10 A first look at supersymmetry breaking

If supersymmetry has anything to do with the real world, it must be a broken symmetry, as we do not see any degeneracy between bosons and fermions in nature. In the globally supersymmetric framework that we have presented so far, this breaking could be spontaneous or explicit. As we will argue later, once we promote the symmetry to a local symmetry, the breaking of supersymmetry must be spontaneous. The signal of such a breaking is a massless fermion, the *goldstino*, whose interactions are governed by low-energy theorems. However, as we will also see, at low energies the theory can appear to be a globally supersymmetric theory with explicit, "soft", breaking of the symmetry. In this chapter we will discuss some features of both spontaneous and explicit breaking.

10.1 Spontaneous supersymmetry breaking

We have seen that supersymmetry breaking is signaled by a non-zero expectation value of an F component of a chiral superfield or a D component of a vector superfield. Models involving only chiral fields with no supersymmetric ground state are referred to as O'Raifeartaigh models. A simple example has three singlet fields, A, B and X, with superpotential

$$W = \lambda A(X^2 - \mu^2) + mBX. \tag{10.1}$$

With this superpotential, the equations

$$F_A = \frac{\partial W}{\partial A} = 0, \quad F_B = \frac{\partial W}{\partial B} = 0 \tag{10.2}$$

are incompatible. To actually determine the expectation values and the vacuum energy, it is necessary to minimize the potential. There is no problem in satisfying the equation $F_X = 0$. So, we need to minimize

$$V_{\text{eff}} = |F_A|^2 + |F_B|^2 = |\lambda^2||X^2 - \mu^2|^2 + m^2|X|^2. \tag{10.3}$$

Assuming that μ^2 and λ are real, the solutions are given by

$$X = 0, \quad X^2 = \frac{2\lambda^2\mu^2 - m^2}{2\lambda^2}. \tag{10.4}$$

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The corresponding vacuum energies are

$$V_0^{(A)} = |\lambda^2\mu^4|, \quad V_0^{(B)} = m^2\mu^2 - \frac{m^4}{4\lambda^2}. \tag{10.5}$$

The vacuum at $X \neq 0$ disappears at a critical value of μ.

Let us consider the spectrum in the first of these (the solution with $X = 0$). We will focus, in particular, on the massless states. First, there is a massless scalar. This arises because at this level not all the fields are fully determined. The equation

$$\frac{\partial W}{\partial X} = 0 \tag{10.6}$$

can be satisfied provided that

$$2\lambda AX + mB = 0. \tag{10.7}$$

This vacuum degeneracy is accidental and, as we will see later, is lifted by quantum corrections.

There is also a massless fermion, ψ_A. This fermion is the goldstino. Replacing the auxiliary fields in the supersymmetry current for this model (Eq. (9.54)) gives

$$j_\mu^\alpha = i\sqrt{2}F_A\sigma^\mu_{\alpha\dot{\alpha}}\psi_A^{*\dot{\alpha}}. \tag{10.8}$$

You should check that the massive states do not form Bose–Fermi degenerate multiplets.

10.1.1 The Fayet–Iliopoulos *D* term

It is also possible to generate an expectation value for a D term. In the case of a $U(1)$ gauge symmetry, we saw that

$$\mu^2 \int d^4\theta \; V = \mu^2 D \tag{10.9}$$

is gauge invariant. Under the transformation $\delta V = \Lambda + \Lambda^\dagger$, the integrals over the chiral and antichiral fields Λ and $\Lambda^\dagger$ are zero. This can be seen either by doing the integrations directly or by noting that differentiation by Grassmann numbers is equivalent to integration (recall our integral table). As a result, for example, $\int d^2\bar{\theta} \propto (\bar{D})^2$. This Fayet–Iliopoulos D term can lead to supersymmetry breaking. For example, if one has two charged fields $\Phi^\pm$ with charges ± 1 and superpotential $m\Phi^+\Phi^-$, one cannot simultaneously make the two auxiliary F fields and the auxiliary D field vanish.

One important feature of both types of model is that at tree level, in the context of global supersymmetry, the spectra are never realistic. They satisfy a sum rule,

$$\sum (-1)^F m^2 = 0. \tag{10.10}$$

Here $(-1)^F = 1$ for bosons and -1 for fermions. This guarantees that there are always light states, and often color and/or electromagnetic symmetry are broken. These statements are not true of radiative corrections or of supergravity, as we will explain later.

