existence results and show no interest in methods of calculating the functions whose existence is asserted. They do achieve thereby some very short and slick proofs, but I question whether this is a worthwhile gain. Despite this, I unhesitatingly recommend this work of great scholarship in a fundamental and still active branch of analysis.

N. J. YOUNG

WEEKS, J. R., *The shape of space: how to visualize surfaces and three-dimensional manifolds* (Pure and Applied Mathematics Vol. 96, Marcel Dekker, New York, 1985), x+324 pp., $59.50.

In this fascinating book the author's aim is to give the reader an intuitive understanding of three-dimensional manifolds with particular reference to the geometry and shape of the universe. It is also his hope that the book will be accessible to the "interested non-mathematician" though, recognising the dearth of nontechnical accounts of these topics, he suggests that mathematicians may also find it a useful general introduction. How accessible the book will be to the wider audience is perhaps a matter of definition but certainly all mathematicians will get from it a clear, nontechnical and intuitive exposition of current developments in the topology and geometry of 3-manifolds.

The author's starting point is E. A. Abbott's classic *Flatland: A Romance of Many Dimensions* first published in 1884. From there he proceeds to describe the construction of surfaces and 3-manifolds by glueing and takes care to distinguish between their topology and geometry. This occupies the first two-thirds of the book and this part ends with an account of the Gauss–Bonnet formula for surfaces with constant curvature. The final part starts by producing examples of 3-manifolds that admit elliptic, Euclidean or hyperbolic geometry and then contains a description of the eight homogeneous geometries that can arise on closed 3-manifolds. In the chapter the author explores possible geometries and global topologies for the universe.

The book is well supplied with clearly drawn diagrams and contains an instructive collection of exercises. At the end there is a useful set of solutions to the exercises as well as a bibliography. The latter refers the reader to a number of the standard texts for which the author has written an admirable complement.

R. M. F. MOSS

GROSSWALD, E., *Representations of integers as sums of squares* (Springer-Verlag, 1985) 251 pp., DM 148.

The study of the representations of a number as a sum of squares has a long history. Diophantus concerned himself with several problems of this type sixteen hundred years ago, but it was only towards the end of the seventeenth century that notable advances were made and valid proofs published. For a positive integer k denote by $r_k(n)$ the number of representations of the non-negative integer n as a sum of k squares of integers; thus $r_k(n)$ is the number of solutions of the Diophantine equation

$$x_1^2+x_2^2+\cdots+x_k^2=n \quad (x_i\in\mathbb{Z},\ 1\leq i\leq k). \tag{1}$$

The order of the x_i is taken into account so that, for example, $r_2(1)=4$, the solutions being $(x_1,x_2)=(\pm 1,0)$ and $(0,\pm 1)$. In his treatment the author follows the historical development of the subject, beginning with the case $k=2$.

Euler proved that, for $k=2$, (1) is soluble if and only if each prime divisor p of n, for which $p\equiv 3\,(\mathrm{mod}\,4)$, occurs in n to an even power. Later, the formula

$$r_2(n)=4\left\{\sum_{\substack{d\mid n\\ d\equiv 1\,(\mathrm{mod}\,4)}} 1 - \sum_{\substack{d\mid n\\ d\equiv 3\,(\mathrm{mod}\,4)}} 1\right\}$$

was established by Gauss using quadratic form theory, and by Jacobi by elliptic functions. By similar methods, using theta functions, Jacobi found formulae for $r_4(n)$, $r_6(n)$ and $r_8(n)$, in each case as sums over divisors of n of different types. The problem for odd k is distinctly harder and Gauss's evaluation of $r_3(n)$ is a tour de force, using his deep theory of classes and genera of quadratic forms.

Theta functions arise naturally since, as is easy to see,

$$\sum_{n=0}^{\infty} r_k(n)z^n = \theta_3^k(z),$$

where

$$\theta_3(z) = \sum_{n=-\infty}^{\infty} z^{n^2} = 1 + 2\sum_{n=1}^{\infty} z^{n^2},$$

the series being absolutely convergent for $|z| < 1$.

A major step forward occurred with the application to the problem of the Hardy–Littlewood circle method. The idea is to write

$$r_k(n) = \frac{1}{2\pi i}\int z^{-n-1}\theta_3^k(z)\,dz,$$

the integral being taken round a circle centered at the origin and of radius $e^{-2\pi\delta}$, where δ is positive and chosen, for best results, to depend on n. The circumference is divided into arcs on each of which an appropriate transformation of the theta function is used to approximate to the integrand. This results in an expression

$$r_k(n) = \rho_k(n) + c_k(n) \quad (k \geq 5),$$

where $\rho_k(n)$ is the so-called singular series, is nonzero and has order $n^{(k/2)-1}$, while the error term $c_k(n)$ has lesser order $O(n^{k/4})$. In fact, $c_k(n) = 0$ for $5 \leq k \leq 8$.

