THE ORIGIN AND STRUCTURE OF THE MAGNETIC FIELDS OF THE CHEMICALLY PECULIAR STARS

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ABSTRACT

Rival theories for the origin of the magnetic fields present in the CP stars are discussed, particular attention being paid to the claims of the 'contemporary dynamo' and 'fossil' theories. The internal structure of the field as predicted by calculations consistent with the fossil theory is discussed at length. It seems that current time dependent models can now give a coherent picture of the fields of the CP stars according to the fossil theory. Dynamo theory modelling has not been developed in such detail. As yet neither the theoretical predictions nor the observational material seem to be detailed enough to allow a decisive comparison between the theories.

1. INTRODUCTION

In recent years much attention has been paid to magnetic fields, activity cycles and related effects in stars of the lower main sequence - the "solar-stellar connection" - and at times those maintaining an interest in the classical magnetic stars of the middle main sequence have almost been made to feel rather old fashioned! Nevertheless there are many unsolved problems connected with such objects and in this review I will discuss some of those, limiting myself to the origin and large scale structure of the fields, and directly related topics. In the limited space available I do not intend to discuss topics such as the cause of the anomalously slow rotations of the magnetic CP stars, nor the short period oscillations or the origins of the anomalous chemical abundances.

The rigid rotator model will be adopted as a working hypothesis to interpret the observations, in common with the usual practice. In almost every case so far investigated field variations can be modelled by a dipole field, maybe with the dipole displaced from the stellar centre. An equivalent representation is as the sum of dipole and (smaller) quadrupole components. One star (HD37776, Thompson and Landstreet, 1985) appears to have a dominant quadrupolar component. The oblique rotator/displaced dipole model has dipole strength, inclinations i and χ

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between the rotation axis and the line of sight and the magnetic axis respectively, and the fractional dipole displacement d as free parameters. The rather meagre observational evidence is consistent with a random distribution of χ (Hensberge et al 1979; Borra & Landstreet, 1980; Didelon, 1984).

An alternative interpretation of the rigid rotator model is the 'perpendicular rotator' in which χ is assumed to be 90°. The surface field is then modelled by a sum of dipolar and quadrupolar components with a common axis in the rotational equator. The ratio of B_s to B_e can now only be kept acceptably small by appealing to a markedly non uniform distribution of elements which have lines sensitive to magnetic fields. This results in a distribution with the apparent values of $\chi < 90°$ when effects of inhomogeneities are ignored (e.g. Oetken, 1977, 1979). Surface inhomogeneities undoubtably are present but fully self consistent models have not yet been calculated.

Theoretical problems in which magnetic fields are involved in CP stars include the following.

Origin of field.
Internal structure - relation between observed (surface) and interior fields.
Stability of field structure.
Explanation of the incidence of stellar magnetism.
Explanation of the χ distribution.
Evolutionary changes - secular effects.
Cause of the low angular velocities.
Cause of the chemical anomalies.
Rapid low amplitude oscillations.

This list is not exhaustive, nor are the topics independent. All but the last three will be touched upon under the main headings of 'Origin' and 'Structure'.

2. ORIGIN OF THE FIELD

Four types of theory have been advanced to explain the fields of the magnetic CP stars: the magnetic oscillator, battery, fossil and dynamo theories. Currently the first two have little favour (although a variant of a battery mechanism might provide an initial 'seed' field, or influence the toroidal field structure near the stellar surface, eg Dolginov, 1977; Mestel & Moss, 1983) and will not be discussed further here (but see, eg, Dolginov, 1984). The only satisfactory way to distinguish between the fossil and the dynamo theories is to develop the theories in detail and try to arrive at testable and distinct sets of predictions.

2.1 Contemporary dynamo theory

This proposes that the observed fields are the surface manifestations of a 'turbulent' dynamo operating in the convective core. Turbulent dynamo theory has been developed in great detail in the last twenty years, following the pioneering work of Krause et al collected in Roberts and Stix (1971). Briefly, dynamo action is possible where the net helicity of the fluid motions is non zero ($<\underline{u} . \nabla \times \underline{u}> \neq 0$) and this condition is likely to be satisfied in rotating convection zones. In this case the normal MHD induction equation is supplemented by the "α-effect" term to give

