area where equilibrium determinacy is guaranteed. Proposition 4, in the next section, will present the solution for the "dual" problem of minimizing the social cost function when the initial social preferences would be in an area of indeterminacy.

Proposition 3 ***[A conservative and interventionist central banker].***
If the social preferences are such that equilibrium determinacy is guaranteed:

(i) The central banker's optimal preferences cannot induce equilibrium indeterminacy;

(ii) When the shock is not highly persistent $\left(\rho_u < \frac{1}{k_\pi}\right)$*, the central banker's preferences that minimize welfare losses are*

$$\tilde{\alpha}_\pi = \frac{1 + \rho_u \frac{k_y^e}{k_y}}{1 - \rho_u k_\pi} \alpha_\pi; \quad \tilde{\alpha}_\theta = \frac{1 + \rho_u \frac{k_y^e}{k_y}}{1 + \rho_u \frac{k_\theta^e}{k_\theta}} \frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2} \alpha_\theta$$

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(iii) *If the shock is sufficiently persistent* $\left(\rho_u > \frac{1}{k_\pi}\right)$, *then the optimal preferences are*

$$\tilde{\alpha}_\pi = +\infty; \quad \tilde{\alpha}_\theta = \frac{1+\rho_u \frac{k_y^e}{k_y}}{1+\rho_u \frac{k_\theta^e}{k_\theta}} \frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2} \alpha_\theta$$

(iv) *The optimal weight for the unconventional instrument is*

$$\tilde{\alpha}_\theta = \frac{1+\rho_u \frac{k_y^e}{k_y}}{1+\rho_u \frac{k_\theta^e}{k_\theta}} \alpha_\theta$$

Proof. See Section A.1 in the Appendix. □

Corollary 1 ***[Optimal preferences].***

Using Proposition 3 it is possible to show how the optimal central bank's preferences deviate from social preferences:

(i) $\tilde{\alpha}_\pi \geq \alpha_\pi$;

(ii) $\tilde{\alpha}_\pi$ *is increasing in the persistence of the shocks and in the effect that future output has on current inflation* k_y^e;[17]

(iii) $\tilde{\alpha}_\theta < \alpha_\theta$ *if* $\frac{k_\theta^e}{k_\theta} > \frac{k_y^e}{k_y}$;

(iv) $\tilde{\alpha}_\theta$ *is decreasing in the persistence of the shock if* $\frac{k_\theta^e}{k_\theta} > \frac{k_y^e}{k_y}$.

Proposition 3 first shows that the optimal central banker always improves credibility and economic stability in the following sense: if the social preferences are such that equilibrium determinacy is guaranteed, then determinacy is also guaranteed under optimal preferences. In addition, determinacy may be obtained under the optimal preferences even when the social preferences are in the indeterminacy area. In other words, when an unconventional instrument is available, the optimal central banker uses it to improve its short-run trade-off and in doing so, she reduces the possibility of indeterminacy.

Proposition 3 and its corollary also show that the weight given to inflation by the optimal central banker is higher than social preferences ($\tilde{\alpha}_\pi \geq \alpha_\pi$). The advantage of appointing a "conservative central banker" even when the target for the output gap is zero was first explained in Clarida et al. (1999); because inflation depends on future output gaps, the central bank has always an incentive to promise strong future actions against inflation before reneging on its promises. Since, under rational expectations, the private sector anticipates this, inflation will be higher under discretionary policy than under commitment. A Rogoff conservative central banker can mitigate this bias. This result is valid in our more general framework.[18] In addition, the more persistent the shock, or the stronger the effect of future output on inflation, the more averse to inflation the central banker should be (if the shocks are one-off, i.e., $\rho_u = 0$, then $\tilde{\alpha}_\pi = \alpha_\pi$ because expected inflation is always zero and thus is unaffected by the commitment technology). The objective is indeed to tackle anticipations of inflation, and inflation expectations create inflation today (and the more so the higher k_π, for instance in presence of a financial friction). For very persistent shocks, when inflation is strongly influenced by expected inflation, the minimization problem (12) does not have an interior solution, and the optimal central banker is Mervyn King (1997)'s "inflation nutter," as she cannot accept any deviation of inflation from her target. Finally, the optimal weight on inflation does not depend on the presence of the second instrument: indeed, the central banker's weight on inflation does not depend on the cost of using this instrument (α_θ) or on its impact in the IS curve or the Phillips curve.

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Proposition 3 also determines what the optimal preferences for the unconventional instrument should be. The central bank should use the secondary instrument more actively than if it were following social preferences ($\tilde{\alpha}_\theta < \alpha_\theta$) if $\frac{k_{y^e}}{k_y} < \frac{k_{\theta^e}}{k_\theta}$. This condition is satisfied when the effect of future unconventional policy on inflation (relative to current policy) is larger than the effect of future conventional policy on inflation (relative to current policy).[19] This would be the case in the model of capital controls presented in Section 2.3, for instance. Using the unconventional instrument aggressively enables the central banker to tackle expectations of high inflation, thus improving the short-run trade-off she faces. The optimal central banker should then not only be conservative but also more interventionist with instruments whose future use affects substantially current economic conditions. It is also worth noting that the difference between social preferences for the use of θ and optimal proferences (i.e., the ratio $\tilde{\alpha}_\theta/\alpha_\theta$) appears to be independent of the cost of inflation α_π.