It is really only when modular form theory is applied that the rather haphazard results for different values of k fall into place. For any value of $k > 4$ (integral or not), θ_3^k is a modular form of weight $\frac{1}{2}k$ belonging to a subgroup Γ of index 3 in the modular group, and is holomorphic in the upper τ-half-plane, where $z = e^{\pi i\tau}$. A fundamental region for Γ has two incongruent cusps, θ_3^k taking the value 1 at one of these cusps (the point $\tau = \infty$, for example) and zero at the other. If E_k is the Eisenstein series associated with ∞, it follows that

$$\theta_3^k = E_k + C_k,$$

where C_k is a cusp form. The nth Fourier coefficient of E_k can be evaluated and is just $\rho_k(n)$, while $c_k(n)$, being the nth coefficient of C_k is, for $k \in \mathbb{Z}$, of the order of $n^{(k/4)-\frac{1}{2}+\varepsilon}$ for any $\varepsilon > 0$ (after Deligne), and vanishes for $k \leq 8$, since for these real values of k the cusp form space reduces to the zero space. This interpretation of the problem stems from the work of Hecke and Petersson on modular forms, although it should be mentioned that Mordell's treatment of the problem in 1919 used similar ideas, despite the fact that there was not then available any general theory of Eisenstein series and cusp forms.

In his book Professor Grosswald leads us through the various methods that have been devised in a lucid and readable manner and includes several other related topics, such as representation of n as a sum of a fixed number of nonvanishing squares, and what he calls essential distinct partitions, where the order and sign of the x_i are not taken into account. These are areas in which he himself has done important work.

Each chapter concludes with a list of problems ranging from the routine to the difficult and unsolved. There is a long concluding chapter summarizing recent work drawn mainly from the area of algebraic number theory and general quadratic form theory. This includes discussions of

Hilbert's 11th and 17th problems and the Hasse principle. From the numerous theorems quoted I select just one to give the flavour, namely Siegel's 1945 result that the only totally real fields in which all totally positive integers can be represented by sums of squares of integers in the field are the rational numbers $\mathbb{Q}$ and the field $\mathbb{Q}(\sqrt{5})$.

As one expects from the publisher, the work is well set out, although there are a number of places where the arguments would be easier to follow if formulae had been displayed and not allowed to run beyond the end of a line of text. As is inevitable in such detailed work, there are a few misprints, but these are easily corrected, including the somewhat unfortunate error on the first page of chapter 1, where, presumably, equation (1.2) should read

$$2x^2-5y^2+3z^2=0.$$

There are two extensive bibliographies, the first giving references to the results discussed, and the second being a selection of more recent papers relevant to the subject and its generalizations.

To sum up: this is an excellent book, which will be found of interest to all workers in the theory of numbers. It is not written for specialists and so can be read with enjoyment by other mathematicians.

R. A. RANKIN

BOLLOBÁS, B., *Random graphs* (London Mathematical Society Monographs, Academic Press, London, 1985), 447 pp., £52 cloth, £27 paper.

This scholarly and encyclopedic work is the first extensive account of the theory of random graphs. The origins of this rich subject, which applies probability ideas to graphs, can be traced back to a paper of Erdös and Rényi in 1959; this book takes us from these beginnings right up to the most recent results in the subject.

In order to explain briefly what the subject is about, let us concentrate on the concept of a hamiltonian cycle. It is well known that it is extremely difficult to determine whether or not a given graph has a hamiltonian cycle, so let us ask: what proportion of graphs have such a cycle? This rather vague question can be made more definite by the use of one of two models of randomness. We can consider $G(n,M)$, the space of all graphs on n vertices with M edges, each graph being assigned the same probability, or we can consider the space $G(n,p)$ of all graphs on n vertices, with each possible edge independently having probability p of being present. In either space we can then ask how likely it is that a graph is hamiltonian. The probability will, of course, increase as M (or p) increases, i.e. as the random graph grows or evolves. The great discovery of Erdös and Rényi was that many important properties of graphs appear quite suddenly. For example, it has been shown that almost every graph on n vertices with at least $cn\log n$ edges ($c>1$) has a hamiltonian cycle, so that the hamiltonian property emerges suddenly since almost every graph with $cn\log n$ edges ($c<\frac{1}{2}$) is not even connected! Surprisingly, the main obstruction to a hamiltonian cycle turns out to be the existence of vertices of degree at most 1.

Many such properties of graphs are dealt with in this masterly account (for example, connectivity and matchings, giant components, degree sequences, cliques and diameter), and the final chapter discusses sorting algorithms. There are exercises at the end of each chapter, and over 750 references to the literature, many of them very recent or still to appear. This book will surely establish itself as *the* reference for the subject. It is not an easy book to read, demanding a high level of concentration and sophistication, and someone new to the subject might be advised to warm up by first looking at the chapter on random graphs in the same author's introductory textbook *Graph theory, an introductory course* (Springer-Verlag, 1979). The reviewer notes the announcement of another related book *Graphical evolution, and introduction to the theory of random graphs* by E. M. Palmer (Wiley, 1985); he has not seen it, but notes that, with only 177 pages, it cannot be as exhaustive as Bollobás's account.

IAN ANDERSON