LIBRATION OF THE MOON :

SHAPE OF THE EARTH AND MOTION OF THE ECLIPTIC PLANE

M. Moons

Dept. of Mathematics, Facultés Universitaires N.D. de la Paix, B-5000 Namur, Belgium

ABSTRACT. Very accurate theories of the libration of the Moon have been recently built by Migus (1980), Eckhardt (1981, 1982) and Moons (1982, 1984). All of them take into account the perturbation due to the Earth and the Sun on the motion of a rigid Moon about its center of mass. Additional perturbations (influence of the planets, shape of the Earth, elasticity of the Moon, ...) are also often included.

We present here the perturbations due to the shape of the Earth and the motion of the ecliptic plane on our theory which already contains planetary perturbations. This theory is completely analytical with respect to the harmonic coefficients of the lunar gravity field which is expanded in spherical harmonics up to the fourth order. The ELP 2000 solution (Chapront and Chapront-Touzé, 1983) supplies us with the motion of the center of mass of the Moon.

1. INTRODUCTION

Any perturbation to the "main problem" of the libration of the Moon (Moons, 1982) is to be introduced in the Hamiltonian in two different ways : directly and indirectly (i.e., through the motion of the center of mass). More precisely, the direct perturbation due to the oblateness of the Earth is the figure-figure interaction and the one for the motion of the ecliptic plane is introduced in the Hamiltonian by the addition of the term $-\vec{L}.\vec{\omega}$.

In both cases, the indirect perturbation reads

$$\delta H = \frac{\partial H_{MP}}{\partial r^*}\,\delta r^* + \frac{\partial H_{MP}}{\partial \gamma^*}\,\delta\gamma^* + \frac{\partial H_{MP}}{\partial \lambda^*}\,\delta\lambda^*$$

with H_{MP} the Hamiltonian of the main problem,

r^* the Earth-Moon distance,

(γ^*,λ^*) the latitude and the longitude of the Earth in the system of the principal axes of inertia of the Moon,

$(\delta r^*,\delta\gamma^*,\delta\lambda^*)$ the perturbations on (r^*,γ^*,λ^*) .

J. Kovalevsky and V. A. Brumberg (eds.), Relativity in Celestial Mechanics and Astrometry, 141–144.

https://doi.org/10.1017/S0074180900148119 Published online by Cambridge University Press

In our theory, the quantities $(r^*, \gamma^*, \lambda^*)$, as well as their perturbations, come from the ELP 2000 solution for the motion of the center of mass of the Moon (Chapront and Chapront-Touzé, 1983).

2. SHAPE OF THE EARTH

The figure-figure interaction is not yet included here. Following Eckhardt (Eckhardt, 1981), the resulting perturbations are

$$\delta p_1 = 0''073 \sin \zeta - 0''005 \sin F ,$$

$$\delta p_2 = 0''073 \cos \zeta - 0''005 \cos F ,$$

$$\delta \tau = 0''007 \sin \Omega$$

where $\zeta = \lambda + p$ and $\Omega = h + p$ with p the precession angle and $\lambda = \ell + g + h = F + h$. If we want to compare both theories, we thus have to substract these quantities from Eckhardt's solution (Eckhardt, 1982).

Our results, computed for the 500 libration set of parameters, are given in Table 1.

A global comparison with Eckhardt's theory will be given at the end of this paper.

SERIES	COS	SIN	TERM
p_1	0.008 0. 0.	8.515 0.104 -0.016	ζ $\zeta - 2F$ $2\zeta - F$
p_2	8.514 -0.104 -0.016	-0.008 0. 0.	ζ $\zeta - 2F$ $2\zeta - F$
τ	0. 0. 0. 0.	7.704 -0.043 -0.013 -0.011	$\zeta - F$ $2\zeta - F$ $\zeta + F - 2\ell$ $2F - 2\ell$

Table 1. Shape of the Earth, indirect effect (terms $\geq 0''005$).

3. MOTION OF THE ECLIPTIC PLANE

For the direct effect, we have to add the term $-\vec{L}.\vec{\omega}$ to the Hamiltonian.