4.2. The stabilization bias when optimal preferences trigger multiple equilibria

The previous results were found assuming that under optimal preferences, equilibrium determinacy is guaranteed. But if this is not the case, who should be appointed as central banker? Assuming that the social costs of indeterminacy are large enough that it needs to be ruled out altogether, the problem can be formalized as follows:

$$\left\{\tilde{\alpha}_\pi^{copt}, \tilde{\alpha}_\theta^{copt}\right\} = \text{argmin } W\left(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\right) \tag{13}$$

subject to:

$$\tilde{\alpha}_\pi^{copt} > \frac{k_\pi - 1}{k_y(k_y + k_{y^e}) + \frac{k_\theta(k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta^{copt}}}$$

and knowing that:

$$\tilde{\alpha}_\pi^{opt} \leq \frac{k_\pi - 1}{k_y(k_y + k_{y^e}) + \frac{k_\theta(k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta^{opt}}}$$

Proposition 4 ***[Optimal preferences in situations of equilibrium indeterminacy].***
If the optimal preferences described in Proposition 3 lead to indeterminacy, then the optimal, constrained, choice $\left\{\alpha_\pi^{copt}, \alpha_\theta^{copt}\right\}$*:*

(i) is located on the determinacy frontier;

(ii) features a higher weight on inflation $\alpha_\pi^{copt} > \alpha_\pi^{opt}$*;*

(iii) features a lower weight on the unconventional instrument $\alpha_\theta^{copt} < \alpha_\theta^{opt}$ *if, and only if,* $\frac{k_\theta^e}{k_\theta} > \frac{k_y^e}{k_y}$.

Proof. See Section A.2 in the Appendix. □

Proposition 4 shows that the optimal preferences are located on the determinacy frontier, to be as close as possible to social preferences. In addition, the optimal, constrained, choice always reinforces the central bank credibility, in the sense that it features a higher inflation weight, and a lower weight on the use of the unconventional instrument if and only if the effect of the future use of the instrument on today's inflation is strong enough. The intuition is similar to that of Proposition (3). If the central banker has an instrument whose future use matters a lot for today's

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inflation, she should be more interventionist with this instrument, even though the constraint on determinacy forces her to adopt "second-best" preferences.

4.3. The inflationary bias

Finally, we undertake a similar analysis to solve for the optimal central banker's preferences in the presence of the traditional inflationary bias, that is, if the social welfare objective function targets a level of output $\bar{y}$ that is higher than its steady state value. The optimization problem is written as:

$$\min_{\{\pi_{H,t}, y_t, \theta_t\}} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\alpha_\pi \pi_{H,t}^2 + \left(y_t - \bar{y}\right)^2 + \alpha_\theta \theta_t^2\right]$$

subject to

$$\pi_{H,t} = k_\pi \pi_{H,t+1}^e + k_y y_t + k_{y^e} y_{t+1}^e + k_\theta \theta_t + k_{\theta^e} \theta_{t+1}^e$$

Proposition 5 ***[The inflationary bias].***
Assume that the optimal preferences are determinate.

(i) If current inflation depends weakly on expected inflation $\left(k_\pi < 1\right)$, the central banker's preferences that minimize welfare losses are

$$\tilde{\alpha}_\pi = \frac{1 + \frac{k_y^e}{k_y}}{1 - k_\pi} \alpha_\pi \quad \tilde{\alpha}_\theta = \frac{1 + \frac{k_y^e}{k_y}}{1 + \frac{k_\theta^e}{k_\theta}} \alpha_\theta \quad ;$$

(ii) If current inflation strongly depends on expected inflation $\left(k_\pi > 1\right)$, the central banker's preferences become

$$\tilde{\alpha}_\pi = +\infty \quad \tilde{\alpha}_\theta = \frac{1 + \frac{k_y^e}{k_y}}{1 + \frac{k_\theta^e}{k_\theta}} \alpha_\theta$$

Proof. Similar to the proof of Proposition 3. □

Since the problem is formally similar to that of the stabilization bias, the results and intuitions developed for Proposition 3 carry over.

5. Conclusion

According to the Tinbergen principle, a policymaker needs as many (independent) instruments as (independent) objectives in order to reach her bliss point. In New Keynesian models where there is divine coincidence, the twin objectives of zero inflation and zero output gap coincide, and one instrument (conventional monetary policy) is sufficient to stabilize the economy perfectly. In practice, policymaking is almost always more challenging than this result would imply, because divine coincidence does not hold; this situation is often captured in theoretical models by the presence of cost-push shocks. The optimal response when the policy interest rate is the only available instrument is then to maintain a positive output gap as long as inflation stays below target.

The recent global crises, however, have forced central banks to explore the use of new instruments, either because the interest rate was constrained (by the zero-lower bound, by fixed currency arrangements) or because new objectives arose (for financial stability, for asset prices, for the balance of payments, or for the exchange rate). These additional instruments, chosen according to

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availability and to the central bank's specific objectives, have included balance sheet operations (quantitative easing), sterilized FX intervention, macroprudential policy, fiscal devaluations, and other measures. The theoretical literature followed suit in justifying the use of such instruments in microfounded models.