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B} + \alpha \underline{B}) - \nabla \times (\eta_T \nabla \times \underline{B}),$$

where η_T is the turbulent resistivity. The additional source term invalidates the 'anti-dynamo' theorems and allows new poloidal field to be generated directly from the toroidal field. (Differential rotation and the α-effect are each capable of generating toroidal field from poloidal). There is a plethora of dynamo models, mostly however linear and kinematic, or limited to a parameterized non-linearity to represent the dynamical feed back of the field on the fluid motions. It is widely (but not universally) accepted that the solar field is generated by an unsteady dynamo, and it is tempting to try to explain the fields of the magnetic CP stars in a similar manner. There are, however, substantial differences between these cases. Any dynamo operating in the CP stars must be steady - both to agree with observations and, perhaps more stringently, to avoid being masked by the skin effect of the overlaying radiative zone. Some very young stars (age maybe of order 5×10^6 years) show kilogauss surface fields. If the dynamo turns on when nuclear reactions become important enough to set the core convecting then the field has to reach the surface within about 5×10^6 years. Classical diffusion times through the envelope are in excess of 10^9 years and even the accelerated 'forcing' discussed by Schüßler and Pähler (1978) only somewhat reduces this time. Many solar dynamo theorists want to have the dynamo just beneath the convection zone in order to prevent buoyancy in the convective region from removing flux too rapidly. There is a suggestion that at spectral type about M5 on the main sequence, where the complete star may become convective so that there is no 'bottom' to the convection zone, the dynamo efficiency declines rapidly. This might imply that a core dynamo is less efficient than predicted by simple turbulent dynamo theory.

If a core dynamo can operate satisfactorily in these stars then a crucial question is the relation between the surface and core fields. Presumably in a star of given structure the core is a function mainly of the angular velocity Ω, at least when a steady state has been reached. Non linear limiting effects must be crucial in determining the final field strength - eg. by modifying the small scale motions and so reducing the effective value of α, or by generating large scale meridional flows, or reducing differential rotation. Crudely it might be imagined that larger angular velocities would give rise to larger fields and this is certainly the usual assumption when modelling lower main sequence dynamos. It is conceivable that non-linear processes seen through the filter of a radiative envelope in which the hydrostatic and energy equations have to be satisfied might give a different result for the observable fields. The only detailed models so far calculated (Moss, 1983) lend some support to this conjecture, but the calculations were only for one, perhaps rather special case. More crudely the idea that for larger values of the angular velocity only oscillatory modes may be available, and so the skin effect will render the fields negligible at the surface, was discussed by Moss (1980).

To agree with the observations the theory must be capable of explaining why stars of the same angular velocity can have very different fields. Rädler (preprint) points out that differential rotation and a field component with symmetry axis perpendicular to the rotation axis cannot persist. Field lines of opposite sense will inevitably be brought close together, and ohmic dissipation of this perpendicular component will be enhanced. If $0 < \chi < 90°$ the reduction of the perpendicular component will reduce the observed value of χ. The Potsdam group have developed in detail the 'perpendicular rotator' dynamo model. They argue that dynamo theory demands that the field axes must be strictly parallel or perpendicular to the rotation axis (ie χ=0 or 90°). As discussed in section 1, they model the field by a sum of dipole and quadrupole components with axis perpendicular to the angular velocity, together with surface element inhomogeneities. Krause (1983) advances the 'fossil rotator' theory in the context of core dynamo theory with strictly perpendicular field and rotation axes. According to this theory, if a star initially has a strong differential rotation, the shear winds up and buries the field beneath the surface, whereas if the initial radial variation of angular velocity is small, or the field is strong enough to remove an initially strong angular velocity shear, the field continues to penetrate the surface and so to be visible. Such mechanisms would introduce some scatter into the relation between surface field and period in the dynamo model. Until calculations can be performed which calculate the inhomogeneities and the observed field in a self consistent manner it is difficult to assess the validity of this model. In general, in contrast to the fossil field model (below), the dynamo model still lacks detailed non-linear calculations which would enable a proper comparison to be made with the observational material.

2.2 Fossil theory

Given the long decay timescales (of order 10^{10} years) associated with large scale fields in a mainly radiative star it is possible that any field initially present when the star first settles on the main sequence could survive throughout the main sequence lifetime. In the simplest form of the theory, the initial field is a relic of the interstellar field which permeated the material from which the star was formed. The first question to be answered is whether such a large scale field can survive the global turbulence of the Hayashi phase (if these stars experience such a phase), or whether it will be mangled and expelled from the star. This is a difficult problem, to which there is no clearcut answer. Numerical calculations of the dynamical interaction between convection and magnetic fields (eg. Galloway, Proctor & Weiss 1978; Galloway & Weiss, 1981; Galloway & Proctor, 1983) do suggest the possibility that a significant amount of flux could survive this phase, having been concentrated into ropes by the turbulence, and so allowing convection to proceed freely in the more or less field free regions between the ropes. Current observations of the solar surface field do seem to indicate an intermittent structure.