It is instructive to prove this sum rule. Consider a theory with chiral fields only (no gauge interactions). The potential is given by

$$V = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 . \tag{10.11}$$

The boson mass matrix has terms of the form $\phi^*_{\bar{i}}\phi_j$ and $\phi_i\phi_j$ + c.c., where we are using indices $\bar{i}$ and $\bar{j}$ for complex conjugate fields. The latter terms, as we will now see, are connected with supersymmetry breaking. The various terms in the mass matrix can be obtained by differentiating the potential:

$$m^2_{i\bar{j}} = \frac{\partial^2 V}{\partial \phi_i \partial \phi^*_{\bar{j}}} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_k} \frac{\partial^2 W^*}{\partial \phi^*_{\bar{k}} \partial \phi^*_{\bar{j}}}, \tag{10.12}$$

$$m^2_{ij} = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} = \frac{\partial W}{\partial \phi^*_{\bar{k}}} \frac{\partial^3 W}{\partial \phi_k \, \partial \phi_i \partial \phi_j}. \tag{10.13}$$

The first term has just the structure of the square of the fermion mass matrix,

$$\mathcal{M}_{\mathrm{F}ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}. \tag{10.14}$$

So, writing the boson mass matrix $\mathcal{M}^2_{\mathrm{B}}$ in the basis $(\phi_i \; \phi^*_{\bar{j}})$ we see that Eq. (10.10) holds.

The theorem is true whenever a theory can be described by a renormalizable effective action. Various non-renormalizable terms in the effective action can give additional contributions to the mass. For example, in our O'Raifeartaigh model, $\int d^4\theta A^\dagger A Z^\dagger Z$ will violate the tree level sum rule. Such terms arise in renormalizable theories when one integrates out heavy fields to obtain an effective action at some scale. In the context of supergravity, such terms are present already at tree level. This is perhaps not surprising, given that these theories are non-renormalizable and must be viewed as effective theories from the very beginning (perhaps as the effective low-energy description of string theory). We will discuss the construction of realistic models shortly. First, however, we turn to the issues of the *goldstino theorem* (the fermionic analog of Goldstone's theorem) and explicit, soft, supersymmetry breaking.

10.2 The goldstino theorem

In each of the examples of supersymmetry breaking there is a massless fermion in the spectrum. We might expect this, by analogy with Goldstone's theorem. The essence of the usual Goldstone theorem is the statement that, for a spontaneously broken global symmetry, there is a massless scalar. There is a coupling of this scalar to the symmetry current j^μ. From Lorentz invariance (see Appendix B),

$$\langle 0 | j^\mu | \pi(p) \rangle = f p^\mu . \tag{10.15}$$

Correspondingly, in the low-energy effective field theory (valid below the scale of symmetry breaking) the current takes the form

$$j^\mu = f\partial^\mu \pi(x). \tag{10.16}$$

Analogous statements for the spontaneous breaking of global supersymmetry are easy to prove. Suppose that the symmetry is broken by the F component of a chiral field (this can be a composite field). Then we can study

$$\int d^4x\, \partial_\mu \left(e^{iq\cdot x} T\langle j^\mu_\alpha(x)\psi_\Phi(0)\rangle\right) = 0, \tag{10.17}$$

where T is the time-ordering operator and j^μ_α is the supersymmetry current; the integral of j^0_α over space is the supersymmetry charge. This expression vanishes because it is an integral of a total derivative. Now evaluating the derivatives, there are two non-vanishing contributions: one from the exponential and one from the action on the time-ordering symbol. Obtaining these derivatives and then taking the limit $q \to 0$ gives

$$\langle\{Q_\alpha, \psi_\Phi(0)\}\rangle = iq_\mu T\langle j^\mu_\alpha(x)\psi_\Phi(0)\rangle_{\rm FT}, \tag{10.18}$$

where FT indicates the Fourier transform. The left-hand side is constant, so the Green's function on the right-hand side must be singular as $q \to 0$. By the usual spectral representation analysis, this shows that there is a massless fermion coupled to the supersymmetry current. In weakly coupled theories we can understand this more simply. Recalling the form of the supersymmetry current, if one of the Fs has an expectation value then

$$j^\mu_\alpha = i\sqrt{2}(\sigma^\mu)_{\alpha\dot\alpha}\psi^{*\dot\alpha}F. \tag{10.19}$$

To leading order in the fields, current conservation amounts to just the massless Dirac equation; F, here, is the goldstino decay constant. We can understand the massless fermion which appeared in the O'Raifeartaigh model in terms of this theorem. It is easy to check that

$$\psi_G \propto F_A\psi_A + F_B\psi_B, \tag{10.20}$$

as in Eq. (10.8) for the case $F_B = 0$.