But the literature is yet to arrive at a consensus on when and how to use these instruments and how to coordinate their use with the central bank's traditional tool, the policy interest rate.[20] The purpose of this paper has been to contribute to this literature by addressing the issue of instruments and objectives in a general but tractable framework of discretionary policy and to examine how some of the key results in the monetary policy literature (determinacy, inflationary bias, discretionary bias, and conservative central banker) carry over to a situation in which the central bank has additional instruments available. We establish that such additional instruments are useful in ensuring equilibrium determinacy and reducing economic volatility in presence of cost-push shocks, although under some specific parameterizations it is possible that committing to *not* use the unconventional instrument is optimal.

We also examined whether the intuition of Rogoff (1985b)'s conservative central banker holds in a model with several instruments. We find that if the future use of the unconventional instrument has relatively more importance for inflation than the future output gap, then the optimal central banker is more interventionist with the instrument than social preferences would imply. In addition, we investigated how a conservative central banker could reduce the risk of equilibrium indeterminacy.

Extensions to our framework could include incorporating an explicitly stochastic setup (though in many situations, models are linearized and stochastic exogenous shocks do not change the results). More important, therefore, may be to allow for nonlinear dynamics. This is particularly relevant for financial stability problems, characterized by abrupt transitions and regime-switching [Woodford (2012)]. Finally, since the policy implications of these models depend on the coefficients that capture the effects of current and future instruments on current inflation, an important task for empirical analysis is to improve our knowledge of the Phillips curve and in particular the impact of unconventional instruments on economic activity and inflation.

Notes

1 According to that argument, asset prices, which adjust at high frequency and thus reflect well the market view of relative prices, should not be part of the inflation measure that guides monetary policy decisions.

2 The interest rate that would prevail at the flexible allocation.

3 We decide to keep the Phillips curve since it is the relevant dynamic equation in the standard New Keynesian model, in which the interest rate allows to control output in the IS curve. However, this is without the loss of generality. The substitution is only possible if the interest rate enters the Phillips curve. If not, the NKPC direclty takes the form of equation (1).

4 It is worth noting that our results would apply to any dynamic optimization problem taking this form, not just models where the state variables are output and inflation. The only important ingredient is that the unconventional instrument affects the variable the central bank wants to stabilize.

5 If the unconventional policy instrument has budgetary implications (for the Treasury, the central bank, or the country as a whole), one may need to take into account an intertemporal budget constraint, although this does not have major consequences on the model results. Such a model is presented in Alla et al. (2020).

6 One case where this is found is when the interest rate enters the Phillips curve by substituting it using the IS curve, but this is not the only situation where this could happen.

7 The risk premiums, which are not required by Home country consumers, can be micro-founded in several ways; see Yakhin (2022)

8 An increase in E_{t+1} captures an expected depreciation of the Home exchange rate and thus requires a higher Home interest rate.

9 Farhi and Werning (2014) model is written in continuous time, but we show an equivalent discrete-time presentation. In addition, they focus on the solution under commitment. A final difference is that we abstract from the steady-state variables in the objective function:

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10 $\alpha_\theta = \frac{\alpha}{1+\phi}\left(\frac{2-\alpha}{1-\alpha}+1-\alpha\right)$

11 And $\alpha_\pi < \frac{k_\pi+1}{k_y(k_{y^e}-k_y)+\frac{k_\theta}{\alpha_\theta}(k_\theta^e-k_\theta)}$ if $k_y(k_{y^e}-k_y)+\frac{k_\theta}{\alpha_\theta}(k_\theta^e-k_\theta)>0$. If the expected impact of output and the second instrument on inflation is larger than the current one (a case that would occur under some parametrization), optimal policy may result in expected inflation being stabilized more efficiently than current inflation, leading to indeterminacy. Even though such a situation appears counterintuitive (in particular, inflation would change sign at each date), it can be avoided by ensuring that the weight on inflation α_π is not too high: the expected impact is then offset by the indexation of current inflation on expected inflation, ensuring determinacy. We omit this condition in the rest of this section.

12 Moreover, the second condition, detailed in the previous footnote, does not apply since $k_{y^e}=k_{\theta^e}=0$.

13 A second condition is $\alpha_\pi < \frac{k_\pi+1}{k_y(k_{y^e}-k_y)+\frac{k_\theta}{\alpha_\theta}(k_\theta^e-k_\theta)}$ if $k_y(k_{y^e}-k_y)+\frac{k_\theta}{\alpha_\theta}(k_\theta^e-k_\theta)>0$.

14 Note that $D(\rho_u)>0 \Leftrightarrow \alpha_\pi > \frac{\rho_u k_\pi - 1}{k_y(k_y+\rho_u k_f)+\frac{k_\theta(k_\theta+\rho_u k_\theta^e)}{\alpha_\theta}} \Leftrightarrow 1+\alpha_\pi\left[k_y^2+\frac{k_\theta^2}{\alpha_\theta}\right] > \rho_u\left[k_\pi - \alpha_\pi\left(k_y k_{y^e}+\frac{k_\theta k_\theta^e}{\alpha_\theta}\right)\right]$, which is always true since the last inequality is verified for $\rho_u=1$ in the Blanchard–Kahn condition (9) (and if the last bracket is negative, the result is trivial).

15 This is always the case, for instance, if the future unconventional instrument does not enter the Phillips curve and the IS curve (in which case $k_\theta^e=0$).

