ON THE CHARACTERISTICS OF ASTRONOMICAL REFRACTION IN THE NORTHERN HEMISPHERE

C. Sugawa and N. Kikuchi
International Latitude Observatory of Mizusawa, 023 Japan

1. EXPONENTIAL REPRESENTATION OF ASTRONOMICAL REFRACTION

The astronomical refraction mainly depends on the vertical structure or the height profile of atmospheric density. Concerning the vertical structure of atmospheric density, various hypotheses were presented during the last century. Among them Newton's hypothesis of equal temperature and Ivory's one that temperature diminishes at a uniform rate with height appear to represent the height profile of temperature approximately in the lower part of the stratosphere and in the troposphere respectively.

Generalizing Newton's hypothesis, G. Teleki (1967) adopted the following exponential representation of refractive index (μ),

$$\mu - 1 = \varepsilon e^{-as}, \tag{1}$$

$$s = \frac{r}{r_0} - 1, \tag{2}$$

where r_0 is the geocentric radius of a station at the ground surface and r that at any height. The values of ε and a are to be determined from aerological data. Gladstone-Dale's law is adopted for the relation between refractive index (μ) and atmospheric density (ρ) as

$$\mu = 1 + c\rho, \tag{3}$$

where the value of c is 0.226. Combining (1) with (3), we can put

$$\rho = \varepsilon' e^{-z/H}, \tag{4}$$

where z is the height above the ground surface and H the height of homogeneous atmosphere or the scale height.

For the purpose of examining the fitness of the exponential representation of atmospheric density, we calculated the monthly mean values of atmospheric density for the year 1965 at 7 aerological stations in

E. Tengström and G. Teleki (eds.), Refractional Influences in Astrometry and Geodesy, 103–117.

Eastern Japan nearly along the meridian at Mizusawa (141°08'E), that is, Wakkanai (WA) (45°25'N, 141°41'E), Sapporo (SA) (43°03'N, 141°20'E), Akita (AK) (39°43'N, 140°06'E), Sendai (SE) (38°16'N, 140°50'E), Tateno (TA) (36°03'N, 140°08'E), Hachijojima (HA) (33°07'N,139°47'E), Torishima (TO) (30°29'N, 140°18'E), by using pressure and temperature data for 23 standard pressure levels (Sfs, 1000, 900, 850, 700, 600, 500, 400, 350, 300, 250, 175, 125, 100, 70, 50, 40, 30, 20, 15 mb).

The atmospheric density (ρ) is derived as

$$\rho = \frac{P}{RT}\,, \tag{5}$$

where P denotes air pressure (mb), T absolute temperature (273°16 + t°C) and R the gas constant for dry air ($2.8704\cdot10^6$). As the effect of water vapour is small for visible ray, dry air is treated thereafter.

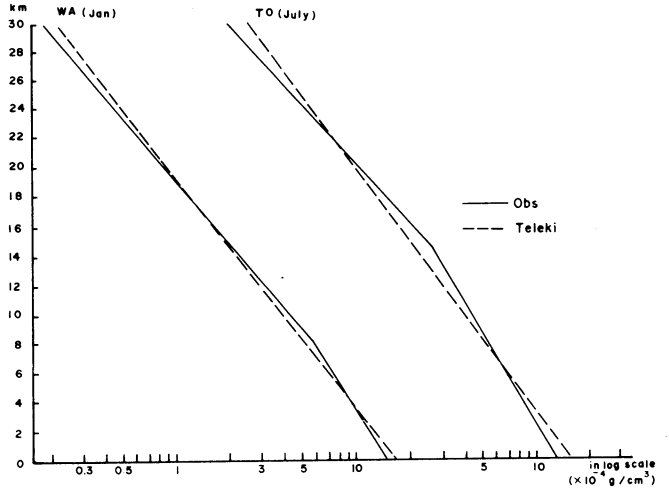

Figure 1. The vertical distribution of the atmospheric density at Wakkanai (Jan.) and Torishima (July) and the fitness with the exponential representations by Teleki.

In Figure 1. the vertical distribution of density at Wakkanai in January and at Torishima in July and the exponential representations by (4) are given in logarithmic scale. As can be seen in Figure 1, the vertical distribution of density in logarithmic scale clearly indicates a tendency of bending near the tropopause between the troposphere and the stratosphere. This tendency was already found by B.R. Bean and E.J. Dutton (1966) for radio wave refraction. In the troposphere the lapse rate of temperature is nearly constant (0°6/100 m) and the lower part of the stratosphere consists of nearly isothermal layers. It seems, therefore, more reasonable to define the scale height separately for the troposphere and the stratosphere. We put the following representations for two spheres model.

$$\rho = \rho_o e^{-z/H^*}, \qquad z^* \geqq z \geqq 0 \text{ (troposphere)} \quad (6)$$

$$\rho = \rho_m e^{-z/H^{**}}, \qquad z \geqq z^* \quad \text{(stratosphere)} \quad (7)$$

where ρ_o, ρ_m, H^* and H^{**} are parameters to be determined from aerological observations. The height of a bending point, z^*, can be determined by the method of successive approximation so as to make coincidence the density representation from the surface in (6) with that in the stratosphere between 5 km (500 mb) and 16 km (100mb). At the same time H^*, H^{**} and ρ_m can be determined.