10.3 Loop corrections and the vacuum degeneracy

We saw that in the O'Raifeartaigh model, at the classical level there is a large vacuum degeneracy. To understand the model fully, we need to investigate the fate of this degeneracy in the quantum theory. Consider the vacuum with $X = 0$. In this case, A is undetermined at the classical level. But A is only an approximate modulus. At one loop, quantum corrections generate a potential for A. Our goal is to integrate out the various

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massive fields to obtain the effective action for A. At one loop, this is particularly easy. The tree level mass spectrum depends on A. The one-loop vacuum energy is

$$\sum_i (-1)^F \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{\vec{k}^2 + m_i^2}. \tag{10.21}$$

Here the sum is over all possible helicity states; again the factor $(-1)^F$ weights bosons with 1 and fermions with -1. In field theory this expression is usually very divergent in the ultraviolet, but in the supersymmetric case it is far less so. If supersymmetry is unbroken, the boson and fermion contributions cancel and the correction simply vanishes. If supersymmetry is broken, the divergence is only logarithmic. To see this we can simply study the integrand at large k, expanding the square root in powers of m^2/k^2. The leading, quartically divergent, term is independent of m^2 and so vanishes. The next term is quadratically divergent, but it vanishes because of the sum rule: $\sum (-1)^F m_i^2 = 0$.

So, at one loop the potential behaves as

$$V(A) = -\sum (-1)^F m_i^4 \int \frac{d^3k}{16(2\pi)^3 k^3} \approx \sum (-1)^F m_i^4 \frac{1}{64\pi^2} \ln \frac{m_i^2}{\Lambda^2}. \tag{10.22}$$

To compute the potential precisely, we need to work out the spectrum as a function of A. We will content ourselves with the limit of large A. Then the spectrum consists of a massive fermion ψ_X, with mass $2\lambda A$, and the real and imaginary parts of the scalar components of X, with masses

$$m_s^2 = 4|\lambda^2 A^2| \pm 2\mu^2\lambda^2 x^2. \tag{10.23}$$

So

$$V(A) = |\lambda^2|\mu^4 \left(1 + \frac{\lambda^2}{8\pi^2} \ln \frac{|\lambda A|^2}{\Lambda^2}\right). \tag{10.24}$$

This result has a simple interpretation. The leading term is the classical energy; the correction corresponds to replacing λ^2 by $\lambda^2(A)$, the running coupling at scale A. In this theory, a more careful study shows that the minimum of the potential is precisely at $A = 0$.

10.4 Explicit soft supersymmetry breaking

Ultimately, if nature is supersymmetric, it is likely that we will want to understand supersymmetry breaking through some dynamical mechanism. But we can be more pragmatic, accept that supersymmetry is broken and parameterize the breaking using the mass differences between the ordinary fields and their superpartners. It turns out that this procedure does not spoil the good ultraviolet properties of the theory. Such mass terms are said to be "soft" for precisely this reason.

We will consider soft breakings in more detail in the next chapter when we discuss the Minimal Supersymmetric Standard Model (MSSM), but we can illustrate the main point simply. Take as a model the Wess–Zumino model, with $m = 0$ in the superpotential. Add

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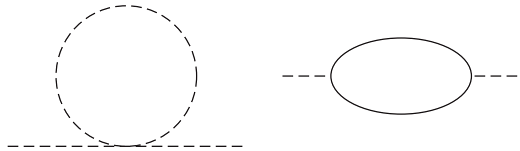

Fig. 10.1 One-loop corrections to scalar masses arising from Yukawa couplings.

to the Lagrangian an explicit mass term $m^2_{\text{soft}}|\phi|^2$. Then we can calculate the one-loop correction to the scalar mass from the two graphs of Fig. 10.1. In the supersymmetric case these two graphs cancel. With the soft breaking term, the cancelation is not exact; instead one obtains

$$\delta m^2 = -\frac{|\lambda|^2}{16\pi^2} m^2_{\text{soft}} \ln \frac{\Lambda^2}{m^2_{\text{soft}}}. \tag{10.25}$$

We can understand this simply on dimensional grounds. We know that for $m^2_{\text{soft}} = 0$ there is no correction. Treating the soft term as a perturbation, the result is necessarily proportional to m^2_{soft}; at most, then, any divergence must be logarithmic.