16 Rogoff's result may seem counterintuitive. Since the central bank under coordination could always choose the same policies as it would under the Nash equilibrium, it would seem by revealed preferences that it could never be worse off under the cooperative equilibrium than under the Nash. Likewise, here since the central bank could always choose not to use the second instrument, it would appear that its availability could never make the central bank worse off. In both examples, the revealed preferences argument breaks down because of the presence of forward-looking private sector and the inability of the central bank to commit to future policies.

17 In Section 2.3, this effect is captured by the financial friction coefficient κ_f.

18 Our results for $\tilde{\alpha}_\pi$ are the same as those in Clarida et al. (1999) when $k_y^e=0$.

19 Output is the reference since its weight is normalized to 1 in the objective function.

20 A case in point is that of the central bank of Sweden, which split over the decision to use interest rate policy to reduce risks to financial stability [Svensson (2011); Svensson (2014)].

21 Since determinacy is ensured, $D>0$.

22 Since it cancels out only once when $\tilde{\alpha}_\pi$ is large enough

23 Fomally, for any couple $\{\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\}$ we have

$$\tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right) > \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)$$

Then, denoting $g(\tilde{\alpha}_\pi) = \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)$,we have

$$g'(\tilde{\alpha}_\pi) = \frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial \tilde{\alpha}_\pi} + \frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}} \frac{\partial \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}}{\partial \tilde{\alpha}_\pi} = \frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial \tilde{\alpha}_\pi}$$

by definition of $\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}}$. We then see that $g(\tilde{\alpha}_\pi) > \lim_{\tilde{\alpha}_\pi \to \infty} g(\tilde{\alpha}_\pi)$, for example:

$$\tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right) > \tilde{W}\left(+\infty, \frac{1}{\tilde{\alpha}_\theta(+\infty)}\right).$$

24 Since the constraint frontier is concave and the optimal point is located below the frontier, the line passing through the origin and the optimal point cuts the frontier once for $\tilde{\alpha}_\theta > \tilde{\alpha}_\theta^{opt}$.

25 Since $a > b\tilde{\alpha}_\pi^{opt}$, the denominator is always stricly positive.

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Appendix A

A.1. Proof of Propositions 3

A.1.1. Planning problem. The central banker has to solve the following problem to determine her optimal preferences:

$$\min W\left(\tilde{\alpha}_\pi,\tilde{\alpha}_\theta\right)=\frac{\alpha_\pi+k_y^2\tilde{\alpha}_\pi^2+\alpha_\theta k_\theta^2\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2}}{\left[1-\rho_u k_\pi+\tilde{\alpha}_\pi\left[k_y\left(k_y+\rho_u k_y^e\right)+\frac{k_\theta\left(k_\theta+\rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]\right]^2}\frac{u_0^2}{1-\beta\rho_u^2}$$

subject to

$$\tilde{\alpha}_\pi>\frac{k_\pi-1}{k_y(k_y+k_{y^e})+\frac{k_\theta\left(k_\theta+k_\theta^e\right)}{\tilde{\alpha}_\theta}}$$

We assume that the constraint is satisfied for the social preferences. We verify *ex post* that the constraint is also satisfied for the optimal preferences. We denote

$$\tilde{W}(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta})=\frac{\alpha_\pi+k_y^2\tilde{\alpha}_\pi^2+\alpha_\theta k_\theta^2\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2}}{\left[1-\rho_u k_\pi+\tilde{\alpha}_\pi\left[k_y\left(k_y+\rho_u k_y^e\right)+\frac{k_\theta\left(k_\theta+\rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]\right]^2}\frac{u_0^2}{1-\beta\rho_u^2}=\frac{N}{D^2}\frac{u_0^2}{1-\beta\rho_u^2}$$

where[21]

$$N=\alpha_\pi+k_y^2\tilde{\alpha}_\pi^2+\alpha_\theta k_\theta^2\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2}\quad,D=1-\rho_u k_\pi+\tilde{\alpha}_\pi\left[k_y\left(k_y+\rho_u k_y^e\right)+\frac{k_\theta\left(k_\theta+\rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]$$

A.1.2. Optimal preferences. We then compute the partial derivatives:

$$\begin{aligned}\frac{\partial\tilde{W}\left(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta}\right)}{\partial\tilde{\alpha}_\pi}&=2\frac{\left[k_y^2+\frac{\alpha_\theta k_\theta^2\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2\tilde{\alpha}_\theta^2}\right]\tilde{\alpha}_\pi D-\left[k_y\left(k_y+\rho_u k_y^e\right)+\frac{k_\theta\left(k_\theta+\rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]N}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}\\&=2\frac{\left[k_y^2+\frac{\alpha_\theta k_\theta^2\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2\tilde{\alpha}_\theta^2}\right]\left[1-\rho_u k_\pi\right]\tilde{\alpha}_\pi-\left[k_y\left(k_y+\rho_u k_y^e\right)+\frac{k_\theta\left(k_\theta+\rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right]\alpha_\pi}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}\end{aligned}$$

We need to consider two cases:

- if $\rho_u k_\pi<1$, there is an interior point where the partial derivative $\frac{\partial\tilde{W}\left(\tilde{\alpha}_\pi,\frac{1}{\tilde{\alpha}_\theta}\right)}{\partial\tilde{\alpha}_\pi}$ is equal to zero.
- if $\rho_u k_\pi>1$, this derivative is negative for any value of $\{\tilde{\alpha}_\pi,\tilde{\alpha}_\theta\}$, the optimal solution is then $\tilde{\alpha}_\pi=+\infty$. The welfare loss converges to a finite value since it is bounded from below by 0.