The height of the bending point (z^*) shows a distinct seasonal variation which is lower in winter and higher in summer. Its amplitude of the seasonal variation becomes larger with the northern latitudes. The height of the bending point is 6-7 km in winter and 11-12 km in summer at Wakkanai and Sapporo in Hokkaido. It is 7-8 km in winter and 12 km in summer at Akita and Sendai including Mizusawa. It is also about 11 km in winter and 13 km in summer at the southern stations, Hachijojima and Torishima. Generally, the height of the tropopause is highest, about 17 km, in the equatorial region, but it decreases till 9 km with higher latitudes. At the same station it is higher in summer and lower in winter. The height of z^* does not exactly coincide with that of the tropopause, but it appears to be very near the boundary between the troposphere and the stratosphere.

Then the scale height H^* shows a similar seasonal variation to that of z^*, lower in winter (9.2 km) and higher (9.6 km) in summer. The seasonal variation of the scale height appears to depend on a regional effect. On the other hand, the scale height H^{**} in the stratosphere gives an opposite seasonal variation, higher (6.5 km) in winter and lower (6.1 km) in summer. The seasonal variation of the scale height is quite opposite between the troposphere and the stratosphere.

The mean density in the stratosphere, ρ_m, shows a similar seasonal variation to the height of the tropopause, small in winter and large in summer.

2. NUMERICAL EXPERIMENTS FOR ASTRONOMICAL REFRACTION.

In Figure 2 the center of the Earth is denoted by C and the observation point at the ground surface 0_1. The direction of the zenith is represented by Z. Assuming the Earth as a sphere, the concentric differential airstrata for every km from the surface to 30 km are taken as i-th layer (i = 1, 2, 3, - - -, 30). Putting observed zenith distance at the ground surface as z_1, and refractive index at the 1st layer as μ_1, we define horizontal and vertical directions, parallel to those at 0_1, at 0_2 where optical path passes the boundary between 1 and 2 layers, and the angle between the tangent to the concentric boundary and the horizontal direction. The angle θ_2 is taken positive clockwisely. The inclination of

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airstrata is taken as S_2, clockwisely positive. Let CO_2 be r_2 and CO_1 be r_1. Then we obtain for the triangle CO_1O_2 as

$$r_1 \sin(z_1 - \theta_1) = r_2 \sin(z_1 - \theta_2). \tag{8}$$

From (8) we can derive as

$$\theta_2 = z_1 - \sin^{-1}\left\{\frac{r_1}{r_2}\sin(z_1 - \theta_1)\right\}. \tag{9}$$

At the ground surface

$$S_1 = 0, \ \theta_1 = 0.$$

Let the zenith distance at O_2 be z_2. From the fundamental relation for refraction we get

$$\mu_1 \sin(z_1 - \theta_2 - S_2) = \mu_2 \sin(z_2 - \theta_2 - S_2). \tag{10}$$

From (10) we can derive z_2 as

$$z_2 = \theta_2 + S_2 + \sin^{-1}\left\{\frac{\mu_1}{\mu_2}\sin(z_1 - \theta_2 - S_2)\right\}. \tag{11}$$

We proceed these procedures successively to the i-th layers (i=3, 4,- -, 30). Thus z_{30}, the last zenith distance, is corrected for refraction. We take the last zenith distance as the result of numerical experiment.

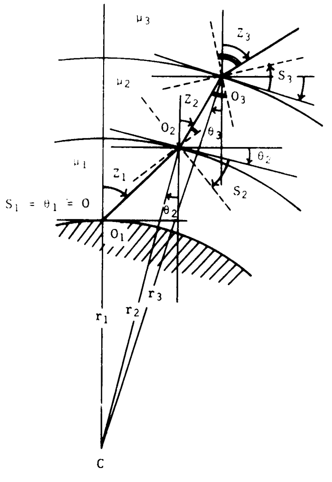

Figure 2

In Figure 3 the profile of the tilting of airstrata of equal density for one combination of two stations nearly along the meridian at Mizusawa, that is, Akita-Sendai (October). The profile derived from directly observed data show a typical character which is north up from the surface to about 10 km (the height of tropopause) and south up from 10 km to 25 km (lower part of stratosphere), as already found by Dines, Harzer, Sugawa and Teleki. It can be seen at once that two spheres model of the exponential representation appears to fit observed results approximately. It has been, therefore, proved that two spheres model is a more reasonable representation.

Moreover, two spheres model expresses the best approach to the actual seasonal variation of surface pressure.