After the convection dies away the field is assumed to diffuse back into a more uniform configuration. A variant of this picture appeals to a dynamo operating in the Hayashi phase to produce a large scale field. When the convection dies away the dynamo ceases to operate, but the relic field again survives over nuclear timescales. The consequences of this 'hybrid' theory are largely indistinguishable from the standard fossil theory, unless dynamo theory can make some definite statements about the initial distribution of the angle χ. For a fossil field to survive over nuclear timescales it must be able to resist a variety of instabilities which tend to reduce the total flux. It is worth emphasizing that, even for the full fossil theory, a certain amount of flux loss sometime in the history of a magnetic star is desirable, otherwise the high magnetic Reynolds number from the later stages of dynamical contraction onwards would ensure a significant flux for all stars. Field freezing may only be important after the 'molecular' phase of star formation - which in the absence of further flux loss would naively result in mean field strengths ($\bar{B}$) through the star of 10^4 to 10^5 gauss on the main sequence. This might be consistent with the observably magnetic stars, and it could be argued (see later) that rapid rotators have small ratios of surface to internal field and so are not seen as magnetic stars. However the slowly rotating HgMn stars do not have observable fields which suggests that further flux loss has occurred in some stars. Possible mechanisms of flux loss include the topological instabilities first investigated in detail by Wright (1973) and Markey and Tayler (1973). They noted that purely poloidal fields near a neutral line locally have the topology of the z-pinch which is known to be unstable, and detailed analysis confirms that purely poloidal (and also purely toroidal) fields are dynamically unstable. If the instability grows into the nonlinear region enhanced ohmic flux destruction can be expected. Linked poloidal and toroidal fields of comparable strength can be expected to be less vulnerable to such effects, although it is difficult to guarantee stability, as has been demonstrated in a series of papers by Goossens, Tayler and their collaborators (eg. Tayler (1982), Goossens et al (1981)). Even if dynamical stability is assured, in a stable sub-adiabatic radiative zone isolated flux tubes tend to rise to the surface at a rate determined by the diffusion of heat into the tube (Parker 1979, Acheson, 1979). This is a secular effect, timescales may well be in excess of 10^7 or 10^8 years, and its relevance to a complex geometry of continuously distributed interlinked poloidal and toroidal fields is hard to assess. Molecular weight gradients could plausibly aid stabilization. There is no evidence for the ratio $\epsilon \sim \frac{\bar{B}^2 R^4}{G M^2}$ of magnetic to gravitational energy ever being anything but a very small number. Even with the largest known surface field of $\sim 3 \times 10^4$g(HD 21544) and associated $\bar{B} \sim 10^6$g, $\epsilon \sim 10^{-5}$. It may well be that structures with larger ϵ are inevitably unstable, and that instabilities induce flux loss until a stable configuration with $\epsilon << 1$ is attained.