In addition to soft masses for scalars, one can add soft masses for gauginos; one can also include trilinear scalar couplings. We can understand how these might arise at a more fundamental level, which also makes clear the sense in which these terms are soft. Suppose that we have a field Z with non-zero F component, as in the O'Raifeartaigh model (but in a more general form). Suppose, further, that at tree level there are no renormalizable couplings between Z and the other fields of the model, which we will denote generically as ϕ. Non-renormalizable couplings, such as

$$\mathcal{L}_Z = \frac{1}{M^2} \int d^4\theta\, Z^\dagger Z \phi^\dagger \phi, \tag{10.26}$$

can be expected to arise as we integrate high-energy processes to obtain the effective Lagrangian; they are not forbidden by any symmetry. Replacing Z by its expectation value, $\langle Z \rangle = \cdots + \theta^2 \langle F_Z \rangle$, gives a mass term for the scalar component of ϕ:

$$\mathcal{L}_Z = \frac{|\langle F \rangle|^2}{M^2} |\phi|^2 + \cdots. \tag{10.27}$$

This is precisely the soft scalar mass we described above; it is soft because it is associated with a high-dimensional operator Similarly, the operator:

$$\int d^2\theta \frac{Z}{M} W_\alpha^2 = \frac{F_Z}{M} \lambda\lambda + \cdots \tag{10.28}$$

gives rise to a mass for gauginos. The term

$$\int d^2\theta \frac{Z}{M} \phi\phi\phi \tag{10.29}$$

leads to a trilinear coupling of the scalars. Simple power counting shows that loop corrections to these couplings due to renormalizable interactions are at most logarithmically divergent.

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To summarize, there are three types of soft-breaking term which can appear in a low-energy effective action:

- soft scalar masses, $m_\phi^2|\phi|^2$ and $\tilde{m}\phi^2\phi\phi + c.c.$;
- gaugino masses, $m_\lambda\lambda\lambda$;
- trilinear scalar couplings, $\Gamma\phi\phi\phi$.

All three types of coupling will play an important role when we think about possible supersymmetry phenomenologies.

10.5 Supersymmetry breaking in supergravity models

We stressed in the last chapter that, since nature includes gravity, if supersymmetry is not simply an accident it must be a local symmetry. If the underlying scale of supersymmetry breaking is high enough, supergravity effects will be important. The potential of a supergravity model will be sufficiently important to us that it is worth writing it down again:

$$V = e^K\left[\left(\frac{\partial W}{\partial\phi_i} + \frac{\partial K}{\partial\phi_i}W\right)g^{i\bar{j}}\left(\frac{\partial W}{\partial\phi_{\bar{j}}^*} + \frac{\partial K}{\partial\phi_{\bar{j}}^*}W^*\right) - 3|W|^2\right]. \tag{10.30}$$

In supergravity the condition for unbroken supersymmetry is that the *Kahler derivative* of the superpotential should vanish:

$$D_iW = \frac{\partial W}{\partial\phi_i} + \frac{\partial K}{\partial\phi_i}W = 0. \tag{10.31}$$

When this is not the case, supersymmetry is broken. If we require the vanishing of the cosmological constant then we have

$$3|W|^2 = \sum_{i,\bar{j}} D_iWD_{\bar{j}}W^*g^{i\bar{j}}. \tag{10.32}$$

In this case the gravitino mass turns out to be

$$m_{3/2} = \langle e^{K/2}W\rangle. \tag{10.33}$$

There is a standard strategy for building supergravity models. One introduces two sets of fields, the hidden-sector fields, denoted by Z_i, and the visible-sector fields, denoted by y_a. The Z_is are assumed to be connected with supersymmetry breaking and to have only very small couplings to the ordinary fields y_a. In other words, one assumes that the superpotential W has the form

$$W = W(Z) + W_y(y), \tag{10.34}$$

at least up to terms suppressed by $1/M$. The y fields should be thought of as the ordinary matter fields and their superpartners.