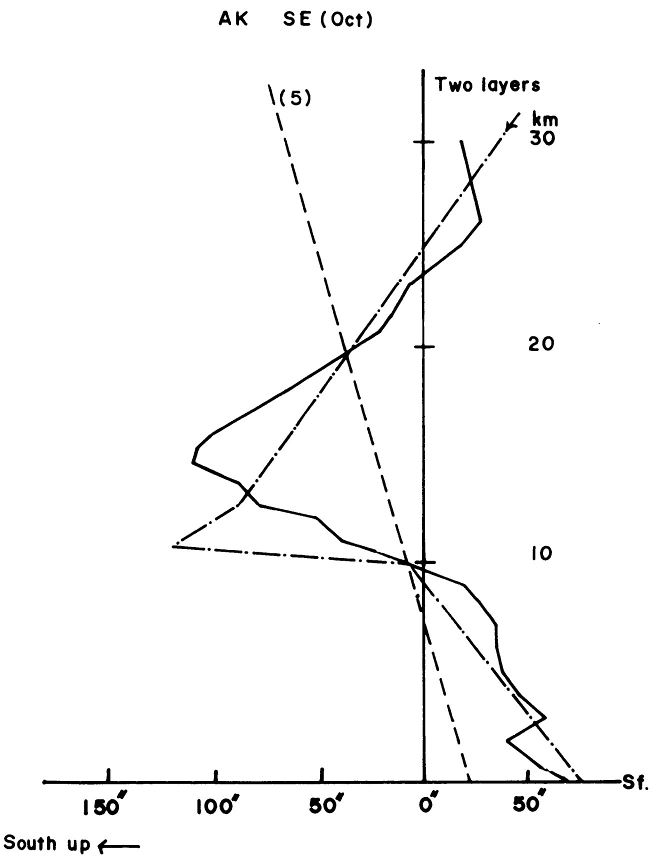

Figure 3 The vertical distribution of the inclination of airstrata
—— obs. —·—·— two spheres model — — — Teleki.

3. THE CHARACTERISTICS OF ASTRONOMICAL REFRACTION IN THE NORTHERN HEMISPHERE

Basic aerological data were obtained at the following standard pressure

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levels; the mean sea level, 1000, 850, 700, 500, 200, 100 and 30 mb respectively. The northern hemisphere is divided into a mesh with 10° in longitude and latitude ranging from 20° to 80°N.

The monthly mean values of aerological data were taken from the following aerological notes for every mesh.
Long Range Forecast, Technical Notes, No.6, Normal values at the heights of constant pressure level in the troposphere (100, 200, 300, 500, 700 and 850 mb) from 1951 to 1960, Japan Meteorological Agency, 1968.
Long Range Forecast, Technical Notes, No.10, Normal values at the heights of constant pressure level and anomalies at 30 mb from 1958 to 1966, Japan Meteorological Agency, 1970.
Normal of Atmospheric Pressure in the Northern Hemisphere (1909-1914 and 1924-1937) for Sfs and 1000 mb, Japan Meteorological Agency, 1966.

Five parameters of two spheres model in each mesh in the northern hemisphere have been calculated for each month. The global charts of the annual mean values on ρ_o, H*, ρ_m, H** and z* are given in Figure 4(a), (b), (c), (d) and (e) respectively. For reference, the charts of the annual mean temperature and pressure at the mean sea level are shown in Figure 5(a), (b) respectively.

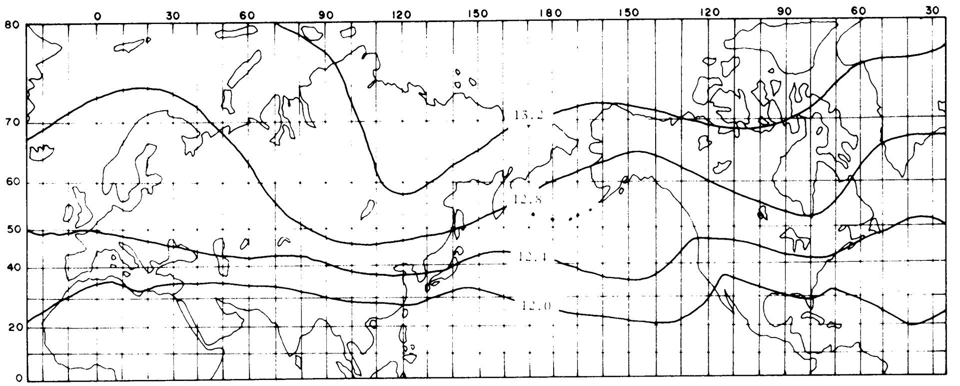

Figure 4 (a) ρ_o 10^{-4}(g/cm^3)