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The second partial derivative can be expressed as follows:

$$\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}} = 2\frac{\tilde{\alpha}_\pi k_\theta \left[\frac{\alpha_\theta k_\theta \beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} D - \left(k_\theta + \rho_u k_\theta^e\right) N\right]}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}$$

$$= \frac{\tilde{\alpha}_\pi k_\theta \left[\frac{\alpha_\theta k_\theta \beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta}\left[1-\rho_u k_\pi + k_y\left(k_y+\rho_u k_y^e\right)\tilde{\alpha}_\pi\right] - \left(k_\theta+\rho_u k_\theta^e\right)\left(\alpha_\pi + k_y^2\tilde{\alpha}_\pi^2\right)\right]}{D^3} \times \frac{u_0^2}{1-\beta\rho_u^2}$$

If $1 > \rho_u k_\pi$, this second derivative necessarily admits an interior cancelation point. Let us first consider this case.

In this situation, each partial derivative cancels and changes signs in one point (for a given value of the other parameter). There is thus only one interior point in which the two derivatives cancel simultaneously. Since they also change signs in this point (from being negative to positive), this is the global minimum.

Using the partial derivatives formulations with N and D, we see that this interior point verifies

$$\left[k_y^2 + \frac{\alpha_\theta k_\theta^2 \beta(1-\rho_u)^2}{(1-\beta\rho_u)^2 \tilde{\alpha}_\theta^2}\right]\tilde{\alpha}_\pi D = \left[k_y\left(k_y + \rho_u k_y^e\right) + \frac{k_\theta\left(k_\theta + \rho_u k_\theta^e\right)}{\tilde{\alpha}_\theta}\right] N$$

$$\frac{\alpha_\theta k_\theta^2 \beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} D = \left(k_\theta + \rho_u k_\theta^e\right) N$$

By dividing the two equations, we find that:

$$\tilde{\alpha}_\theta^{opt} = \frac{\left(k_y + \rho_u k_y^e\right) k_\theta}{k_y\left(k_\theta + \rho_u k_\theta^e\right)}\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\alpha_\theta = \frac{1+\rho_u\frac{k_y^e}{k_y}}{1+\rho_u\frac{k_\theta^e}{k_\theta}}\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\alpha_\theta;$$

We then substitue for $\tilde{\alpha}_\theta^{opt}$ in any of the above equations and find that the optimal choice for inflation is

$$\tilde{\alpha}_\pi^{opt} = \frac{1+\rho_u\frac{k_y^e}{k_y}}{1-\rho_u k_\pi}\alpha_\pi;$$

If $1 < \rho_u k_\pi$, we saw that the optimal choice for the inflation coefficient is $\tilde{\alpha}_\pi^{opt} = +\infty$. Using the second equality for the partial derivative[22] $\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}}$, we find that the optimal choice for $\tilde{\alpha}_\theta^{opt}$ verifies[23]

$$\tilde{\alpha}_\theta^{opt}\left(\tilde{\alpha}_\pi\right) = \frac{\frac{\alpha_\theta k_\theta \beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\tilde{\alpha}_\pi\left[1-\rho_u k_\pi + k_y\left(k_y+\rho_u k_y^e\right)\tilde{\alpha}_\pi\right]}{\left(k_\theta + \rho_u k_\theta^e\right)\left(\alpha_\pi + k_y^2\tilde{\alpha}_\pi^2\right)} \xrightarrow[\tilde{\alpha}_\pi\to+\infty]{} \frac{1+\rho_u\frac{k_y^e}{k_y}}{1+\rho_u\frac{k_\theta^e}{k_\theta}}\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\alpha_\theta$$

The coefficient for the unconventional tool is then unchanged, and the optimal choice is

$$\tilde{\alpha}_\pi^{opt} = +\infty, \quad \tilde{\alpha}_\theta^{opt} = \frac{1+\rho_u\frac{k_y^e}{k_y}}{1+\rho_u\frac{k_\theta^e}{k_\theta}}\frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2}\alpha_\theta;$$

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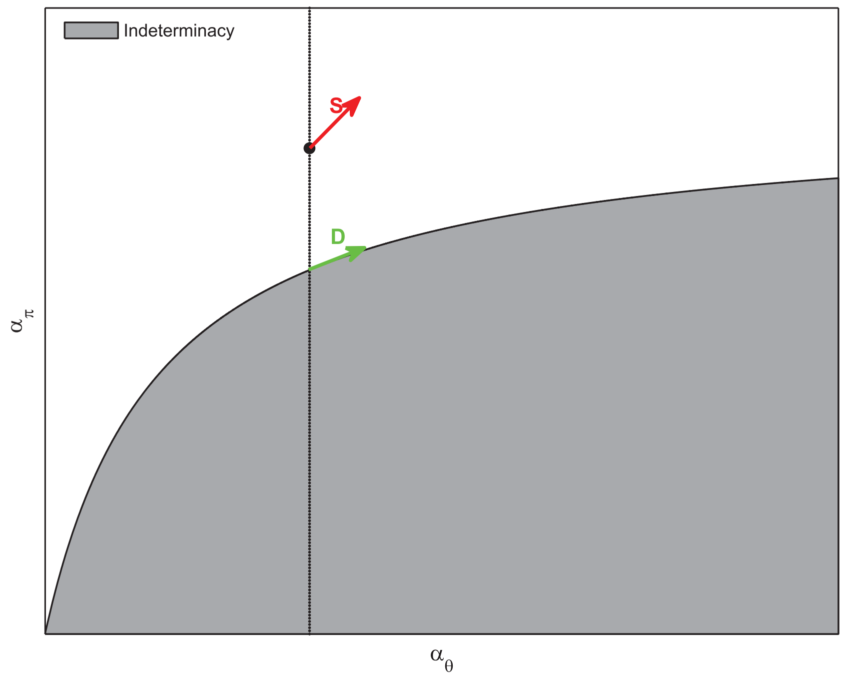