The adopted value for $\vec{\omega}$ is

$$\vec{\omega} = [s_1 \sin(g'+h') + c_1 \cos(g'+h')]\, \vec{e}_1 + \\ + [s_1 \cos(g'+h') - c_1 \sin(g'+h')]\, \vec{e}_2$$

https://doi.org/10.1017/S0074180900148119 Published online by Cambridge University Press

with $c_1 = -46''.8093$,

$s_1 = 4''.1997$,

$g' + h' = 282° \ 56' \ 14.42753$.

The resulting perturbations are given in Table 2, those for the indirect effect are given in Table 3.

SERIES	COS	SIN	TERM
p_1	6.040 -0.045 0.010	-0.544 0.004 -0.001	T + D T + D - ℓ T + D - 2 F
p_2	-0.544 0.002 0.001	-6.034 0.023 0.009	T + D T + D - ℓ T + D - 2 F
τ	0.275	-0.025	T + D - F

Table 2. Motion of the ecliptic plane, direct effect (terms $\geq 0''.005$).

SERIES	COS	SIN	TERM
p_1	1.391	-0.125	T + D
p_2	-0.125	-1.391	T + D
τ	0.375	-0.034	T + D - F

Table 3. Motion of the ecliptic plane, indirect effect (terms $\geq 0''.005$).

4. COMPARISONS

In order to compare our solution with the one of Eckhardt, we have to sum up the results of Tables 1, 2 and 3 and besides to include the planetary perturbations if any.

The results are given in Table 4 for the terms for which the amplitude is greater than $0''.010$ in p_1 , p_2 or in τ .

https://doi.org/10.1017/S0074180900148119 Published online by Cambridge University Press

SERIES	TERM	MOONS		ECKHARDT		DIFFERENCES (M-E)	
		COS	SIN	COS	SIN	COS	SIN
p_1	ζ	0.008	8.515	0.	8.069	0.008	0.446
	T+D	7.561	-0.723	7.447	-0.711	0.114	-0.012
	ζ-2F	0.	0.104	0.	0.047	0.	0.057
	T+D-ℓ	-0.045	0.004	-0.046	0.004	0.001	0.
	2ζ-F	0.	-0.016	0.	-0.015	0.	-0.001
	T+D-2F	0.014	-0.001	0.003	0.	0.011	-0.001
p_2	ζ	8.514	-0.008	8.069	0.	0.445	-0.008
	T+D	-0.722	-7.556	-0.711	-7.447	-0.011	-0.109
	ζ-2F	-0.104	0.	-0.047	0.	-0.057	0.
	T+D-ℓ	0.002	0.023	0.002	0.022	0.	0.001
	2ζ-F	-0.016	0.	-0.015	0.	-0.001	0.
	T+D-2F	0.001	0.014	0.	0.003	0.001	0.011
τ	ζ-F	0.	7.704	0.	8.176	0.	-0.472
	T+D-F	0.654	-0.059	0.736	-0.070	-0.082	0.011
	2ζ-2F	0.	-0.043	0.	-0.043	0.	0.
	ζ+F-2ℓ	0.	-0.013	?	?	?	?
	2F-2ℓ	0.	-0.011	?	?	?	?

Table 4. Comparisons (terms ≥ 0".01).

The " ? " denote terms that do not appear in Eckhardt's solution (truncated at 0".01).

The differences between both theories are much more important than in the main problem, or, even, in the planetary perturbations. The most important of them is coming from the perturbation due to the non-sphericity of the Earth. It will be very interesting to compute the figure-figure interaction to see exactly where it comes from. Let us remark finally that, if we drop the perturbation in distance, the value 7.704 sin(ζ-F) in τ is increased to 7.790 sin(ζ-F) and the remaining difference is -0.393 but is still important.

REFERENCES

Chapront, J., Chapront-Touzé, M. (1983). Astron. Astrophys. **124**, 50-62.
Eckhardt, D.H. (1981). The Moon and the Planets **25**, 3-49.
Eckhardt, D.H. (1982). In "High-Precision Earth Rotation and Earth-Moon Dynamics", Ed. O. Calame, 193-198.
Migus, A. (1980). The Moon and the Planets **23**, 391-427.
Moons, M. (1982). The Moon and the Planets **27**, 257-284.
Moons, M. (1984). Celestial Mechanics **34**, 263-273.