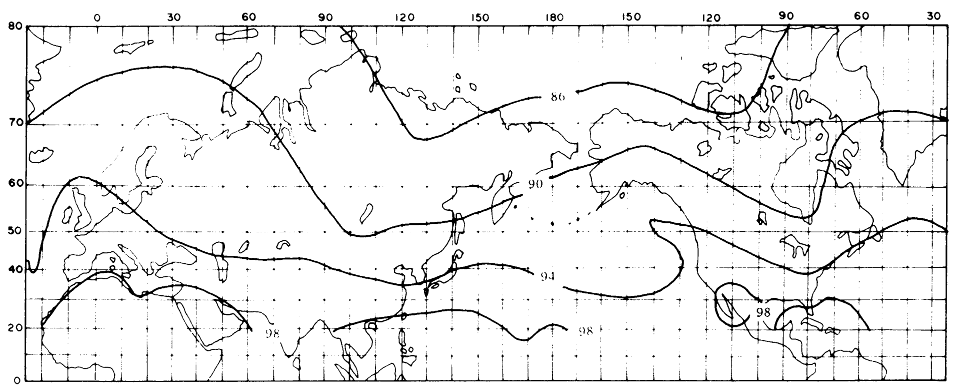

Figure 4 (b) H* (10^2m)

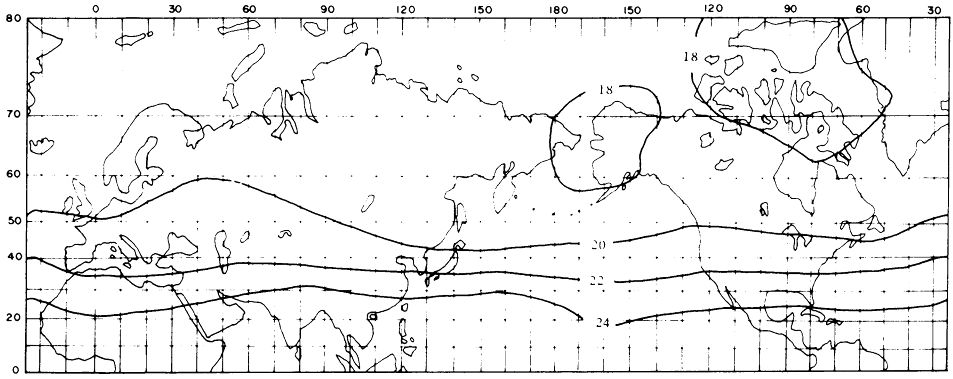

Figure 4 (c) ρ_m $(10^{-4} g/cm^3)$

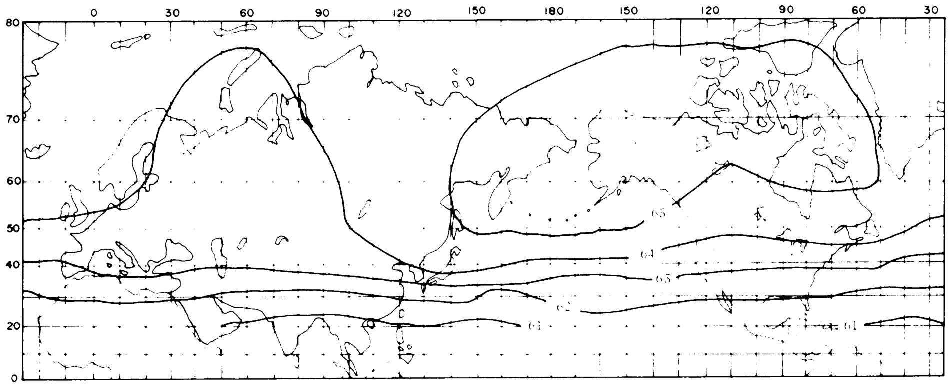

Figure 4 (d) H** $(10^2 m)$

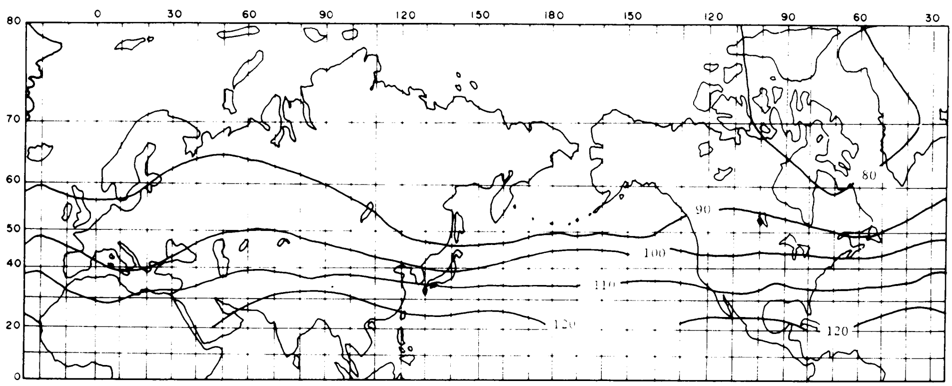

Figure 4 (e) Z* $(10^2 m)$

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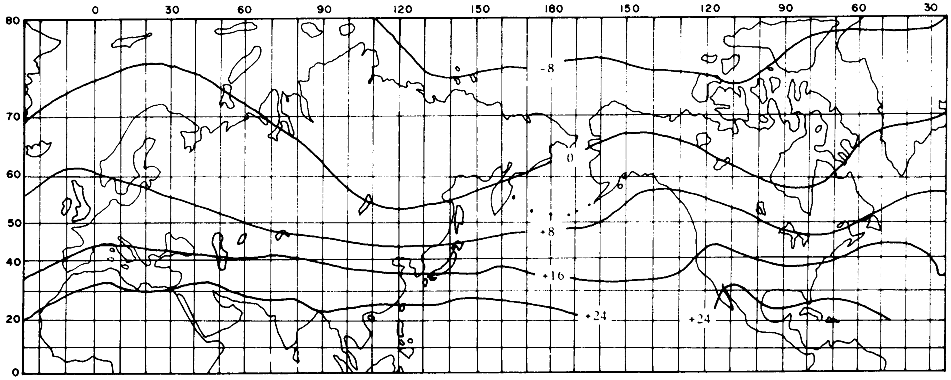