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One also usually assumes that the Kahler potential has a "minimal" form,

$$K = \sum Z_i^\dagger Z_i + \sum y_a^\dagger y_a. \tag{10.35}$$

One chooses (i.e. tunes) the parameters of W_Z in such a way that

$$\langle F_Z \rangle \approx M_W M \tag{10.36}$$

and

$$\langle V \rangle = 0. \tag{10.37}$$

Note that this means that

$$\langle W \rangle \approx M_W M^2. \tag{10.38}$$

The simplest model of the hidden sector is known as the *Polonyi model*. In this model

$$W = m^2(Z + \beta), \tag{10.39}$$

$$\beta = (2 + \sqrt{3})M. \tag{10.40}$$

In global supersymmetry, with only renormalizable terms, this would be a rather trivial superpotential, but this is not so in supergravity. The minimum of the potential for Z lies at

$$Z = (\sqrt{3} - 1)M, \tag{10.41}$$

and

$$m_{3/2} = (m^2/M)e^{(\sqrt{3}-1)^2/2}. \tag{10.42}$$

This symmetry breaking also leads to soft-breaking mass terms for the fields y, i.e. terms of the form

$$m_0^2 |y_i|^2. \tag{10.43}$$

These arise from the $|(\partial_i K)\ W|^2 = |y_i|^2 |W|^2$ terms in the potential. For the simple Kahler potential,

$$m_0^2 = 2\sqrt{3} m_{3/2}^2, \quad A = (3 - \sqrt{3}) m_{3/2}. \tag{10.44}$$

If we now allow for a non-trivial W_y, we also find supersymmetry-violating quadratic and cubic terms in the potential. These are known as the B and A terms and have the form

$$B_{ij} m_{3/2} \phi_i \phi_j + A_{ijk} m_{3/2} \phi_i \phi_j \phi_k. \tag{10.45}$$

For example, if W is homogeneous and of degree three, there are terms in the supergravity potential of the form

$$e^K \frac{\partial W}{\partial y_a} \frac{\partial K}{\partial y_a^*} \langle W \rangle + \text{c.c.} = 3 m_{3/2} W(y). \tag{10.46}$$

Additional contributions arise from

$$e^K \left\langle \frac{\partial W}{\partial z_i} \right\rangle \langle z_i^* \rangle W^* + \text{c.c.} \tag{10.47}$$

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There are analogous contributions to the B terms. In the exercises, these are worked out for specific models.

Gaugino masses m_λ (both in local and global supersymmetry) can arise from a non-trivial gauge coupling function

$$f^a = c\frac{Z}{M}, \tag{10.48}$$

which gives

$$m_\lambda = \frac{cF_z}{M}. \tag{10.49}$$

These models have just the correct structure to build a theory of TeV-scale supersymmetry, provided that $m_{3/2} \sim$ TeV. They have soft breakings of the correct order of magnitude. We will discuss their phenomenology further when we discuss the Minimal Supersymmetric Standard Model (MSSM) in the next chapter.

Even without a deep understanding of local supersymmetry, there are a number of interesting observations we can make. Most important, our arguments for the non-renormalization of the superpotential in global supersymmetry remain valid here. This will be particularly important when we come to string theory, which is a locally supersymmetric theory.

Suggested reading

It was Witten (1981) who most clearly laid out the issues of supersymmetry breaking. His paper remains extremely useful and readable today. The notion that one should consider adding soft-breaking parameters to the MSSM was developed by Dimopoulos and Georgi (1981). Good introductions to models with supersymmetry breaking in supergravity are provided by a number of review articles and textbooks, for example those of Mohapatra (2003) and Nilles (1984).

Exercises

(1) Work out out the spectrum of the O'Raifeartaigh model. Show that the spectrum is not supersymmetric, but verify the sum rule $\sum(-1)^F m^2 = 0$.
(2) Work out the spectrum of a model with a Fayet–Iliopoulos D term and supersymmetry breaking. Again verify the sum rule.
(3) Check Eqs. (10.40)–(10.44) for the Polonyi model.

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