3. STRUCTURE OF THE FIELD

With the above considerations in mind we can discuss models for the large scale structure of the field throughout the star. Reflecting the detailed work done the discussion will be almost entirely concerned with models consistent with the fossil field theory. Early attempts at modelling, consistent with both the hydrostatic and energy equations, were "quasi-static" in that those which explicitly included meridional motions and the magnetohydrodynamic equation wrote the latter in a way equivalent to introducing a small source term to offset the overall decay of the field predicted by the simpler forms of the anti-dynamo theorems (eg. Cowling 1934). See Mestel & Moss (1977) and references therein. Models can be found for a variety of field geometries (dipole, even and odd multipole) and topologies (poloidal, linked poloidal and toroidal) some of which plausibly are stable against the more obvious dynamical instabilities. All these models predict an increasing ratio of internal to surface field for a given total flux as the angular velocity increases and for plausible parameters the field is strongly concentrated to the interior. At the time this seemed quite satisfactory, and was certainly in accord with what then seemed to be quite a strong anti-correlation between B_e and Ω. The thrust of the observational evidence then shifted somewhat, weakening (but not destroying) this anti-correlation, and also to suggest that the distribution of the angle χ was more nearly random. This was in contrast to what had appeared to be an approximately bimodal distribution, with a weaker peak near $\chi = 0$ and a stronger one near $\chi = 90°$. On the theoretical front Mestel et al (1981) worked out the details of what they called internal "ξ - motions", resulting from the rotation of a compressible body which is not symmetric about its rotation axis - for example an oblique rotator. In order to conserve angular momentum the rotation axis must precess about the axis of symmetry of the field with angular velocity $\omega \sim \epsilon \Omega$. These motions change the pressure-density field, and equilibrium can only be maintained by dynamically driven "ξ-motions", which are also of frequency ω. These distort the field and dissipate energy, predominantly by ohmic dissipation , and the body eventually adopts the minimum energy configuration for its fixed angular momentum, and so rotates about its axis of greatest moment of inertia. Thus $\chi \to 0$ if the star is dynamically oblate about the magnetic axis, and $\chi \to 90°$ if it is dynamically prolate. Poloidal fields give oblate configurations, toroidal prolate; the mixed fields so far calculated all gave oblate configurations, but the existence of prolate configurations was by no means ruled out. More importantly, the time scale for alignment or anti-alignment, τ_χ, is sensitive to the interior field concentration. Whereas for modest ratios of maximum interior radial field to surface field (0(10) say) $\tau_\chi << \tau_{ms}$, for significantly greater ratios such as found in the theoretical models discussed above $\tau_\chi << \tau_{ms}$ unless the period is greater than about 100 days. This implies that we should rarely see values of χ other than 0 or 90°, in contradiction to the simplest interpretation of the observations according to the oblique rotator/fossil field model which give a nearly random χ distribution (but note that if surface inhomogeneities can reconcile the perpendicular rotator version of dynamo theory with observations, then they can do the same for a fossil field in the asymptotic state $\chi = 90°$). Note also that ξ - motion theory is also relevant to dynamo models, with the additional complication that if the structure were such as to make χ tend to 0 from 90°, the new flux continuously being generated would tend to restore the original value of χ. Other mechanisms which might change χ include magnetic stellar wind torques (Mestel and Selley, 1970), and the purely kinematic effect of the rotationally driven circulation on the surface flux distribution in an initially oblique rotator (Moss, 1977).

Recently it has become possible to calculate improved, time dependent, magnetic models and to follow the evolution of the field over nuclear timscales (but, so far, ignoring the effects of stellar evolution on the underlying model). The calculations show that the previous quasi-static models are rather special cases selected from a much wider range of possible models (Moss, 1984, 1985). The evolution of models with fairly arbitrary initial flux distributions now can be followed over nuclear timescales. For models with $\chi = 0$ and relatively slow rotation ($P > P_c \sim 4^d$ for plausible values of mass and luminosity) the initial flux distribution survives with little change, except for slight ohmic decay, over a nuclear time scale. In particular a field that has small initial concentration to the interior never becomes concentrated below the surface. Stars with shorter periods and fields with initially low interior concentration will bury their fields, but on a time scale governed by the time scale of the of Eddington-Sweet circulation - typically of order 10^8 year or slightly more. The critical period is such that the magnetic Reynolds number of the Eddington-Sweet circulation is of order unity through the bulk of the radiative zone.

In the more general case of the oblique rotator, with respect to a spherical polar coordinate system (r,θ,λ) with axis the magnetic axis, the radial component of the Eddington-Sweet circulation can be written

$$v_r = V_{ES}P_2(\cos\theta)P_2(\cos\chi) - \frac{1}{4}V_{ES}\,\mathrm{Sin}^2\chi\,P_2^2(\cos\theta)\cos2\lambda + + \frac{1}{2}V_{ES}\sin2\chi\,P_2^1(\cos\theta)\sin\lambda, \qquad (1)$$

where $V_{ES}(r)$ is the radial component of the Eddington-Sweet circulation along the rotation axis. When $\chi = 90°$ the λ independent part is in the opposite sense with respect to the field axis than when $\chi = 0$. Simple kinematic considerations (supported by detailed calculations) suggest that in this case the field will never be buried, but that if $P < P_c$ then the surface field will tend to be concentrated towards the magnetic poles. Further, equation (1) suggests that there is a critical value of χ, $\chi_c \simeq 55°$, such that for $\chi < \chi_c$ these models will behave quantitatively like aligned rotators and so bury their surface field if P is small enough (allowing for the reduction of V_r ($\propto \Omega^2$) by the factor $P_2(\cos\chi)$); and will never bury the surface field for any P if $\chi > \chi_c$. The existence of a critical value, $\chi_c \simeq 55°$, is also predicted from study of the hydrostatic equation by Galea and Wood (preprint). These conclusions also apply to displaced dipole models (Moss 1985). The interaction of a velocity field of the form (1) with an initially axisymmetric magnetic field when $\chi \neq 0$ can be expected to generate non axisymmetric components, although this aspect has not yet been investigated.