Figure 2. Optimal preferences determinacy.

A.1.3. Optimal preferences determinacy. Let us finally prove that if the determinacy constraint is satisfied for the social preferences $\{\alpha_\pi, \alpha_\theta\}$, it is then satisfied for the optimal preferences chosen by the central banker.

Given that the frontier is concave [see Figure 1 and equation (9)], and since $\alpha_\pi^{opt} \geq \alpha_\pi$, we see that if $\alpha_\theta^{opt} \leq \alpha_\theta$, then the optimal preferences are also determined.

We then consider the case when the unconventional instrument is less forward-looking than ouput (i.e., $\frac{k_y^e}{k_y} > \frac{k_\theta^e}{k_\theta}$), potentially inducing an optimal cost that is higher than the social one. The slope of the optimal deviation is then:

$$S = \frac{\alpha_\pi^{opt} - \alpha_\pi}{\alpha_\theta^{opt} - \alpha_\theta} = \frac{\frac{k_y^e}{k_y} + k_\pi}{\left(\frac{k_y^e}{k_y} - \frac{k_\theta^e}{k_\theta}\right)\left(1 - \rho_u k_\pi\right)} \frac{\alpha_\pi}{\alpha_\theta} \geq \frac{\alpha_\pi}{\alpha_\theta}$$

We want to compare this slope to the frontier derivative for $\alpha_\theta = \tilde{\alpha}_\theta$. Since the frontier is stricly concave, if S is greater than its derivative, the optimal preferences are in the determinacy area. Figure 2 illustrates the proof.

The frontier can be parametrized as follows:

$$\tilde{\alpha}_\pi^{fr}\left(\tilde{\alpha}_\theta\right) = \frac{a\tilde{\alpha}_\theta}{1 + b\tilde{\alpha}_\theta}$$

where $a = \frac{k_\pi - 1}{k_\theta(k_\theta + k_\theta^e)}$ and $b = \frac{k_y(k_y + k_{ye})}{k_\theta(k_\theta + k_\theta^e)}$. Its derivative for $\tilde{\alpha}_\theta = \alpha_\theta$ is then equal to $D = \frac{a}{(1 + b\tilde{\alpha}_\theta)^2}$

Since the social preferences are located above the determinacy frontier, we have

$$\alpha_\pi \geq \frac{a\alpha_\theta}{1 + b\alpha_\theta}$$

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We finally get that:

$$S \geq \frac{\alpha_\pi}{\alpha_\theta} \geq \frac{a}{1+b\tilde{\alpha}_\theta} \geq \frac{a}{\left(1+b\tilde{\alpha}_\theta\right)^2} = D$$

This proves that if the social preferences are determinate, the optimal ones are too. In this sense, the optimal central banker preferences strenghten its credibility.

A.2. Proof of Proposition 4

We consider that the optimal choice, as defined in Section A.1, leads to indeterminacy, for example:

$$\tilde{\alpha}_\pi^{opt} \leq \frac{k_\pi - 1}{k_y(k_y + k_{y^e}) + \frac{k_\theta(k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta^{opt}}} \tag{14}$$

The determinacy constraint (9) assumes that the inflation weight should be stricly above the frontier. However, we show below that the solution to the problem that includes the border is unique and located on the border.

It is then easy to see that the solution to the strict inequality problem will be in the neighborhood of the above point (there would be no solution *per se*, but a sequence converging to this point). We will then consider that the solution to the problem (13) is located on the border.

A.2.1. Solution location. Let us first prove that the solution to the constrained problem is located on the determinacy frontier. To that end, we reformulate the partial derivatives:

$$\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right)}{\partial \tilde{\alpha}_\pi} = 2\frac{k_y^2}{1-\rho_u k_\pi}\frac{\tilde{\alpha}_\pi - \tilde{\alpha}_\pi^{opt} + \frac{k_\theta\left(k_\theta + \rho_u k_\theta^e\right)}{k_y\left(k_y + \rho_u k_y^e\right)}\left[\frac{\tilde{\alpha}_\theta^{opt}}{\tilde{\alpha}_\theta}\tilde{\alpha}_\pi - \tilde{\alpha}_\pi^{opt}\right]}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}$$