Figure 5 (a) T_O (°C)

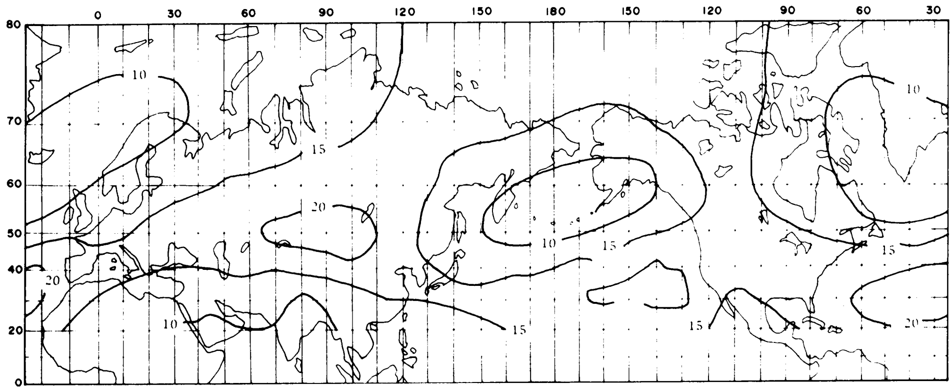

Figure 5 (b) P_O (+1000 mb)

As can be readily seen in Figure 4(a), (b) and Figure 5(a), the world-wide distribution of surface density (ρ_O) and the scale height in the troposphere (H*) appear to be fairly similar to that of surface temperature.

The distribution of ρ_m corresponding to the equivalent density of the stratosphere shows two lower regions in Bering Sea and North West Territory in Canada in Figure 4(c). In the middle latitudes, the contour line of equal density appear to be nearly parallel to the equator. In Figure 4(d) the distribution of the scale height in the stratosphere (H**) appears to be also nearly parallel to the equator in the middle latitudes, but it shows some higher region in high latitudes. In figure 4(e) the distribution of the height of the bending point (z*) nearly corresponding to the height of the tropopause appears to give similar chart to that of ρ_m except Bering Sea region.

4. EXPERIMENTAL FORMULA FOR NORMAL OR PURE REFRACTION.

The fundamental integral of normal refraction is, as known well,

$$R = a\mu_O \sin z \int_1^{\mu_O} \frac{d\mu}{\mu(r^2 - a^2 \sin^2 z)^{1/2}}. \qquad (12)$$

This integral can be developed approximately in the following principal terms.

$$R = (\mu_o - 1)\tan z + B \tan z(1 + \tan^2 z) = A \tan z + B \tan^3 z, \quad (13)$$

where

$$A = (\mu_o - 1) + B. \quad (14)$$

The value of B is nearly -0\".0669 under normal, 0°C and 760 mm Hg or 1013.25 mb. In the refraction table of "Connaissance des temp", A is put as

$$A = R_o \frac{\mu_o - 1}{\mu_s - 1} \quad (15)$$

where μ_s represents refractive index at the surface under normal condition. The value of R_o is adopted as 60\".154.

Now the difference of the amount of refraction appears to depend on that of the scale height, as shown in Figure 6. Let ρ_o be constant but the scale height be different and the incident angle be equal in right and left sides. However, those at the uppermost layer are different due to the difference of the scale height in their way.

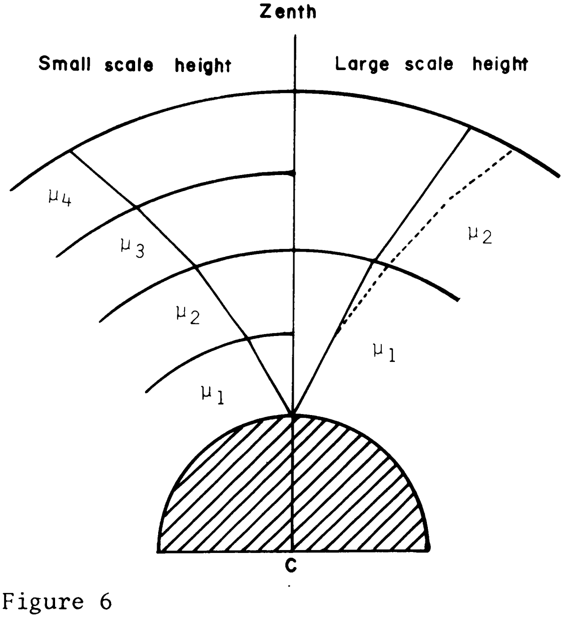

Figure 6

Putting R_o as the amount of refraction derived by numerical experiments for each mesh, we get