It is only quite recently that changes with time of the stellar fields have attracted serious study. Borra (1981) and North and Cramer (1984) find evidence for a decline in field strength on timescales of order 10^8 years. The calculations just discussed follow the evolution of the field under the influence of ohmic diffusion and the magneto-centrifugal circulation. A number of other effects can be mentioned. A linked poloidal - toroidal field topology seems to be essential for stability. If the field is a fossil, decay inevitably occurs. In the simplest axisymmetric case the torque free condition will hold approximately which implies $B_t = F(\psi)/(r\sin\theta)$, where the poloidal field $\underline{B}_p = \nabla \times (\psi/r \sin\theta\,\hat{\underline{t}})$, $\hat{\underline{t}}$ the unit toroidal vector. In general $\underline{B}_t$ and $\underline{B}_p$ will decay at different rates. Torques will arise, generating differential rotation, until a new torque free configuration is found. In reality continuing adjustment will occur. Similar effects can be expected to occur in non axisymmetric configurations. If a core dynamo operates and the diffusion of the core field to the surface has not reached a steady state, then the surface field may increase for some time (eg. Schüßler & Pähler, 1978). Correspondingly, both for dynamo and fossil fields, braking (eg. by a magnetic stellar wind) on the main sequence will cause a continuing adjustment of the magnetic structure. Finally the central condensation of the star will increase with time. This effect has been investigated in detail by Moss (1983), for initial field configurations corresponding to the quasi-static fields described above. Both these calculations, and the time dependent models of Moss (1984, 1985) suggest that in many cases surface fields will decline on roughly an evolutionary timescale, which is consistent with the very limited observational material.

4. CONCLUSIONS

The central issue remains the origin of the magnetic fields. It is not yet possible to give an unambiguous answer to this question. Fossil theory models have been worked out in considerable detail, which has not been matched by the dynamo models. Contemporary dynamo theory does appear to have problems in explaining the detailed distribution of field strength with period, in explaining the appearance of strong fields at the surface of very young stars, and the incidence of fields (eg. why don't the slowly rotating HgMn stars display fields?). The fossil theory has an extra degree of freedom in that the initial fluxes are fairly arbitrary, and it does now appear possible to begin to put together a coherent picture according to this theory. Suppose that stars lose nearly all their primeval flux before settling on the main sequence, and that these residual fluxes have, for example, a Maxwellian distribution. Assume that the initial fluxes are not strongly concentrated to the interior and that the high flux tail of this distribution contains the stars whose fields we see today. If $P > P_c$ then the initial configuration survives more or less unchanged, apart from some ohmic decay. If $P < P_c$ and the initial value of χ is less than χ_c then the field is buried, over a time scale roughly of order of the main sequence lifetime, depending on the period and initial χ value. If the initial $\chi > \chi_c$ the field is not buried, but may be concentrated towards the poles on a similar timescale. The observed value of χ may change because of kinematic effects of the predominantly rotationally driven circulation on the field, or because of wind torques. Magnetic braking may occur. This picture suggests that rapid rotators with strong fields should either be young or have large χ. This general scheme does not appear to be contradicted by the rather limited observational material, but clearly greater detail in both theory and observations is needed to provide a decisive test.

To finish on a note of caution: if the fossil field is a direct descendant of a large scale quasi-uniform primeval field which permeated the interstellar medium then, naively, the observed fields should be of odd parity - that is they should be represented approximately by the displaced dipole model (Moss, 1985). If the dynamo theory (either contemporary or hybrid) produces a combination of dipolar and quadrupolar models then, prima facie, it might be expected that we should sometimes see a predominantly quadrupolar field. Until recently the displaced dipole (ie. predominantly dipolar) model was adequate to approximate the field structure of all known stars. Thompson and Landstreet (1985) have recently modelled HD 37776 with a predominantly quadrupolar field. As yet this is an isolated case, and it is unclear how much weight should be given to it. Further evidence of this nature would be of great interest.

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