We then see that if $\tilde{\alpha}_\pi > \tilde{\alpha}_\pi^{opt}$ and $\tilde{\alpha}_\pi > \frac{\tilde{\alpha}_\theta}{\tilde{\alpha}_\theta^{opt}}\tilde{\alpha}_\pi^{opt}$, the welfare loss is strictly increasing with $\tilde{\alpha}_\pi$. Similarly,

$$\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}} = 2\frac{\tilde{\alpha}_\pi k_\theta k_y\left(k_\theta + \rho_u k_\theta^e\right)}{k_y + \rho_u k_y^e}\frac{\left(\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta}\tilde{\alpha}_\theta^{opt} - \tilde{\alpha}_\pi^{opt} + k_y^2\tilde{\alpha}_\pi^2\tilde{\alpha}_\pi^{opt}\left[\frac{\tilde{\alpha}_\theta^{opt}}{\tilde{\alpha}_\theta} - 1\right]\right)}{D^3}\frac{u_0^2}{1-\beta\rho_u^2}$$

Since $\frac{\partial W(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta)}{\partial \tilde{\alpha}_\theta} = -\tilde{\alpha}_\theta^2\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}}$, the welfare loss is stricly decreasing (resp. increasing) with $\tilde{\alpha}_\theta$ when $\tilde{\alpha}_\pi > \frac{\tilde{\alpha}_\pi^{opt}}{\tilde{\alpha}_\theta^{opt}}\tilde{\alpha}_\theta$ (resp. $\tilde{\alpha}_\pi < \frac{\tilde{\alpha}_\pi^{opt}}{\tilde{\alpha}_\theta^{opt}}\tilde{\alpha}_\theta$) and $\tilde{\alpha}_\theta < \tilde{\alpha}_\theta^{opt}$ (resp. $\tilde{\alpha}_\theta > \tilde{\alpha}_\theta^{opt}$).

To get some intuition, let us represent graphically the above dynamics (Figure 3). The red arrows represent the gradient of $W\left(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\right)$ along its partial derivatives.

We then see that if the optimal preferencs are located below the curve,[24] starting from any point located above the frontier, it is optimal to move along a direction that brings you back to the frontier or to the red part of the line passing through the origin and the optimal point.

Along this line, denoting its slope $S^{opt} = \frac{\tilde{\alpha}_\pi^{opt}}{\tilde{\alpha}_\theta^{opt}}$ and the ratio $R = \frac{k_\theta\left(k_\theta + \rho_u k_\theta^e\right)}{k_y\left(k_y + \rho_u k_y^e\right)}$, the welfare loss can be expressed as follows:

$$W\left(\tilde{\alpha}_\pi\right) = \frac{\frac{\alpha_\pi}{k_y^2} + RS^{opt}\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^2}{\left[\frac{\alpha_\pi}{k_y^2} + RS^{opt}\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}\tilde{\alpha}_\pi\right]^2}$$

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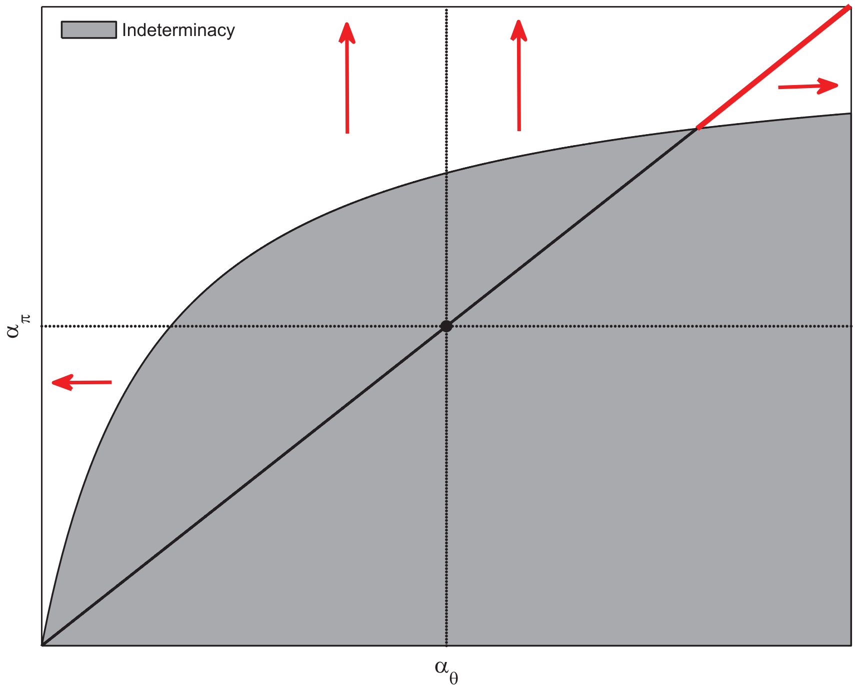

Figure 3. Welfare loss variations in the determinacy area.

This function derivative is

$$W'\left(\tilde{\alpha}_\pi\right) = \frac{\left(\frac{\alpha_\pi}{k_y^2} + RS^{opt}\tilde{\alpha}_\pi^{opt}\right)\left(\tilde{\alpha}_\pi - \tilde{\alpha}_\pi^{opt}\right)}{\left[\frac{\alpha_\pi}{k_y^2} + RS^{opt}\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}\tilde{\alpha}_\pi\right]^3}$$

We see that the welfare loss is strictly increasing along this ray for $\tilde{\alpha}_\pi > \tilde{\alpha}_\pi^{opt}$. It is then optimal to get back to the frontier on the red part of the line too.

We then proved that the solution to the problem featuring a lower or equal sign is located on the determinacy border.