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$$R_o = R/(\frac{\rho_o}{\rho_s} \tan z), \qquad (16)$$

where R denotes equivalent refraction constant, ρ_s the standard value of density (0.00129 g/cm^3) and z a zenith distance. R can be approximated as

$$R_o = \alpha + \beta \tan^2 z, \qquad (17)$$

where

$$\alpha = A + B \frac{H_o}{H_s},$$

$$\beta = C + D \frac{H_o}{H_s},$$

where H_o is the scale height and H_s the standard scale height (7,999 m). From (17) we have

$$R = R_o \frac{\rho_o}{\rho_s} \tan z$$

$$= (\alpha + \beta \tan^2 z) \frac{\rho_o}{\rho_s} \tan z$$

$$= (A + B \frac{H_o}{H_s}) \frac{\rho_o}{\rho_s} \tan z + (C + D \frac{H_o}{H_s}) \tan^3 z. \qquad (18)$$

As H_o is proportional to T_o, we obtain

$$R = (A + B \frac{T_o}{T_s}) \frac{\rho_o}{\rho_s} \tan z + (C + D \frac{T_o}{T_s}) \frac{\rho_o}{\rho_s} \tan^3 z, \qquad (19)$$

where T_s is 237°16 k.
On the other hand, ρ_o, T_o and P_o are mutually related as

$$\frac{\rho_o}{\rho_s} = \frac{T_s}{P_s} \frac{P_o}{T_o} \qquad (20)$$

where P_s is 1013.25 mb.
Thus we can derive the following formula which contains P_o as the whole atmospheric mass.

$$R = (A \frac{\rho_o}{\rho_s} + B \frac{\rho_o}{\rho_s}) \tan z + (C \frac{\rho_o}{\rho_s} + D \frac{P_o}{P_s}) \tan^3 z. \quad (21)$$

From the values of astronomical refraction obtained by numerical experiments the following experimental formula was obtained by the method of least squares (number of equations of condition is 9,828).

$$R = \underset{\pm 45}{60''23622} \frac{T_s}{P_s}\frac{P_o}{T_o} - \underset{\pm 44}{0''06719} \frac{P_o}{P_s} \tan z$$

$$+ \underset{\pm 161}{0''01249} \frac{T_s}{P_s}\frac{P_o}{T_o} - \underset{\pm 156}{0''07502} \frac{P_o}{P_s} \tan^3 z. \qquad (22)$$

For reference the corresponding formula in "Connaissance des Temps" is shown as

$$R = 60''154 \frac{T_s}{P_s}\frac{P_o}{T_o} \tan z - 0''0669 \tan^3 z. \qquad (23)$$

Numerical experiments based on a single layer model of exponential representation were made and compared with (23). Using 600 samples in the range of ρ_0 ($0.0012 \sim 0.00137$ g/cm^3), H_0 ($6 \sim 10$ km), zenith distance ($1° \sim 45°$) and latitude ($0° \sim 90°$), numerical experiments were performed from surface to ∞ km.

Then the following formula was obtained.

$$R = (\underset{\pm 8}{60''24067} - \underset{\pm 8}{0''07539} \frac{H_o}{H_s}) \frac{\rho_o}{\rho_s} \tan z$$

$$+ (\underset{\pm 9}{0''00804} - \underset{\pm 9}{0''07415} \frac{H_o}{H_s}) \frac{\rho_o}{\rho_s} \tan^3 z. \qquad (24)$$

The latitude effect can be neglected according to (25). The value of $60''24067$ exactly corresponds to $c\rho_s$ ($c = 0.226$). The differences between the values derived by (25) and those by numerical experiments remain within 10^{-5} of arc up to $z = 45°$.

5. ASTRONOMICAL REFRACTION IN A CASE OF THE TILTING OF AIRSTRATA OF EQUAL DENSITY.

The tilting of airstrata of equal density in a mesh with 10 in longitude and latitude was computed using five parameters in the vartical distribution from surface to 30 km. Tracing successively a refracted ray path of a fictitious star which is observed as zenith at the ground surface its zenith distance at 30 km is computed by numerical experiments. The results are given in Figure 7 (a), (b). For reference, the horizontal pressure gradients are shown in Figure 8 (a), (b). In these figures a positive sign indicates refracted directions towards south and east from the zenith. For the pressure gradient a positive sign indicates north and west up.

It has been accepted widely that the general tendency of the tilting of airstrata of equal density shows north up from the surface to about 9 km and south up about 10 km to about 30 km.

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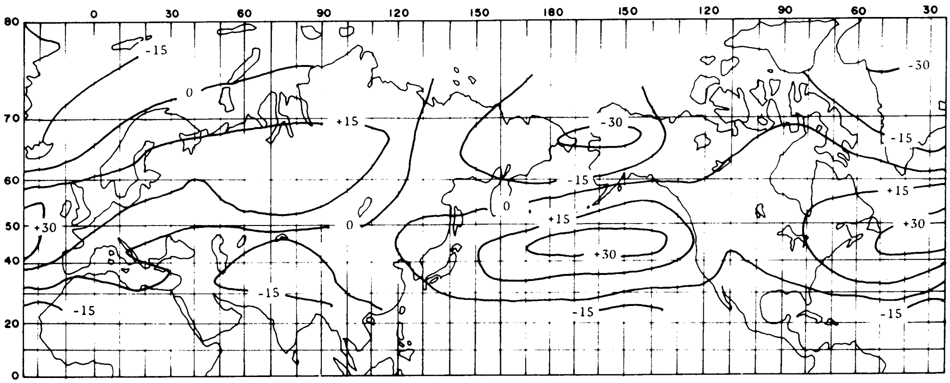

Figure 7 (a) z_{NS} (unit: "10^{-4})

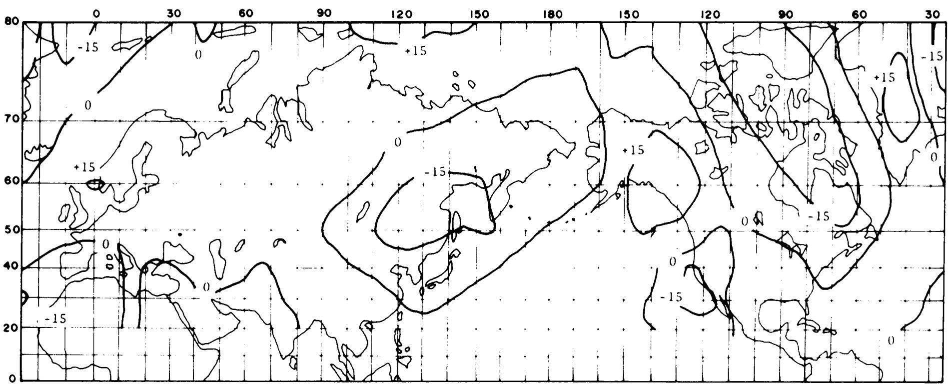

Figure 7 (b) z_{EW} (unit: "10^{-4})

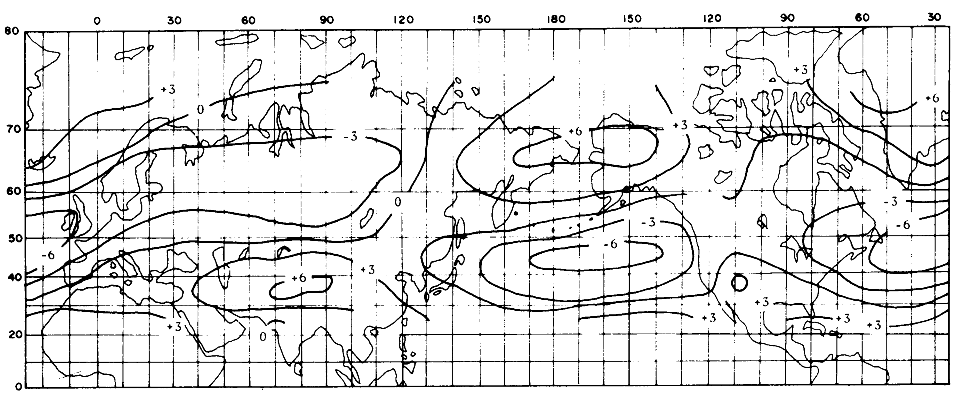

Figure 8 (a) P_{NS} ($P_N - P_S$) (unit: mb)

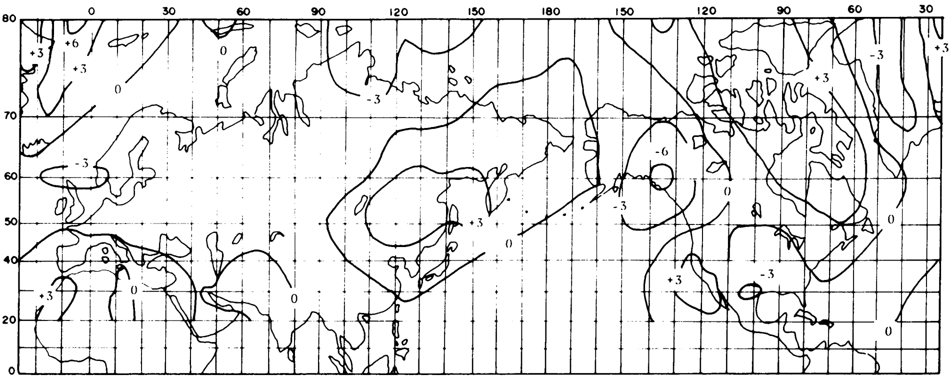

Figure 8 (b) P_{EW} (P_W - P_E) (unit: mb)

Then it has been discussed which is more important for the total effect, troposphere or stratosphere, but it has been confirmed from the chart of the annual mean zenith refraction that the total effect follows the horizontal gradient of P_0 as the total mass of the whole atmosphere. Namely, it is refracted towards higher pressure direction. The amount of refraction is about 0".003 corresponding to the pressure difference of 7 mb in 10° of latitude (about 1,110 km). The correlation coefficients between Δz and ΔP are -0.971 for NS direction and -0.916 for EW direction when the value of P_0 is used for ΔP. They are also -0.994 for NS direction and -0.993 for EW direction when $P_0 = \int_0^\infty \rho(z)\,dz$ (g = 980.665 gal) is used for ΔP_0.