A.2.2. Solution determination. Since the solution of the constrained problem is located on the determinacy frontier, using the frontier parametrization introduced in Section A.1, the optimal parameters are linked by the following relation:

$$\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} = a - b\tilde{\alpha}_\pi \tag{15}$$

Using the above notations, the welfare loss can then be expressed as follows:

$$W\left(\tilde{\alpha}_\pi\right) = \frac{\frac{\alpha_\pi}{k_y^2} + R\tilde{\alpha}_\theta^{opt}\left(a - b\tilde{\alpha}_\pi\right)^2 + \tilde{\alpha}_\pi^2}{\left[\frac{\alpha_\pi}{k_y^2} + \tilde{\alpha}_\pi^{opt}\left[\tilde{\alpha}_\pi + R\left(a - b\tilde{\alpha}_\pi\right)\right]\right]^2} = \frac{\frac{\alpha_\pi}{k_y^2} + R\tilde{\alpha}_\theta^{opt}a^2 - 2abR\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi + \left(1 + Rb^2\tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\pi^2}{\left[\frac{\alpha_\pi}{k_y^2} + aR\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}\left(1 - bR\right)\tilde{\alpha}_\pi\right]^2}$$

Its derivative is then equal to:

$$W'(\tilde{\alpha}_\pi) = \frac{\left[\frac{\alpha_\pi}{k_y^2}\left(1 + Rb^2\tilde{\alpha}_\theta^{opt}\right) + aR\tilde{\alpha}_\pi^{opt}\left(1 + b\tilde{\alpha}_\theta^{opt}\right)\right]\tilde{\alpha}_\pi - \left[\frac{\alpha_\pi}{k_y^2}\left[\tilde{\alpha}_\pi^{opt} + bR\left(a\tilde{\alpha}_\theta^{opt} - \tilde{\alpha}_\pi^{opt}\right)\right] + a^2R\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi^{opt}\right]}{\left[\frac{\alpha_\pi}{k_y^2} + aR\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}\left(1 - bR\right)\tilde{\alpha}_\pi\right]^3}$$

The cancelation point, that corresponds to the constrained optimal, is then unique and defined by:

$$\tilde{\alpha}_\pi^{copt} = \frac{\frac{\alpha_\pi}{k_y^2}\left[\tilde{\alpha}_\pi^{opt} + bR\left(a\tilde{\alpha}_\theta^{opt} - \tilde{\alpha}_\pi^{opt}\right)\right] + a^2R\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi^{opt}}{\frac{\alpha_\pi}{k_y^2}\left(1 + Rb^2\tilde{\alpha}_\theta^{opt}\right) + aR\tilde{\alpha}_\pi^{opt}\left(1 + b\tilde{\alpha}_\theta^{opt}\right)}$$

We want to compare this constrained optimal to the unconstrained optimal choice. After some algebra, we get

$$\tilde{\alpha}_\pi^{copt} - \tilde{\alpha}_\pi^{opt} = \frac{R\left(b\frac{\alpha_\pi}{k_y^2} + a\tilde{\alpha}_\pi^{opt}\right)\left[a\tilde{\alpha}_\theta^{opt} - \left(1 + b\tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\pi^{opt}\right]}{\frac{\alpha_\pi}{k_y^2}\left(1 + Rb^2\tilde{\alpha}_\theta^{opt}\right) + aR\tilde{\alpha}_\pi^{opt}\left(1 + b\tilde{\alpha}_\theta^{opt}\right)}$$

Since the optimal preferences are indeterminate, following equation (14), we have

$$a\tilde{\alpha}_\theta^{opt} > \left(1 + b\tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\pi^{opt}$$

The optimal constrained inflation choice is then always above the optimal unconstrained point.

We now want to determine the location of the constrained optimum for the unconventionnal instrument. Using the frontier equation (15) and the above formula for $\tilde{\alpha}_\pi^{sb}$, we get after some algebra:[25]

$$\tilde{\alpha}_\theta^{copt} = \frac{\frac{\alpha_\pi}{k_y^2}\left[\tilde{\alpha}_\pi^{opt} + bR\left(a\tilde{\alpha}_\theta^{opt} - \tilde{\alpha}_\pi^{opt}\right)\right] + a^2R\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi^{opt}}{\frac{\alpha_\pi}{k_y^2}\left[a + b(bR - 1)\tilde{\alpha}_\pi^{opt}\right] + a^2R\tilde{\alpha}_\pi^{opt}}$$

We finally compute the difference between the constrained optimal and the unconstrained optimal for the unconventional instrument. We get

$$\tilde{\alpha}_\theta^{copt} - \tilde{\alpha}_\theta^{opt} = \frac{\frac{\alpha_\pi}{k_y^2}(bR - 1)\left[a\tilde{\alpha}_\theta^{opt} - \left(1 + b\tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\pi^{opt}\right]}{\frac{\alpha_\pi}{k_y^2}\left[a + b(bR - 1)\tilde{\alpha}_\pi^{opt}\right] + a^2R\tilde{\alpha}_\pi^{opt}}$$

$bR = \frac{1+\rho_u\frac{k_y^e}{k_y}}{1+\rho_u\frac{k_\theta^e}{k_\theta}}$ is simply the ratio of output and unconventional instrument forward-looking impacts.

When it is smaller (resp. larger) than 1, for example, the unconventional instrument is more (resp. less) forward-looking than output, the constrained optimum uses more (resp. less) aggressively this instrument.

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