Assuming a linear relation, we put

$$z_0('') = u \times P_0 \text{ (mb/km)},$$

where $z_0('')$ is the amount of refraction of a zenith star in the second of arc and ΔP_0 the horizontal gradient of P_0 in the unit of mb/km.

$$u = -0.583 \text{ for NS direction,}$$

$$u = -0.477 \text{ for EW direction.}$$

The resultant relation is represented by the following formula

$$z_0('') = -0.48 \times P_0 \text{ (mb/km)}, \qquad (25)$$

where a negative sign is due to an opposite refracted direction with the tilt of airstrat in a polar coordinate system.

Now the following charasteristics have been obtained for the case when the zenith distance is not zero. Taking a half sum of the values derived by numerical experiments for positive and negative zenith distances, 1°, 5°, 10°, 20°, 30° and 45° an experimental formula derived by using many samples can be approximated by

$$\Delta z = \Delta z_0 \sec^2 z, \qquad (26)$$

where Δz is an amount of refraction at a zenith distance of z. However, the expressions (25) and (26) can hold for z within 1".

6. CONCLUDING NOTES.

It has been noticed that the exponential representation of refractive index gives some bending near the tropopause between the troposphere and the stratosphere. Therefore, we proposed the two spheres model as the improved exponential representation of refractive index.

The distribution of surface density (ρ_0) and scale height in the troposphere (H^*) appears to be nearly similar to that of surface temperature. The distribution of ρ_m corresponding to the equivalent density in the stratosphere shows two lower regions, that is, in Bering Sea and North West Territory in Canada. In the middle latitudes the contour lines of equal density appear to be nearly parallel to the equator. The distribution of the scale height in the stratosphere appears to be also parallel to the equator in the middle latitudes, but it shows some higher region in high latitudes. The distribution of the height of bending point (z^*) nearly corresponding to the height of the tropopause appears to give similar chart to that of ρ_m except Bering Sea region.

In this note two new methods of correction for astronomical refraction are proposed.
The effect of vertical gradient of atmospheric density has been so far neglected. However, it may be better to add a term for P_0 (total mass of the atmosphere), as given in (22). This effect is about 0".001.
It may be possible to correct an anomalous refraction with an accuracy of 0".1 using (25) and (26), where horizontal gradient of P_0 corresponds to a total integrated amount of the tilting of airstrata of equal density.

It is to be remarked that astronomical refraction is corrected by using P_0 common to two above-mentiond effects. The important element on astronomical refraction has been hitherto considered as air temperature at the surface (T_0), but the contribution of air pressure at the surface (P_0) should be anew recognized.

The above mentioned methods are based on the exponential distribution of the atmospheric density. If an exponential representation would be applicable for the general field of the atmosphere, the pressure difference 0.02 mb for a distance of 100 m in the scale height corresponding to that of about 3°.4 k in temperature may produce the effect of about 0".08

in the amount of refraction from surface to 3 km for 20° of zenith distance even when there is no tilting of airstrata of equal density. However, the inversion of atmospheric temperature often occurs near the surface and several kilometers in height during clear nights. Moreover, considering the irregularity of the vertical distribution of atmospheric density due to the advection, it may be remained as a future problem to what extent our conception would enable us to correct anomalous refraction precisely.

We have usually called astronomical refraction with the tilting of airstrata of equal density as an anomalous refraction. However, it actually appears to occur every time and everywhere. Therefore, it would be more exact and convenient to call it as apparent refraction.

At the end of this note we express our hearty thanks to Prof. G. Teleki, a Chairman of Working Group on Astronomical Refraction (WGAR) of Commission 8 of the IAU for his kind advices and encouragements.

REFERENCES

Bean, B.R. and Dutton, E.J., 1966. "Radio Meteorology" pp,65-76.
Teleki, G., 1967. "Publ. Astron. Obs. Beograd," 13, pp, 1-147.

DISCUSSION

G. Teleki: stressed that Sugawa's and Kikuchi's investigation shows that for the calculation of new refraction tables we have to use a real atmosphere and not a mathematical spherical symmetric atmosphere.

B. Garfinkel: Is it possible to use also the polytropic atmosphere for this kind of investigation?

C. Sugawa: answered that it is possible. We have actually used the exponential representation for a two spheres model, which is a kind of the polytropic atmosphere.

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